Solutions Manual

to accompany

Communication Systems

An Introduction to Signals and Noise in Electrical Communication

Fourth Edition

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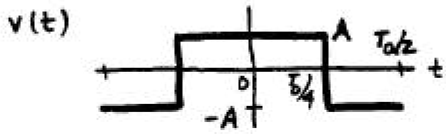
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Chapter 2

2.1-1

$$c_n = \frac{Ae^{jf}}{T_0}\int_{-T_0/2}^{T_0/2} e^{j2p(m-n)f_0 t}\,dt = Ae^{jf}\,\text{sinc}(m-n) = \begin{cases} Ae^{jf} & n=m \\ 0 & \text{otherwise} \end{cases}$$

2.1-2

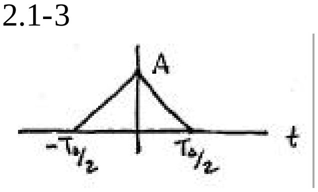


$c_0 \left\langle v(t) \right\rangle = 0$

$$c_n = \frac{2}{T_0}\int_0^{T_0/4} A\cos\frac{2p\,nt}{T_0}dt + \int_{T_0/4}^{T_0/2}(-A)\cos\frac{2p\,nt}{T_0}\,dt = \frac{2A}{p\,n}\sin\frac{p\,n}{2}$$

n	0	1	2	3	4	5	6	7
$\lvert c_n \rvert$	0	$2A/p$	0	$2A/3p$	0	$2A/5p$	0	$2A/7p$
$\arg c_n$		0		$\pm 180°$		0		$\pm 180°$

2.1-3

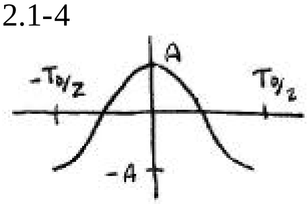


$c_0 = \left\langle v(t) \right\rangle = A/2$

$$c_n = \frac{2}{T_0}\int_0^{T_0/2}\left(A - \frac{2At}{T_0}\right)\cos\frac{2p\,nt}{T_0}dt = \frac{A}{p\,n}\sin p\,n - \frac{A}{(p\,n)^2}(\cos p\,n - 1)$$

n	0	1	2	3	4	5	6
$\lvert c_n \rvert$	$0.5A$	$0.2A$	0	$0.02A$	0	$0.01A$	0
$\arg c_n$	0	0		0		0	

2.1-4



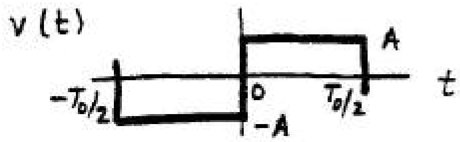
$$c_0 = \frac{2}{T_0}\int_0^{T_0/2} A\cos\frac{2p\,t}{T_0} = 0$$

(cont.)

$$c_n = \frac{2}{T_0}\int_0^{T_0/2} A\cos\frac{2pt}{T_0}\cos\frac{2p nt}{T_0}\,dt = \frac{2A}{T_0}\left[\frac{\sin(p - p n)2t/T_0}{4(p - p n)/T_0} + \frac{\sin(p + p n)2t/T_0}{4(p + p n)/T_0}\right]_0^{T_0/2}$$

$$= \frac{A}{2}\left[\mathrm{sinc}(1-n) + \mathrm{sinc}(1+n)\right] = \begin{cases} A/2 & n = \pm 1 \\ 0 & \text{otherwise} \end{cases}$$

2.1-5

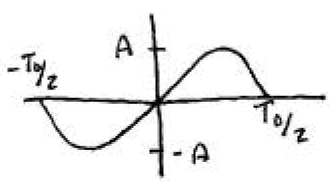


$$c_0 = \langle v(t) \rangle = 0$$

$$c_n = -j\frac{2}{T_0}\int_0^{T_0/2} A\sin\frac{2p nt}{T_0}dt = -j\frac{A}{p n}(1 - \cos p n)$$

n	1	2	3	4	5
$\lvert c_n \rvert$	$2A/p$	0	$2A/3p$		$2A/5p$
$\arg c_n$	$-90°$		$-90°$		$-90°$

2.1-6



$$c_0 = \langle v(t) \rangle = 0$$

$$c_n = -j\frac{2}{T_0}\int_0^{T_0/2} A\sin\frac{2pt}{T_0}\sin\frac{2p nt}{T_0}\,dt = -j\frac{2A}{T_0}\left[\frac{\sin(p - p n)2t/T_0}{4(p - p n)/T_0} - \frac{\sin(p + p n)2t/T_0}{4(p + p n)/T_0}\right]_0^{T_0/2}$$

$$= -j\frac{A}{2}\left[\mathrm{sinc}(1-n) - \mathrm{sinc}(1+n)\right] = \begin{cases} \mp jA/2 & n = \pm 1 \\ 0 & \text{otherwise} \end{cases}$$

2.1-7

$$c_n = \frac{1}{T_0}\left[\int_0^{T_0/2} v(t)e^{-jnw_0 t}dt + \int_{T_0/2}^{T_0} v(t)e^{-jnw_0 t}\,dt\right]$$

where $\int_{T_0/2}^{T_0} v(t)e^{-jnw_0 t}dt = \int_0^{T_0/2} v(l + T_0/2)\,e^{-jnw_0 l}\,e^{-jnw_0 T_0/2}d\,l$

$$= -e^{jnp}\int_0^{T_0/2} v(t)e^{-jnw_0 t}dt$$

since $e^{jnp} = 1$ for even n, $c_n = 0$ for even n

2.1-8

$$P = |c_0|^2 + 2\sum_{n=1}^{\infty}|c_n|^2 = |Af_0t|^2 + 2|Af_0t\ \mathrm{sinc}\ f_0t|^2 + 2|Af_0t\ \mathrm{sinc}2f_0t|^2 + 2|Af_0t\ \mathrm{sinc}3f_0t|^2 + \cdots$$

where $\frac{1}{t} = 4f_0$

$$|f| > \frac{1}{t} \quad P = \frac{A^2}{16}\left[1 + 2\mathrm{sinc}^2\frac{1}{4} + 2\mathrm{sinc}^2\frac{1}{2} + 2\mathrm{sinc}^2\frac{3}{4}\right] = 0.23A^2$$

$$|f| > \frac{2}{t} \quad P = \frac{A^2}{16}\left[1 + 2\mathrm{sinc}^2\frac{1}{4} + 2\mathrm{sinc}^2\frac{1}{2} + 2\mathrm{sinc}^2\frac{3}{4} + 2\mathrm{sinc}^2\frac{5}{4} + 2\mathrm{sinc}^2\frac{3}{2} + 2\mathrm{sinc}^2\frac{7}{4}\right] = 0.24A^2$$

$$|f| > \frac{1}{2t} \quad P = \frac{A^2}{16}\left[1 + 2\mathrm{sinc}^2\frac{1}{4} + 2\mathrm{sinc}^2\frac{1}{2}\right] = 0.21A^2$$

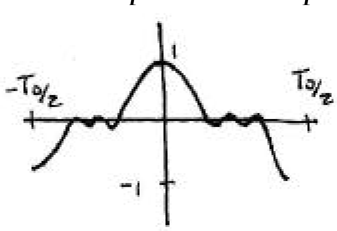
2.1-9

$$c_n = \begin{cases} 0 & n \text{ even} \\ \left(\frac{2}{p\mathrm{n}}\right)^2 & n \text{ odd} \end{cases}$$

a) $$P = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2}\left(1 - \frac{4|t|}{T_0}\right)^2 dt = \frac{2}{T_0}\int_0^{T_0/2}\left(1 - \frac{4t}{T_0}\right)^2 dt = \frac{1}{3}$$

$$P' = 2\left(\frac{4}{p^2}\right)^2 + 2\left(\frac{4}{9p^2}\right)^2 + 2\left(\frac{4}{25p^2}\right)^2 = 0.332 \quad \text{so} \quad P'/P = 99.6\%$$

b) $$v'(t) = \frac{8}{p^2}\cos w_0t + \frac{8}{9p^2}\cos 3w_0t + \frac{8}{25p^2}\cos 5w_0t$$



2.1-10

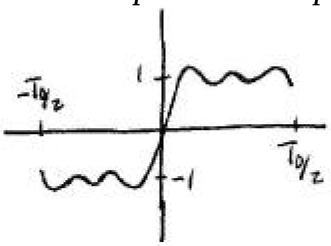
$$c_n = \begin{cases} 0 & n \text{ even} \\ \frac{-j2}{pn} & n \text{ odd} \end{cases}$$

a) $$P = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2}(1)^2\, dt = 1 \quad P' = 2\left[\left(\frac{2}{p}\right)^2 + \left(\frac{2}{3p}\right)^2 + \left(\frac{2}{5p}\right)^2\right] = 0.933 \ \text{ so } \ P'/P = 93.3\%$$

(cont.)

b) $v'(t) = \frac{4}{p}\cos\left(w_0 t - 90°\right) + \frac{4}{3p}\cos\left(3w_0 t - 90°\right) + \frac{4}{5p}\cos\left(5w_0 t - 90°\right)$

$\quad = \frac{4}{p}\sin\left(w_0 t\right) + \frac{4}{3p}\sin\left(3w_0 t\right) + \frac{4}{5p}\sin\left(5w_0 t\right)$



2.1-11

$$P = \frac{1}{T_0}\int_0^{T_0}\left(\frac{t}{T_0}\right)^2 dt = \frac{1}{3} \qquad \left|c_n\right| = \begin{cases} 1/2 & n = 0 \\ 1/2p\,n & n \neq 0 \end{cases}$$

$$P = 2\sum_{n\ \text{odd}}^{\infty}\left(\frac{2}{p\,n}\right)^4 = 2\left(\frac{2}{p}\right)^4\left(\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \cdots\right) = \frac{1}{3}$$

Thus, $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots = \frac{4p^2}{2}\left(\frac{1}{3} - \frac{1}{4}\right) = \frac{p^2}{6}$

2.1-12

$$P = \frac{2}{T_0}\int_0^{T_0/2}\left(1 - \frac{4t}{T_0}\right)^2 dt = \frac{1}{3} \qquad \left|c_n\right| = \begin{cases} 0 & n\ \text{even} \\ (2/p\,n)^2 & n\ \text{odd} \end{cases}$$

$$P = \left(\frac{1}{2}\right)^2 + 2\sum_{n=1}^{\infty}\left(\frac{1}{2p\,n}\right)^2 = \frac{1}{4} + \frac{2}{4p^2}\left(\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots\right) = \frac{1}{3}$$

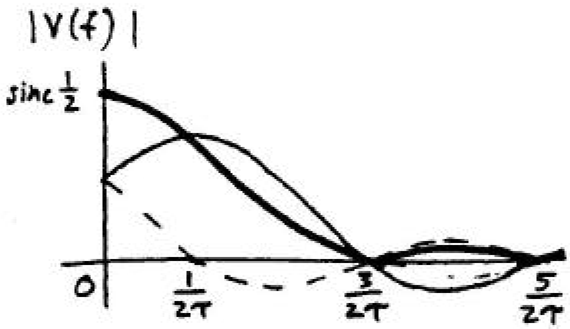
Thus, $\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \cdots = \frac{p^4}{2 \cdot 2^4}\frac{1}{3} = \frac{p^4}{96}$

2.2-1

$$V(f) = 2\int_0^{t/2} A\cos\frac{pt}{t}\cos 2p\,ft\,dt$$

$$= 2A\left[\frac{\sin\left(\frac{p}{t} - 2p\,f\right)\frac{t}{2}}{2\left(\frac{p}{t} - 2p\,f\right)} + \frac{\sin\left(\frac{p}{t} + 2p\,f\right)\frac{t}{2}}{2\left(\frac{p}{t} + 2p\,f\right)}\right] = \frac{At}{2}\left[\text{sinc}(f t - 1/2) + \text{sinc}(f t + 1/2)\right]$$

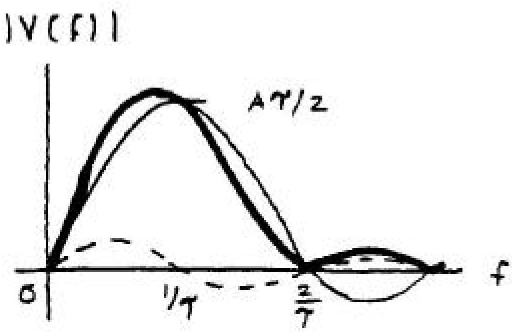
(cont.)



2.2-2

$$V(f) = -j2\int_0^{\tau/2} A\sin\frac{2\pi t}{\tau}\cos 2\pi ft\,dt$$

$$= -j2A\left[\frac{\sin\left(\frac{2\pi}{\tau}-2\pi f\right)\frac{\tau}{2}}{2\left(\frac{2\pi}{\tau}-2\pi f\right)} - \frac{\sin\left(\frac{2\pi}{\tau}+2\pi f\right)\frac{\tau}{2}}{2\left(\frac{2\pi}{\tau}+2\pi f\right)}\right] = -j\frac{A\tau}{2}\left[\text{sinc}(f\tau-1)-\text{sinc}(f\tau+1)\right]$$



2.2-3

$$V(f) = 2\int_0^{\tau}\left(A - A\frac{t}{\tau}\right)\cos\omega t\,dt = \frac{2A\tau}{(\omega\tau)^2}\left[2\sin^2\left(\frac{\omega\tau}{2}\right)-1+1\right] = A\tau\ \text{sinc}^2\ f\tau$$

2.2-4

$$V(f) = -j2\int_0^{\tau} A\frac{t}{\tau}\sin\omega t\,dt = -j\frac{2A\tau}{(\omega\tau)^2}(\sin\omega\tau - \omega\tau\cos\omega\tau)$$

$$= -j\frac{A}{\pi f}(\text{sinc}2f\tau - \cos 2\pi f\tau)$$

2.2-5

$$v(t) = \text{sinc}2Wt \leftrightarrow \frac{1}{2W}\Pi\left(\frac{f}{2W}\right)$$

$$\int_{-\infty}^{\infty}\left|\text{sinc}2Wt\right|^2 dt = \int_{-\infty}^{\infty}\left|\frac{1}{2W}\Pi\left(\frac{f}{2W}\right)\right|^2 df = \int_{-\infty}^{\infty}\frac{1}{4W^2}df = \frac{1}{2W}$$

2.2-6

$$E = \int_0^\infty \left(Ae^{-bt}\right)^2 dt = \frac{A^2}{2b} \qquad E' = 2\int_0^W \frac{A^2}{b^2 + (2p\,f)^2}\,df = \frac{A^2}{p\,b}\arctan\frac{2pW}{b}$$

$$\frac{E'}{E} = \frac{2}{p}\arctan\frac{2pW}{b} = \begin{cases} 50\% & W = b/2p \\ 84\% & W = 2b/p \end{cases}$$

2.2-7

$$\int_{-\infty}^{\infty} v(t)w(t)dt = \int_{-\infty}^{\infty} v(t)\left[\int_{-\infty}^{\infty} W(f)e^{jwt}df\right]dt$$

$$= \int_{-\infty}^{\infty} W(f)\left[\int_{-\infty}^{\infty} v(t)e^{-j(-w)t}dt\right]df = \int_{-\infty}^{\infty} W(f)V(-f)df$$

$V(-f) = V^*(f)$ when $v(t)$ is real, so $\int_{-\infty}^{\infty} v^2(t)dt = \int_{-\infty}^{\infty} V(f)V^*(f)df = \int_{-\infty}^{\infty} \left|V(f)\right|^2 df$

2.2-8

$$\int_{-\infty}^{\infty} w^*(t)e^{-j2p\,ft}dt = \left[\int_{-\infty}^{\infty} w(t)e^{j2p\,ft}dt\right]^* = \left[\int_{-\infty}^{\infty} w(t)e^{-j2p(-f)t}dt\right]^* = W^*(f)$$

Let $z(t) = w^*(t)$ so $Z(f) = W^*(-f)$ and $W^*(f) = Z(-f)$

Hence $\int_{-\infty}^{\infty} v(t)z(t)dt = \int_{-\infty}^{\infty} V(f)Z(-f)df$

2.2-9

$\Pi\left(\frac{t}{A}\right) \leftrightarrow A\,\mathrm{sinc}\,Af$ so $\mathrm{sinc}\,At \leftrightarrow \frac{1}{A}\Pi\left(\frac{f}{A}\right)$

$v(t) = \mathrm{sinc}\,\frac{2t}{t} \leftrightarrow V(f) = \frac{t}{2}\Pi\left(\frac{ft}{2}\right)$ for $A = \frac{2}{t}$

2.2-10

$$B\cos\frac{p\,t}{t}\Pi\left(\frac{t}{t}\right) \leftrightarrow \frac{Bt}{2}\left[\mathrm{sinc}(f\,t - 1/2) + \mathrm{sinc}(f\,t + 1/2)\right]$$

so $\frac{Bt}{2}\left[\mathrm{sinc}(tt - 1/2) + \mathrm{sinc}(tt + 1/2)\right] \leftrightarrow B\cos\frac{p(-f)}{t}\Pi\left(\frac{-f}{t}\right) = B\cos\frac{p\,f}{t}\Pi\left(\frac{f}{t}\right)$

Let $B = A$ and $t = 2W \quad \Rightarrow \quad z(t) = AW\left[\mathrm{sinc}(2Wt - 1/2) + \mathrm{sinc}(2Wt + 1/2)\right]$

2.2-11

$$B\sin\frac{2p\,t}{t}\Pi\left(\frac{t}{t}\right) \leftrightarrow -j\frac{Bt}{2}\left[\mathrm{sinc}(f\,t - 1) + \mathrm{sinc}(f\,t + 1)\right]$$

so $-j\frac{Bt}{2}\left[\mathrm{sinc}(tt - 1) + \mathrm{sinc}(tt + 1)\right] \leftrightarrow B\sin\frac{2p(-f)}{t}\Pi\left(\frac{-f}{t}\right) = -B\sin\frac{2p\,f}{t}\Pi\left(\frac{f}{t}\right)$

Let $B = -jA$ and $t = 2W \quad \Rightarrow \quad z(t) = AW\left[\mathrm{sinc}(2Wt - 1) + \mathrm{sinc}(2Wt + 1)\right]$

2.2-12

$$e^{-b|t|} \leftrightarrow \frac{2b}{b^2+(2p\,f)^2} \Rightarrow e^{-2pa|t|} \leftrightarrow \frac{4pa}{(2pa)^2+(2p\,f)^2} = \frac{a/p}{a^2+f^2}$$

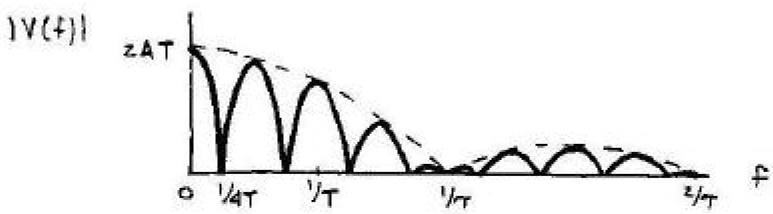
$$\int_{-\infty}^{\infty}\left(e^{-2pa|t|}\right)^2 dt = \frac{1}{2pa} = \int_{-\infty}^{\infty}\left|\frac{a/p}{a^2+f^2}\right| df = 2\left(\frac{a}{p}\right)^2 \int_0^{\infty}\frac{df}{\left(a^2+f^2\right)^2}$$

Thus, $\displaystyle \int_0^{\infty}\frac{dx}{\left(a^2+x^2\right)^2} = \frac{1}{2}\left(\frac{p}{a}\right)^2 \frac{1}{2pa} = \frac{p}{4a^3}$

2.3-1

$z(t) = v(t-T) + v(t+T)$ where $v(t) = A\Pi(t/t) \leftrightarrow At \text{ sinc } ft$

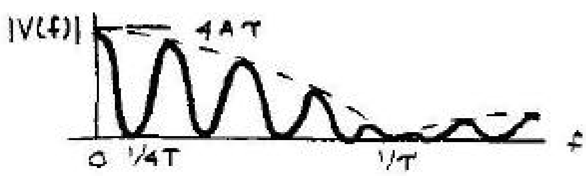
so $Z(f) = V(f)e^{-jwT} + V(f)e^{jwT} = 2At \text{ sinc } f t \cos 2p\, fT$



2.3-2

$z(t) = v(t-2T) + 2v(t) + v(t+2T)$ where $v(t) = a\Pi(t/t) \leftrightarrow At \text{ sinc } f t$

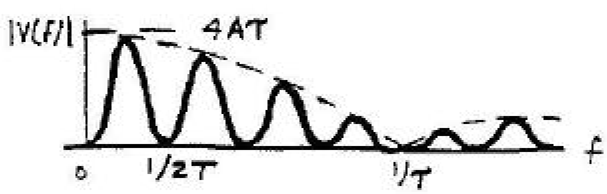
$Z(f) = V(f)e^{-j2wT} + V(f) + V(f)e^{j2wT} = 2At(\text{sinc } f t)(1+\cos 4p\, fT)$



2.3-3

$z(t) = v(t-2T) - 2v(t) + v(t+2T)$ where $v(t) = a\Pi(t/t) \leftrightarrow At \text{ sinc } f t$

$Z(f) = V(f)e^{-j2wT} - 2V(f) + V(f)e^{j2wT} = 2At(\text{sinc } f t)(\cos 4p\, fT - 1)$



2.3-4

$$v(t) = A\Pi\left(\frac{t-T}{2T}\right) + (B-A)\Pi\left(\frac{t-T/2}{T}\right)$$

$$V(f) = 2AT \text{ sinc} 2\, fTe^{-jwT} + (B-A)T \text{ sinc } fTe^{-jwT/2}$$

2.3-5

$$v(t) = A\Pi\left(\frac{t-2T}{4T}\right) + (B-A)\Pi\left(\frac{t-2T}{2T}\right)$$

$$V(f) = 4AT\,\mathrm{sinc}\,4fTe^{-j2wT} + 2(B-A)T\,\mathrm{sinc}\,2fTe^{-j2wT}$$

2.3-6

$$\text{Let } w(t) = v(at) \leftrightarrow W(f) = \frac{1}{|a|}V(f/a)$$

$$\text{Then } z(t) = v[a(t - t_d/a)] = w(t - t_d/a) \text{ so } Z(f) = W(f)e^{-jwt_d/a} = \frac{1}{|a|}V(f/a)e^{-jwt_d/a}$$

2.3-7

$$\mathbf{F}\left[v(t)e^{jw_c t}\right] = \int_{-\infty}^{\infty} v(t)e^{jw_c t}e^{-jwt}dt = \int_{-\infty}^{\infty} v(t)e^{-j2p(f-f_c)t}dt = V(f - f_c)$$

2.3-8

$$v(t) = A\Pi(t/t)\cos w_c t \quad \text{with} \quad w_c = 2p f_c = p/t$$

$$V(f) = \frac{At}{2}\mathrm{sinc}(f - f_c)t + \frac{At}{2}\mathrm{sinc}(f + f_c)t = \frac{At}{2}\left[\mathrm{sinc}(ft - 1/2) + \mathrm{sinc}(ft + 1/2)\right]$$

2.3-9

$$v(t) = A\Pi(t/t)\cos(w_c t - p/2) \quad \text{with} \quad w_c = 2p f_c = 2p/t$$

$$V(f) = \frac{e^{-jp/2}}{2}At\,\mathrm{sinc}(f - f_c)t + \frac{e^{jp/2}}{2}At\,\mathrm{sinc}(f + f_c)t$$

$$= -j\frac{At}{2}\left[\mathrm{sinc}(ft - 1) - \mathrm{sinc}(ft + 1)\right]$$

2.3-10

$$z(t) = v(t)\cos w_c t \qquad v(t) = Ae^{-|t|} \leftrightarrow \frac{2A}{1 + (2p f)^2}$$

$$Z(f) = \frac{1}{2}V(f - f_c) + \frac{1}{2}V(f + f_c) = \frac{A}{1 + 4p^2(f - f_c)^2} + \frac{A}{1 + 4p^2(f + f_c)^2}$$

2.3-11

$$z(t) = v(t)\cos(w_c t - p/2) \qquad v(t) = Ae^{-t} \quad \text{for } t \ge 0 \quad \leftrightarrow \frac{A}{1 + j2p f}$$

$$Z(f) = \frac{e^{-jp/2}}{2}V(f - f_c) + \frac{e^{jp/2}}{2}V(f + f_c) = \frac{-jA/2}{1 + j2p(f - f_c)} + \frac{jA/2}{1 + j2p(f + f_c)}$$

$$= \frac{A/2}{j - 2p(f - f_c)} - \frac{A/2}{j - 2p(f + f_c)}$$

2.3-12

$$v(t) = t\, z(t) \quad z(t) = \frac{A}{\tau}\Pi\left(\frac{t}{\tau}\right) \leftrightarrow 2A\,\text{sinc}\,2f\tau$$

$$\frac{d}{df}Z(f) = 2A\frac{d}{df}\left[\frac{\sin 2\pi f\tau}{2\pi f\tau}\right] = \frac{2A}{(2\pi f\tau)^2}\left[(2\pi\tau)^2 f\cos 2\pi f\tau - 2\pi\tau\sin 2\pi f\tau\right]$$

$$V(f) = \frac{1}{-j2\pi}\frac{d}{df}Z(f) = \frac{-jA}{\pi f}\left(\text{sinc}\,2f\tau - \cos 2\pi f\tau\right)$$

2.3-13

$$z(t) = tv(t) \quad v(t) = Ae^{-b|t|} \leftrightarrow \frac{2Ab}{b^2 + (2\pi f)^2}$$

$$Z(f) = \frac{1}{-j2\pi}\frac{d}{df}\left[\frac{2Ab}{b^2 + (2\pi f)^2}\right] = \frac{j2Abf}{\left[b^2 + (2\pi f)^2\right]^2}$$

2.3-14

$$z(t) = t^2 v(t) \quad v(t) = Ae^{-t} \ \text{ for } t \ge 0 \quad \leftrightarrow \frac{A}{b + j2\pi f}$$

$$Z(f) = \frac{1}{\left(-j2\pi f\right)^2}\frac{d}{df}\left[\frac{A}{b + j2\pi f}\right] = \frac{2A}{\left[b + j2\pi f\right]^3}$$

2.3-15

$$v(t) = e^{-\pi(bt)^2} \leftrightarrow V(f) = \frac{1}{b}e^{-\pi(f/b)^2}$$

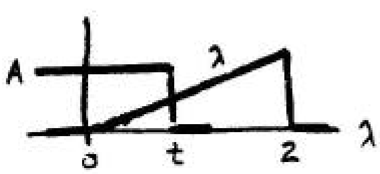
$$(a)\ \frac{d}{dt}v(t) = -2\pi b^2 te^{-\pi(bt)^2} \leftrightarrow \frac{j2\pi f}{b}e^{-\pi(f/b)^2}$$

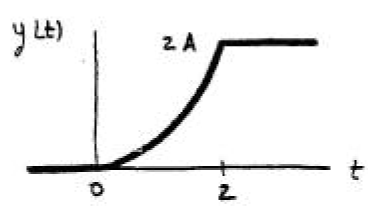
$$(b)\ te^{-\pi(bt)^2} \leftrightarrow \frac{1}{-j2\pi}\frac{d}{df}V(f) = \frac{f}{jb}e^{-\pi(f/b)^2}$$

Both results are equivalent to $bte^{-\pi(bt)^2} \leftrightarrow -jf\, e^{-\pi(f/b)^2}$

2.4-1

$$\begin{aligned} y(t) &= 0 & t &< 0 \\ &= \int_0^t A\lambda\, d\lambda = \frac{At^2}{2} & 0 < t &< 2 \\ &= \int_0^2 A\lambda\, d\lambda = 2A & t &> 2 \end{aligned}$$





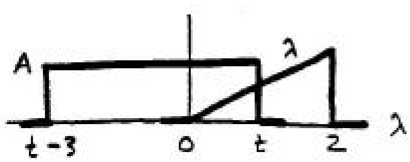
2.4-2

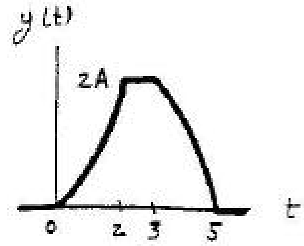
$$y(t) = 0 \qquad t < 0, t > 5$$

$$= \int_0^t Al\, d\,l = \frac{At^2}{2} \qquad 0 < t < 2$$

$$= \int_0^2 Al\, d\,l = 2A \qquad 2 < t < 3$$

$$= \int_{t-3}^2 Al\, d\,l = \frac{A}{2}\left[4 - (t-3)^2\right] \qquad 3 < t < 5$$





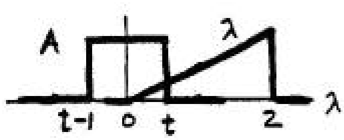
2.4-3

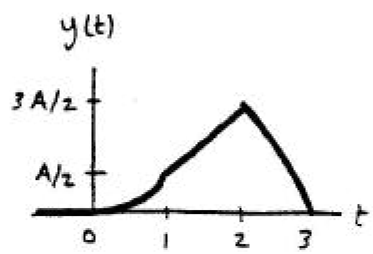
$$y(t) = 0 \qquad t < 0,\ t > 3$$

$$= \int_0^t Al\, d\,l = \frac{At^2}{2} \qquad 0 < t < 1$$

$$= \int_{t-1}^t Al\, d\,l = \frac{A}{2}(2t-1) \qquad 1 < t < 2$$

$$= \int_{t-1}^2 Al\, d\,l = \frac{A}{2}\left[4 - (t-1)^2\right] \qquad 2 < t < 3$$



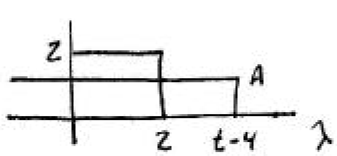


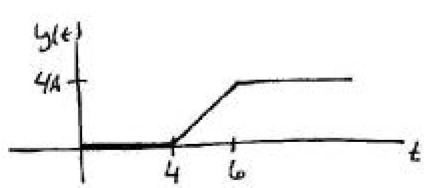
2.4-4

$$y(t) = 0 \qquad t < 4$$

$$= \int_4^t 2Ad\,l = 2At - 8A \qquad 4 \le t \le 6$$

$$= \int_6^8 2Ad\,l = 4A \qquad t > 6$$



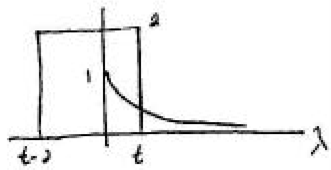


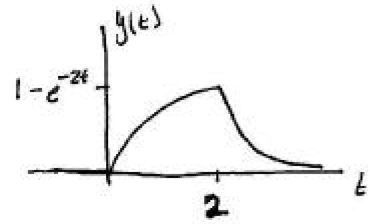
2.4-5

$$y(t) = 0 \qquad\qquad t < 0$$

$$= \int_0^t 2e^{-2l}\,d\,l = 1 - e^{-2t} \qquad 0 \le t \le 2$$

$$= \int_{t-2}^t 2e^{-2l}\,d\,l = e^{-2t}\left[e^4 - 1\right] \qquad t > 2$$





2.4-6

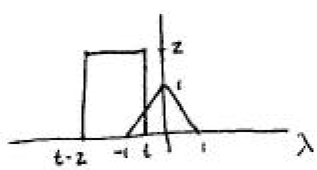
$$y(t) = 0 \qquad\qquad t < -1,\ t \ge 3$$

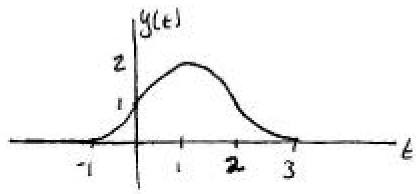
$$= \int_{-1}^t 2(1 + l)\,d\,l = t^2 + 2t + 1 \qquad -1 \le t < 0$$

$$= \int_{-1}^0 2(1 + l)\,d\,l + \int_0^t 2(1 - l)\,d\,l = -t^2 + 2t + 1 \qquad 0 \le t < 1$$

$$= \int_{t-2}^0 2(1 + l)\,d\,l + \int_0^1 2(1 - l)\,d\,l = -t^2 + 2t + 1 \qquad 1 \le t < 2$$

$$= \int_{t-2}^1 2(1 - l)\,d\,l = t^2 - 6t + 9 \qquad 2 \le t < 3$$





2.4-7

$$y(t) = 0 \qquad\qquad t \le 0$$

$$= \int_0^t Ae^{-a\,l}\,Be^{-b(t-l)}\,d\,l = [AB/(a - b)][e^{-bt} - e^{-at}] \qquad t > 0$$

2.4-8

$$v(t) = Ae^{-at} \qquad w(t) = \sin pt = \frac{j}{2}e^{jpt} - \frac{j}{2}e^{-jpt} = B_1 e^{-b_1 t} + B_2 e^{-b_2 t}$$

$$y(t) = v * w_1(t) + v * w_2(t) = [AB_1/(a - b_1)][e^{-b_1 t} - e^{-at}] + [AB_2/(a - b_2)][e^{-b_2 t} - e^{-at}]$$

Let $B_1 = j/2,\ b_1 = -jp,\ B_2 = -j/2,\ b_2 = jp$ and simplify

2.4-9

$$v * w(t) = \int_{-\infty}^{\infty} v(l)w(t - l)\,d\,l \qquad \text{let } m = t - l$$

$$= -\int_{\infty}^{-\infty} v(t - m)w(m)\,d\,m = \int_{-\infty}^{\infty} w(m)v(t - m)\,d\,m = w * v(t)$$

2.4-10

Let $y(t)=\int_{-\infty}^{\infty} v(l)w(t-l)dl$ where $v(-t)=v(t),\ w(-t)=w(t)$

$$y(-t)=\int_{-\infty}^{\infty} v(l)w(-t-l)\,dl=\int_{-\infty}^{\infty} v(l)w(t+l)\,dl$$

$$=-\int_{-\infty}^{\infty} v(-m)w(t-m)\,dm=\int_{-\infty}^{\infty} v(m)w(t-m)dm=y(t)$$

2.4-11

Let $y(t)=\int_{-\infty}^{\infty} v(l)w(t-l)dl$ where $v(-t)=-v(t),\ w(-t)=-w(t)$

$$y(-t)=\int_{-\infty}^{\infty} v(l)w(-t-l)\,dl=-\int_{-\infty}^{\infty} v(l)w(t+l)\,dl$$

$$=\int_{-\infty}^{\infty} v(-m)w(t-m)\,dm=\int_{-\infty}^{\infty} v(m)w(t-m)dm=y(t)$$

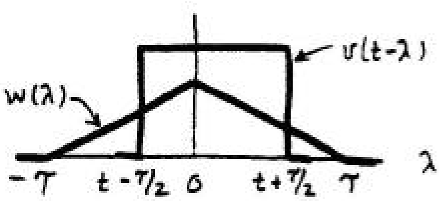
2.4-12

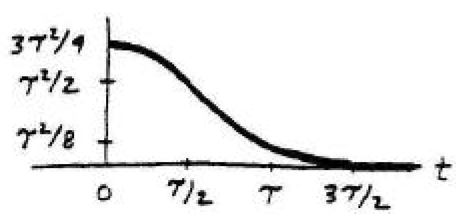
Let $w(t)=v*v(t)=t\Lambda(t/t)$

$$v*w(t)=\int_{t-t/2}^{0}(t+l)\,dl+\int_{0}^{t+t/2}(t-l)\,dl=\frac{3}{4}t^2-t^2 \qquad 0\le t<t/2$$

$$=\int_{t-t/2}^{t}(t-l)\,dl=\frac{1}{2}\left(t-\frac{3}{2}t\right)^2 \qquad t/2\le t<3t/2$$

Thus
$$v*v*v(t)=\begin{cases}\frac{3}{4}t^2-t^2 & |t|<t/2\\ \frac{1}{2}\left(|t|-\frac{3}{2}t\right)^2 & t/2\le|t|<3t/2\\ 0 & |t|\ge 3t/2\end{cases}$$





2.4-13

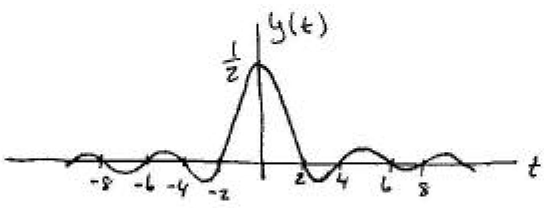
$$\mathbf{F}\{v(t)*[w(t)*z(t)]\}=V(f)[W(f)Z(f)]=[V(f)W(f)]Z(f)$$

so $v(t)*[w(t)*z(t)]=\mathbf{F}^{-1}\{[V(f)W(f)]Z(f)\}=[v(t)*w(t)]*z(t)$

2.4-14

$$V(f)=\frac{1}{4}\Pi\left(\frac{f}{4}\right) \quad W(f)=4\Pi(2f)$$

$$Y(f)=V(f)W(f)=\Pi(2f)\leftrightarrow y(t)=(1/2)\mathrm{sinc}(t/2)$$



2.5-1

$$z(t)=A\Pi\left(\frac{t}{t}\right)\cos w_c t \quad Z(f)=\frac{At}{2}\mathrm{sinc}(f-f_c)t+\frac{At}{2}\mathrm{sinc}(f+f_c)t$$

As $t\to 0$ the cosine pulse $z(t)$ gets narrower and narrower while maintaining height A. This is not the same as an impulse since the area under the curve is also getting smaller. As $t\to 0$ the main lobe and side lobes of the spectrum $Z(f)$ get wider and wider, however the height gets smaller and smaller. Eventually the spectrum will cover all frequencies with almost zero energy at each frequency. Again this is different from what happens in the case of an impulse.

2.5-2

$$W(f)=v(f)e^{-j2p\,ft_d}=\left[\sum_n c_v(nf_0)d(f-nf_0)\right]e^{-j2p\,ft_d}$$

$$=\sum_n\left[c_v(nf_0)e^{-j2p\,nf_0t_d}\right]d(f-nf_0)\Rightarrow c_w(nf_0)=c_v(nf_0)e^{-j2p\,nf_0t_d}$$

2.5-3

$$W(f)=j2p\,fV(f)=j2p\,f\left[\sum_n c_v(nf_0)d(f-nf_0)\right]=\sum_n\left[j2p\,nf_0\;c_v(nf_0)\right]d(f-nf_0)$$

$$\Rightarrow c_w(nf_0)=j2p\,nf_0\;c_v(nf_0)$$

2.5-4

$$W(f)=\frac{1}{2}[V(f-mf_0)]=\frac{1}{2}\left[\sum_n c_v(nf_0)d(f-kf_0-mf_0)+\sum_n c_v(nf_0)d(f-kf_0+mf_0)\right]$$

$$=\frac{1}{2}\left[\sum_k c_v[(k-m)f_0]d(f-kf_0)+\sum_k c_v[(k+m)f_0]d(f-kf_0)\right]$$

$$=\sum_n\frac{1}{2}\left\{c_v[(n-m)f_0]+c_v[(n+m)f_0]\right\}d(f-nf_0)$$

so $c_w(nf_0)=\frac{1}{2}\left\{c_v[(n-m)f_0]+c_v[(n+m)f_0]\right\}$

2.5-5

$v(t) = Au(t) - Au(t-2\tau)$

$$V(f) = A\left\{\frac{1}{j2\pi f} + \frac{1}{2}\delta(f) - \left[\frac{1}{j2\pi f} + \frac{1}{2}\delta(f)\right]e^{-j4\pi f\tau}\right\}$$

But $\delta(f)e^{-j4\pi f\tau} = e^{-j0}\delta(f)$, so

$$V(f) = \frac{A}{j2\pi f}\left(1 - e^{-j4\pi f\tau}\right) = 2A\tau\,\text{sinc}\,2f\tau\, e^{-j2\pi f\tau}$$

Agrees with $v(t) = \Pi\left(\frac{t-\tau}{2\tau}\right) \leftrightarrow 2A\tau\,\text{sinc}\,2f\tau\, e^{-j2\pi f\tau}$

2.5-6

$v(t) = A - Au(t+\tau) + Au(t-\tau)$

$$V(f) = A\left\{\delta(f) - \left[\frac{1}{j2\pi f} + \frac{1}{2}\delta(f)\right]e^{j2\pi f\tau} - \left[\frac{1}{j2\pi f} + \frac{1}{2}\delta(f)\right]e^{-j2\pi f\tau}\right\}$$

But $\delta(f)e^{j2\pi f\tau} = \delta(f)e^{-j2\pi f\tau} = e^{j0}\delta(f)$, so

$$V(f) = A\left[\delta(f) - \frac{1}{j2\pi f}\left(e^{j2\pi f\tau} - e^{-j2\pi f\tau}\right)\right] = A\delta(f) - 2A\tau\,\text{sinc}\,2f\tau$$

Agrees with $v(t) = A - A\Pi(t/2\tau) \leftrightarrow A\delta(f) - 2A\tau\,\text{sinc}\,2f\tau$

2.5-7

$v(t) = A - Au(t+T) - Au(t-T)$

$$V(f) = A\left\{\delta(f) - \left[\frac{1}{j2\pi f} + \frac{1}{2}\delta(f)\right]e^{j2\pi fT} - \left[\frac{1}{j2\pi f} + \frac{1}{2}\delta(f)\right]e^{-j2\pi fT}\right\}$$

But $\delta(f)e^{j2\pi fT} = \delta(f)e^{-j2\pi fT} = e^{j0}\delta(f) = \delta(f)$, so

$$V(f) = \frac{-A}{j2\pi f}\left(e^{j2\pi fT} + e^{-j2\pi fT}\right) = \frac{-A}{j\pi f}\cos 2\pi fT$$

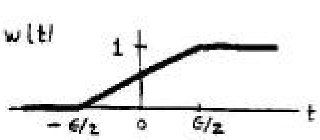
If $T \to 0$, $\quad v(t) = -A\,\text{sgn}\,t \leftrightarrow V(f) = \frac{-A}{j\pi f}$, which agrees with Eq. (17)

2.5-8

$V(f) = \text{sinc}\,f\varepsilon$ and $V(0) = 1$, so

$$W(f) = \frac{\text{sinc}\,f\varepsilon}{j2\pi f} + \frac{1}{2}\delta(f)$$

If $\varepsilon \to 0$, $\quad w(t) = u(t)$ and $W(f) = \frac{1}{j2\pi f} + \frac{1}{2}\delta(f)$, which agrees with Eq. (18)

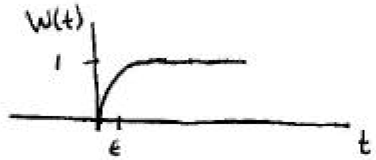


2.5-9

$V(f) = \dfrac{1/e}{1/e + j2p\,f}$ and $V(0) = 1$, so

$W(f) = \dfrac{1/e}{(j2p\,f)(1/e + j2p\,f)} + \dfrac{1}{2} d(f)$

If $e \to 0$, $w(t) = u(t)$ and $W(f) = \dfrac{1}{j2p\,f} + \dfrac{1}{2} d(f)$, which agrees with Eq. (18)



2.5-10

$z(t) = A\Pi(t/t) * [d(t-T) + d(t+T)]$

so $Z(f) = (At \text{ sinc } ft)\left(e^{-jwT} + e^{jwT}\right) = 2At \text{ sinc } ft \cos 2p\,fT$

2.5-11

$z(t) = A\Pi(t/t) * [d(t-2T) + 2d(t) + d(t+2T)]$

so $Z(f) = (At \text{ sinc } ft)\left(e^{-jw2T} + 2 + e^{jw2T}\right) = 2At \text{ sinc } ft\,(1 + \cos 4p\,fT)$

2.5-12

$z(t) = A\Pi(t/t) * [d(t-2T) - 2d(t) + d(t+2T)]$

so $Z(f) = (At \text{ sinc } ft)\left(e^{-jw2T} - 2 + e^{jw2T}\right) = 2At \text{ sinc } ft\,(\cos 4p\,fT - 1)$

2.5-13

n	0	1	2	3	4	5	6	7	8
$\sin(p\,t)d(t-0.5n)$	0	1	0	1	0	1	0	1	0
$v(t)$	0	1	1	2	2	3	3	4	4

2.5-14

n	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0
$\cos(2p\,t)d(t-0.1n)$	1	0.81	0.31	-0.31	-0.81	-1	-0.81	-0.31	0.31	0.81	1
$v(t)$	1	1.81	2.12	1.81	1	0	-0.81	-1.12	-0.81	0	1
$v(t)$ for $n = 1,10$	1.81	2.12	1.81	1	0	-0.81	-1.12	-0.81	0	1	

Chapter 3

3.1-1

$$y(t) = h(t) * A[d(t+t_d) - d(t-t_d)] = A[h(t+t_d) - h(t-t_d)]$$

$$Y(f) = H(f)A\left(e^{jwt_d} - e^{-jwt_d}\right) = j2AH(f)\sin 2p\, ft_d$$

3.1-2

$$y(t) = h(t) * A\left[d\left(t+t_d\right) + d\left(t\right)\right] = A\left[h\left(t+t_d\right) + h\left(t\right)\right]$$

$$Y(f) = H(f)A\left(e^{jwt_d} + 1\right) = 2AH(f)\cos p\, ft_d e^{jp\, ft_d}$$

3.1-3

$$y(t) = h(t) * Ah(t-t_d) = Ah(t) * h(t-t_d)$$

$$Y(f) = H(f)AH(f)e^{-jwt_d} = AH^2(f)e^{-jw\, ft_d}$$

3.1-4

$$y(t) = h(t) * Au(t-t_d) = A\int_{\infty}^{t-t_d} h(l)\,dl$$

$$Y(f) = H(f)A\left[\frac{1}{j2p\, f} + \frac{1}{2}d(f)\right]e^{-jwt_d} = \frac{A}{j2p\, f}H(f)e^{-j2p\, ft_d} + \frac{A}{2}H(0)d(f)$$

3.1-5

$$\mathbf{F}[g(t)] = H(f)\left[\frac{1}{j2p\, f} + \frac{1}{2}d(f)\right] = \frac{1}{j2p\, f}H(f) + \frac{1}{2}H(0)d(f)$$

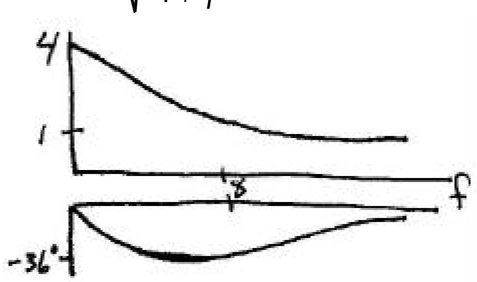
$$\mathbf{F}\left[dg(t)/dt\right] = j2p\, f\,\mathbf{F}\left[g(t)\right] = H(f) = \mathbf{F}\left[h(t)\right] \quad \text{Thus } h(t) = dg(t)/dt$$

3.1-6

$$j2p\, fY(f) + 4pY(f) = j2p\, fX(f) + 16p\, X(f)$$

$$H(f) = \frac{Y(f)}{X(f)} = \frac{j2p\, f + 2p8}{j2p\, f + 2p2} = \frac{8+jf}{2+jf}$$

$$|H(f)| = \sqrt{\frac{64+f^2}{4+f^2}} \quad \arg H(f) = \arctan\frac{f}{8} - \arctan\frac{f}{2}$$

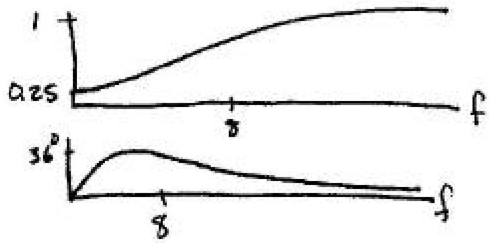


3.1-7

$$j2\pi fY(f)+16\pi Y(f)=j2\pi fX(f)+4\pi X(f)$$

$$H(f)=\frac{Y(f)}{X(f)}=\frac{j2\pi f+2\pi 2}{j2\pi f+2\pi 8}=\frac{2+jf}{8+jf}$$

$$\left|H(f)\right|=\sqrt{\frac{4+f^2}{64+f^2}} \quad \arg H(f)=\arctan\frac{f}{2}-\arctan\frac{f}{8}$$

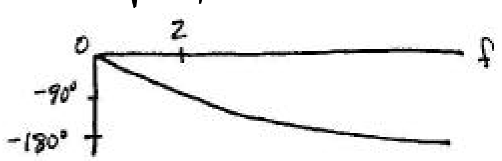


3.1-8

$$j2\pi fY(f)-4\pi Y(f)=-j2\pi fX(f)+4\pi X(f)$$

$$H(f)=\frac{Y(f)}{X(f)}=\frac{-j2\pi f+2\pi 2}{j2\pi f-2\pi 2}=\frac{2-jf}{2+jf}$$

$$\left|H(f)\right|=\sqrt{\frac{2+f^2}{2+f^2}}=1 \text{ for all } f \quad \arg H(f)=-2\arctan\frac{f}{2}$$



3.1-9

$$H(f)\approx\frac{B}{jf} \quad \text{for } \left|f\right|\geq W \gg B$$

$$\text{Thus } Y(f)\approx\frac{B}{jf}X(f)=2\pi B\frac{1}{j2\pi f}X(f) \quad \text{for } \left|f\right|\geq W$$

$$\text{and } y(t)\approx 2\pi B\int_{-\infty}^{t}x(\lambda)d\lambda \quad \text{since } X(0)\approx 0$$

3.1-10

$$H(f)\approx\frac{jf}{B} \quad \text{for } \left|f\right|\leq W \ll B$$

$$\text{Thus } Y(f)\approx\frac{jf}{B}X(f)=\frac{1}{2\pi B}j2\pi fX(f) \quad \text{for } \left|f\right|\leq W$$

$$\text{so } y(t)\approx\frac{1}{2\pi B}\frac{dx(t)}{dt}$$

3.1-11

$$x(t)=2\operatorname{sinc}4Wt \leftrightarrow X(f)=\frac{2}{4W}\Pi\left(\frac{f}{4W}\right)=\frac{1}{2W}\Pi\left(\frac{f}{4W}\right)$$

$$E_x=\int_{-\infty}^{\infty}\left|X(f)\right|^2 df=\int_{-2W}^{2W}\frac{1}{4W^2}df=\frac{1}{W}$$

$$Y(f)=\frac{1}{1+j(f/B)}\frac{1}{2W}\Pi\left(\frac{f}{4W}\right)$$

$$E_y=2\int_0^{2W}\frac{1/4W^2}{1+(f/B)^2}df=\frac{B}{2W^2}\arctan\frac{2W}{B}$$

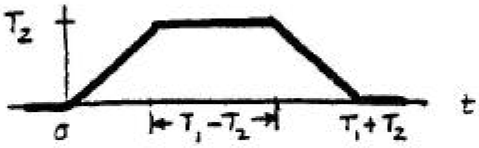
$$\frac{E_y}{E_x}=\frac{B}{2W}\arctan\frac{2W}{B}$$

3.1-12

$$h(t)=\mathbf{F}^{-1}\left[H_1(f)H_2(f)\right]=h_1(t)*h_2(t)$$

where $h_1(t)=u(t)-u(t-T_1)$

$h_2(t)=u(t)-u(t-T_2)$

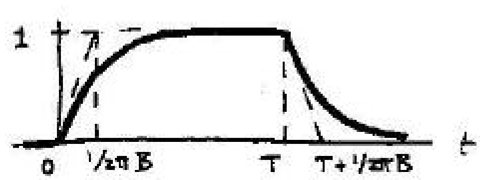


3.1-13

$$h(t)=\mathbf{F}^{-1}\left[H_1(f)H_2(f)\right]=h_1(t)*h_2(t)$$

where $h_1(t)=2\pi Be^{-2\pi Bt}u(t)$

$h_2(t)=u(t)-u(t-T)$



3.1-14

$$H(f)=\frac{j2\pi f}{1+jK2\pi f}=\frac{1}{K}j2\pi f\frac{1}{1/K+j2\pi f}$$

$$h(t)=\frac{1}{K}\frac{d}{dt}\left[e^{-t/K}u(t)\right]=\frac{1}{K}\delta(t)-\frac{1}{K^2}e^{-t/K}u(t)$$

$$g(t)=\mathbf{F}^{-1}\left\{\frac{1/K}{1/K+j2\pi f}\right\}=\frac{1}{K}e^{-t/K}u(t)$$

3-1-15

$$H(f) = \frac{K}{1 + Kj2p\,f} = \frac{1}{\frac{1}{K} + j2p\,f}$$

so $h(t) = e^{-t/K}u(t)$

$$g(t) = \int_{-\infty}^{t} h(l)\ dl\ =\ K\left(1 - e^{-t/K}\right)u(t)$$

3.1-16

Since $h(t)$ is real, $H_r(f) = H_e(f)$ and $H_i(f) = H_o(f)$, so

$$h(t) = \int_{-\infty}^{\infty} [H_e(f) + jH_o(f)]e^{jwt}df = 2\int_0^{\infty} H_r(f)\cos wt\ df + j2\int_0^{\infty} jH_i(f)\sin wt\ df$$

$$= 2\left[\int_0^{\infty} H_r(f)\cos wt\ df - \int_0^{\infty} H_i(f)\sin wt\ df\right]$$

$h(t) = 0$ for $t < 0 \Rightarrow \int_0^{\infty} H_i(f)\sin wt\ df = \int_0^{\infty} H_r(f)\cos wt\ df$

Hence, for $t > 0$, $\quad -\int_0^{\infty} H_i(f)\sin wt\ df = \int_0^{\infty} H_r(f)\cos wt\ df$

so $\quad h(t) = \int_0^{\infty} H_i(f)\sin wt\ df = \int_0^{\infty} H_r(f)\cos wt\ df$

3.2-1

$$|H(f)| = \left[1 + (f/B)^2\right]^{-1/2} = 1 - \frac{1}{2}(f/B)^2 + \cdots \approx 1$$

$$\arg H(f) = -\arctan\frac{f}{B} = -\frac{f}{B} + \frac{1}{3}\left(\frac{f}{B}\right)^3 + \cdots \approx -\frac{f}{B}$$

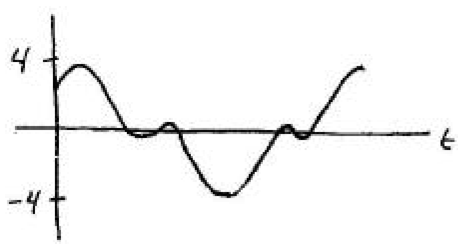
for $|f| \le W \ll B$

3.2-2

$|H(nf_0)| = \left[1 + (n/3)^2\right]^{-1/2} \qquad \arg H(nf_0) = -\arctan(n/3)$

$$y(t) = (0.95)(4)\cos(w_0 t - 18°) + (0.71)(4/9)\cos(3w_0 t - 45°) + (0.5)(4/25)\cos(5w_0 t - 59°)$$

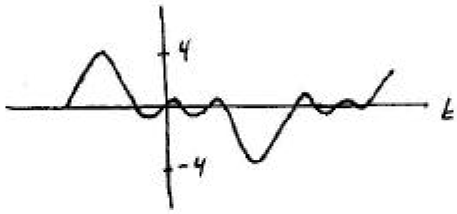
$$= 3.79\cos(w_0 t - 18°) + 0.31\cos(3w_0 t - 45°) + 0.08\cos(5w_0 t - 59°)$$



3.2-3

$$\left|H\left(nf_0\right)\right| = \frac{n/3}{\sqrt{1+\left(n/3\right)^2}} \qquad \arg H\left(nf_0\right) = 90° - \arctan(n/3)$$

$$y(t) = (0.32)(4)\cos\left(w_0 t - 72°\right) + (0.71)(4/9)\cos\left(3w_0 t - 45°\right) + (0.86)(4/25)\cos\left(5w_0 t - 31°\right)$$
$$= 1.28\cos\left(w_0 t - 72°\right) + 0.31\cos\left(3w_0 t - 45°\right) + 0.14\cos\left(5w_0 t - 31°\right)$$

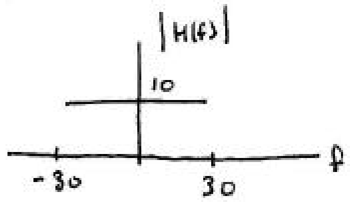


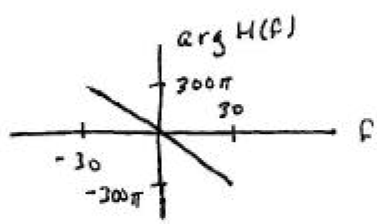
3.2-4

$$X(f) = \frac{2}{40}\Pi\left(\frac{f}{40}\right) = \frac{1}{20}\Pi\left(\frac{f}{40}\right)$$

$$Y(f) = \frac{20}{40}\Pi\left(\frac{f}{40}\right)e^{-jw5}$$

$$H(f) = \frac{Y(f)}{X(f)} = \frac{\frac{1}{2}\Pi\left(\frac{f}{40}\right)e^{-jw5}}{\frac{1}{20}\Pi\left(\frac{f}{40}\right)} = 10e^{-jw5}$$





Note that $300p = 2p$ and the phase actually wrapped around several times. Under normal plotting conventions we would go from $-p$ to p and repeat this pattern 300 times.

3.2-5

$$t_d(f) = \frac{-\arctan(f/B)}{2p\, f} \qquad B = 2\text{kHz} \qquad \lim_{f \to 0} t_d(f) = \frac{1}{2p\, B}$$

f kHz	$t_d(f)$, ms
0	-0.08
0.5	-0.078
1	-0.074
2	-0.062

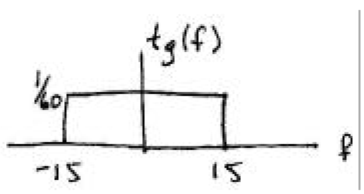
3.2-6

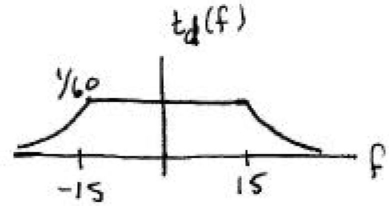
$$\arg H(f) = \begin{cases} -\frac{p\, f}{30} & |f| \le 15 \\ -\frac{p}{2} & |f| > 15 \end{cases}$$

$$t_g(f) = -\frac{1}{2p}\frac{d}{df}\arg H(f) = \begin{cases} -\frac{1}{2p}\left(-\frac{p}{30}\right) = \frac{1}{60} & |f| \le 15 \\ 0 & |f| > 15 \end{cases}$$

$$t_d(f) = \frac{-\arg H(f)}{2p\, f} = \begin{cases} \frac{p\, f/30}{2p\, f} = \frac{1}{60} & |f| \le 15 \\ \frac{p\,/2}{2p\, f} = \frac{1}{4f} & |f| > 15 \end{cases}$$

so $t_d(f) = t_g(f)$ for $|f| \le 15$



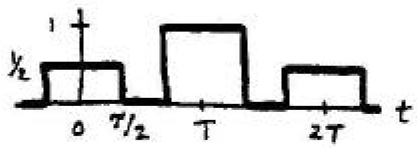


3.2-7

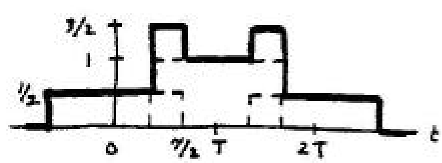
(a) $H_c(f) = \left[1 + 2a\,\frac{1}{2}\left(e^{jwT} + e^{-jwT}\right)\right]e^{-jwT} = a + e^{-jwT} + a\,e^{-jw2T}$

Thus, $y(t) = a\,x(t) + x(t - T) + a\,x(t - 2T)$

(b) $t = \frac{2T}{3}$



$t = \frac{4T}{3}$



3.2-8

$$\exp\left[-j\left(wT - a\sin wT\right)\right] = e^{-jwT}e^{ja\sin wT} = e^{-jwT}\left(1 + ja\sin wT - \frac{a^2}{2}\sin^2 wT + \cdots\right)$$

If $|a| \ll p/2$, $H_c(f) \approx e^{-jwT} + ja\sin wT\, e^{-jwT} = e^{-jwT} + \frac{a}{2}\left(e^{jwT} - e^{-jwT}\right)e^{-jwT}$

$$\approx \frac{a}{2} + e^{-jwT} - \frac{a}{2}e^{-jw2T}$$

Thus, $y(t) \approx \underbrace{\frac{a}{2}x(t)}_{\text{leading echo}} + x(t-T) - \underbrace{\frac{a}{2}x(t-2T)}_{\text{inverted trailing echo}}$

3.2-9

$$H_{eq}(f) = Ke^{-jw(t_d - T)}e^{j0.4\sin wT}$$

$$e^{j0.4\sin wT} = 1 + j0.4\sin wT - 0.8\sin^2 wT + \cdots \approx 1 + 0.2\left(e^{jwT} - e^{-jwT}\right)$$

so $H_{eq}(f) \approx Ke^{-jw(t_d - T)}\left(1 + 0.2e^{jwT} - 0.2e^{-jwT}\right)$

Take $K = 1$ and $t_d = 2T$, so $H_{eq}(f) \approx \left(0.2e^{jwT} + 1 - 0.2e^{-jwT}\right)e^{-jwT}$

Hence, $\Delta = T$, $M = 1$, $c_{-1} = 0.02$, $c_0 = 1$, $c_1 == -0.2$

3.2-10

$$H_{eq}(f) = Ke^{-jw(t_d - T)}\left(1 + 0.8\cos wT\right)^{-1}$$

Expanding using the first 3 terms

$$\left(1 + 0.8\cos wT\right)^{-1} = 1 - 0.8\cos wT + 0.64\cos^2 wT - 0.51\cos^3 wT$$

where $\cos wT = \frac{1}{2}\left(e^{jwT} + e^{-jwT}\right)$, $\cos^2 wT = \frac{1}{2} + \frac{1}{2}\cos 2wT = \frac{1}{2} + \frac{1}{4}\left(e^{j2wT} + e^{-j2wT}\right)$

$$\cos^3 wT = \frac{1}{4}\left(3\cos wT + \cos 3wT\right) = \frac{3}{8}\left(e^{jwT} + e^{-jwT}\right) + \frac{1}{8}\left(e^{j3wT} + e^{-j3wT}\right)$$

Take $K = 1$ and $t_d = 4T$, so

$$H_{eq}(f) = \left[-\frac{0.13}{2}e^{j3wT} + \frac{0.64}{4}e^{j2wT} - \left(\frac{0.8}{2} + \frac{0.38}{2}\right)e^{jwT} + 1 + \frac{0.64}{2} - \left(\frac{0.8}{2} + \frac{0.38}{2}\right)e^{-jwT} + \frac{0.64}{4}e^{-j2wT} - \frac{0.13}{2}e^{-j3wT}\right]e^{-j3wT}$$

Hence, $\Delta = T$, $M = 3$, $c_{-3} = c_3 = -0.065$, $c_{-2} = c_2 = 0.16$, $c_{-1} = c_1 = -0.59$, $c_0 = 1.32$

3.2-11

$$y(t) = 2A\cos w_0 t - 3A^3\cos^3 w_0 t \qquad 3A^3\cos^3 w_0 t = \frac{9A^3}{4}\cos w_0 t + \frac{3A^3}{4}\cos 3w_0 t$$

so $y(t) = \left(2A - \frac{9A^3}{4}\right)\cos w_0 t - \frac{3A^3}{4}\cos 3w_0 t$

2nd harmonic distortion = 0

3rd harmonic distortion = $\left|\frac{\frac{3A^3}{4}}{2A - \frac{9A^3}{4}}\right| \times 100 = \begin{cases} 300\% & A = 1 \\ 42\% & A = 2 \end{cases}$

3.2-12

$$y(t) = 5A\cos w_0 t - 2A^2\cos^2 w_0 t + 4A^3\cos^3 w_0 t$$

$$2A^2\cos^2 w_0 t = A^2 + A^2\cos 2w_0 t \qquad 4A^3\cos^3 w_0 t = 3A^3\cos w_0 t + A^3\cos 3w_0 t$$

so $y(t) = -A^2 + \left(5A + 3A^3\right)\cos w_0 t - A^2\cos 2w_0 t + A^3\cos 3w_0 t$

2nd harmonic distortion = $\left|\frac{A^2}{5A + 3A^3}\right| \times 100 = \begin{cases} 12.5\% & A = 1 \\ 11.8\% & A = 2 \end{cases}$

3rd harmonic distortion = $\left|\frac{A^3}{5A + 3A^3}\right| \times 100 = \begin{cases} 12.5\% & A = 1 \\ 23.5\% & A = 2 \end{cases}$

3.3-1

$P_{in} = 0.5\text{W} = 27\text{dBm} \quad \ell = 50\text{km} \quad a = 2\text{dB/km}$

$P_{out} = 50\text{mW} = 17\text{dBm} \quad 20\mu\text{W} = -17\text{dBm}$

$27\text{dBm} - 2\ell_1 = -17\text{dBm} \Rightarrow \ell_1 = 22\text{km} \Rightarrow \ell_3 = 50 - 22 = 28\text{km}$

$-17\text{dBm} + g_2 - 2\times 28 = -17\text{dBm} \Rightarrow g_2 = 56\text{dB}$

$-17\text{dBm} + g_4 = 17\text{dBm} \Rightarrow g_4 = 34\text{dB}$

3.3-2

$P_{in} = 100\text{mW} = 20\text{dBm} \quad \ell = 50\text{km} \quad a = 2\text{dB/km}$

$P_{out} = 0.1\text{W} = 20\text{dBm} \quad 20\mu\text{W} = -17\text{dBm}$

$20\text{dBm} - 2\ell_1 = -17\text{dBm} \Rightarrow \ell_1 = 18.5\text{km} \Rightarrow \ell_3 = 40 - 18.5 = 21.5\text{km}$

$-17\text{dBm} + g_2 - 2\times 21.5 = -17\text{dBm} \Rightarrow g_2 = 43\text{dB}$

$-17\text{dBm} + g_4 = 20\text{dBm} \Rightarrow g_4 = 37\text{dB}$

3.3-3

$$\frac{P_{\text{out}}}{P_{\text{in}}} = \frac{50\times 10^{-3}}{2} = -16 \text{ dB}, \quad mL_i = 0.4\times 400 = 160 \text{ dB}, \ g_i \le 30 \text{ dB}$$

$$m\times 30 \text{ dB} - 160 \ge -16 \quad \Rightarrow \quad m \ge 4.8 \quad \text{so} \ \ m = 5$$

$$g = (160-16)/5 = 28.8 \text{ dB}$$

3.3-4

$$L_i = 0.5\times 3000/\,m = 1500/m \text{ dB} \qquad P_{in} = 5\text{mW} = 7\text{dBm}$$

$$\frac{P_{in}}{L_1} \ge 67\mu\text{W} = -11.75\text{dBm}$$

$$7\text{dBm} - \frac{1500}{m} \ge -11.75 \Rightarrow m \ge 80$$

$$\frac{P_{out}}{P_{in}} = \frac{mg_i}{mL_i} = 1 \Rightarrow g_i = L_i = \frac{1500}{80} = 18.75\text{dB}$$

3.3-5

$$L_i = 2.5\times 3000/\,m = 7500/m \text{ dB} \qquad P_{in} = 5\text{mW} = 7\text{dBm}$$

$$\frac{P_{in}}{L_1} \ge 67\mu\text{W} = -11.75\text{dBm}$$

$$7\text{dBm} - \frac{7500}{m} \ge -11.75 \Rightarrow m \ge 400$$

$$\frac{P_{out}}{P_{in}} = \frac{mg_i}{mL_i} = 1 \Rightarrow g_i = L_i = \frac{7500}{400} = 18.75\text{dB}$$

3.3-6

$$\frac{P_{\text{out}}}{P_{\text{in}}} = 2\times 10^{-6}/5 = -64 \text{ dB}, \quad L = 92.4\text{-}6 + 26 = 112.4 \text{ dB}$$

$$\frac{g^2}{L} = \frac{P_{\text{out}}}{P_{\text{in}}} \quad \Rightarrow \quad g = (112.4-64)/2 = 24.2 \text{ dB} = 263$$

$$\text{so} \quad \frac{4p\left(p\,r^2\right)\left(0.5\times 10^9\right)^2}{\left(3\times 10^5\right)^2} = 263 \quad \Rightarrow \quad r = 1.55\times 10^{-3} \text{ km} = 1.55 \text{ m}$$

3.3-7

$$\frac{P_{out}}{P_{in}} = \frac{2\times10^{-6}}{5} = -64\text{dB} \qquad L = 92.4 - 14 + 20 = 98.4$$

$$\frac{g^2}{L} = \frac{P_{out}}{P_{in}} \Rightarrow g = \frac{98.4-64}{2} = 17.2\text{dB} = 52.5$$

so $\dfrac{4p\left(p r^2\right)\left(0.2\times10^9\right)^2}{\left(3\times10^5\right)^2} = 52.5 \Rightarrow r = 1.7\times10^{-3}\,\text{km} = 1.7\text{m}$

3.3-8

With repeater $P_{out} = \dfrac{g_T g_R g_{rpt}}{L_1 L_2} P_{in}$ Without repeater $P_{out} = \dfrac{g_T g_R}{L} P_{in}$

$$\frac{g_T g_R g_{rpt}}{L_1 L_2} = 1.2\frac{g_T g_R}{L} \Rightarrow g_{rpt} = 1.2\frac{L_1 L_2}{L}$$

$L_{1_{dB}} = 92.4 - 20\log f_{GHz} + 20\log 25\text{km} = 120 + 20\log f$

$L_2 = L_1$

$L = 92.4 - 20\log f + 20\log 50 = 126 + 2 - \log f$

Let $f = 1\text{GHz}$

$L_1 = L_2 = 120\text{dB} \Rightarrow 10^{12} \qquad L = 126\text{dB} \Rightarrow 3.98\times10^{12}$

$$g_{rpt} = 1.2\frac{10^{12}\times10^{12}}{3.98\times10^{12}} = 0.3\times10^{12} = 115\text{dB}$$

3.3-9

$L_u = 92.4 + 20\log 17 + 20\log 3.6\times10^4 = 208$

$L_d = 92.4 + 20\log 12 + 20\log 3.6\times10^4 = 205$

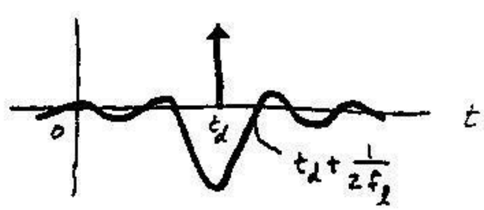
$P_{in} = 30\text{dBW}$ so $P_{sat_{in}} = 30 + 55 - 208 + 20 = -103\text{dBW}$

based on parameters from Example 3.3-1 $g_{amp} = 18 + 144 = 162\text{dB}$

$P_{sat_{out}} = -103 + 162 = 59\text{dBW}$ so $P_{out} = 59 + 16 - 205 + 51 = -79\text{dBW} \Rightarrow 1.26\times10^{-8}\text{W}$

3.4-1

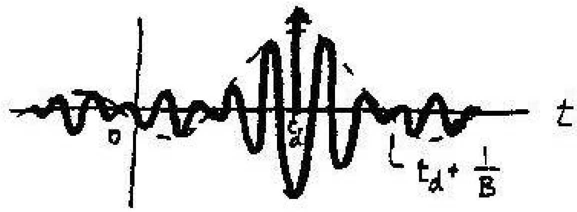
$$H(f) = Ke^{-jwt_d} - K\Pi\left(\frac{f}{2f_\ell}\right)e^{-jwt_d} \qquad h(t) = Kd(t-t_d) - 2Kf_\ell\,\text{sinc}\,2f_\ell(t-t_d)$$



3.4-2

$H(f) = Ke^{-jwt_d} - H_{BP}(f)$ where $H_{BP}(f) = \text{Eq. (1)}$

Thus, from Exercise 3.4-1, $h(t) = Kd(t - t_d) - 2BK \operatorname{sinc} B(t - t_d) \cos w_c (t - t_d)$



3.4-3

$|H(0.7B)|^2 = \left[1 + (0.7)^{2n}\right]^{-1} \geq 10^{-1/10} = 1/1.259$

so $1 + (0.7)^{2n} \leq 1.259$ or $(0.7)^{2n} \leq 0.259$

n	$(0.7)^{2n}$	
1	0.49	$\Rightarrow$ select $n = 2$
2	0.24	

$|H(3B)| = \left[1 + 3^{2n}\right]^{-1/2} = \left[1 + 3^4\right]^{-1/2} = 0.11 = -19\text{dB}$

3.4-4

$|H(0.9B)|^2 = \left[1 + (0.9)^{2n}\right]^{-1} \geq 10^{-1/10} = 1/1.259$

so $1 + (0.9)^{2n} \leq 1.259$ or $(0.9)^{2n} \leq 0.259$

n	$(0.9)^{2n}$	
6	0.282	$\Rightarrow$ select $n = 7$
7	0.229	

$|H(3B)| = \left[1 + 3^{2n}\right]^{-1/2} = \left[1 + 3^{14}\right]^{-1/2} = 4.6 \times 10^{-4} = -66.8\text{dB}$

3.4-5

$$H(f) = \left[1 + j\sqrt{2}\frac{f}{B} - \left(\frac{f}{B}\right)^2\right]^{-1} \quad \text{from Table 3.4-1}$$

$$H(s) = H(f)|_{f=s/j2p} = \left[1 + j\sqrt{2}\frac{s}{2pB} - \left(\frac{s}{2pB}\right)^2\right]^{-1}$$

$$= \frac{2b^2}{(s+b)^2 + b^2} \qquad b = 2pB/\sqrt{2}$$

so $h(t) = 2be^{-bt} \sin bt\, u(t)$

3.4-6

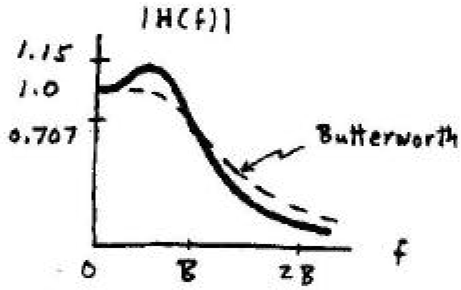
(a) $H(f) = \dfrac{Z_{RC}}{Z_{RC} + jwL}$ where $Z_{RC} = \dfrac{R/jwC}{R + 1/jwC} = \dfrac{\sqrt{LC}}{1 + jw\sqrt{LC}}$

Thus, $H(f) = \dfrac{1}{1 + jw\sqrt{LC} - w^2 LC}$

so $|H(f)|^2 = \left[\left(1 - w^2 LC\right)^2 + \left(w^2 LC\right)^2\right]^{-1} = 1 - \left(f/f_0\right)^2 + \left(f/f_0\right)^4$ with $f_0 = \dfrac{1}{2p\sqrt{LC}}$

(b) $|H(B)|^2 = 1/2 \Rightarrow 1 - \left(B/f_0\right)^2 + \left(B/f_0\right)^4 = 2$

so $\left(\dfrac{B}{f_0}\right)^2 = \dfrac{1}{2}\left(1 + \sqrt{5}\right) \qquad B = \sqrt{\dfrac{1}{2}\left(1 + \sqrt{5}\right)}\, f_0 = 1.27\, f_0$



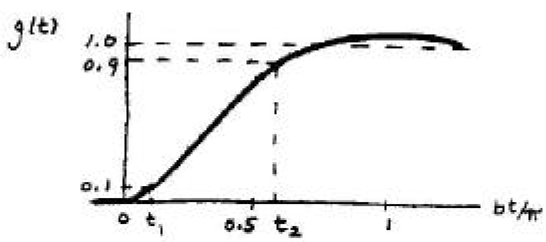
3.4-7

$$\left.\begin{aligned} 1 - e^{-2p Bt_1} &= 0.1 \Rightarrow t_1 = 0.11/2p B \\ 1 - e^{-2p Bt_2} &= 0.9 \Rightarrow t_2 = 2.30/2p B \end{aligned}\right\} \quad t_r = t_2 - t_1 = \frac{2.30 - 0.11}{2p B} = \frac{1}{2.87B}$$

3.4-8

$g(t) = \int_{-\infty}^{t} h(l)\,dl = 2b\int_0^t e^{-bl}\sin bl\; dl = 1 - e^{-bt}\left(\sin bt + \cos bt\right)$ for $t \geq 0$

$bt_1/p \approx 0.1, \quad bt_2/p \approx 0.6 \qquad t_r \approx \dfrac{0.5p}{b} = \dfrac{0.5p\sqrt{2}}{2p B} = \dfrac{1}{2.8B}$

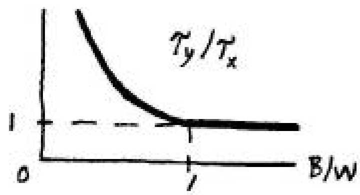


3.4-9

$$x(t) = A\,\text{sinc}\,2Wt \;\Rightarrow t_x = \frac{1}{W}, \quad X(f) = \frac{A}{2W}\Pi\left(\frac{f}{2W}\right)$$

$$Y(f) = \Pi\left(\frac{f}{2B}\right)X(f) = \begin{cases} \frac{A}{2W}\Pi\left(\frac{f}{2W}\right) & \text{for } B > W \\ \frac{A}{2W}\Pi\left(\frac{f}{2B}\right) & \text{for } B < W \end{cases}$$

$$y(t) = \begin{cases} A\,\text{sinc}\,2Wt \;\Rightarrow\; t_y = 1/W \;\; \text{for } B > W \\ \frac{B}{W}A\,\text{sinc}\,2Bt \Rightarrow t_y = 1/B \;\; \text{for } B < W \end{cases}$$



3.4-10

$$H(0) = \int_{-\infty}^{\infty} h(t)e^{-jwt}dt\Big|_{f=0} = \int_{-\infty}^{\infty} h(t)\,dt$$

$$|h(t)| = \left|\int_{-\infty}^{\infty} H(f)e^{jwt}df\right| \le \int_{-\infty}^{\infty}\left|H(f)e^{jwt}\right|df = \int_{-\infty}^{\infty}|H(f)|\,df$$

$$\text{Thus } \; t_{\text{eff}} = \frac{H(0)}{|h(t)|_{\max}} \ge \frac{H(0)}{\int_{-\infty}^{\infty}|H(f)|\,df} = \frac{1}{2B_{\text{eff}}}$$

3.4-11

(a) $H(f) = 2KB\int_0^{2t_d}\text{sinc}\,2B(t-t_d)e^{-jwt}dt = 2KBe^{-jwt_d}\int_{-t_d}^{t_d}\text{sinc}\,2Bl\,e^{-jwl}d\,l$

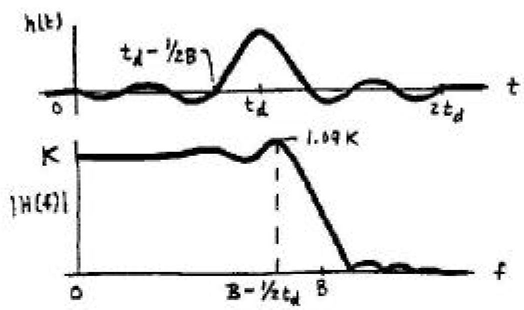
where $\sin 2p\,Bl\,\cos 2p\,f\,l = \frac{1}{2}\left[\sin 2p\,(f+B)\,l - \sin 2p\,(f-B)\,l\right]$

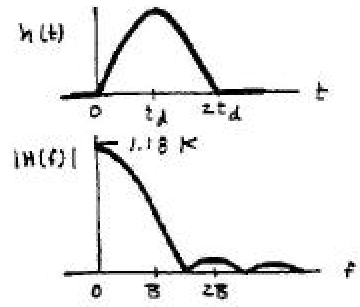
and $\int_0^{t_d}\frac{\sin 2p\,(f\pm B)\,l}{2p\,Bl}\,d\,l = \frac{1}{2p\,B}\int_0^{2p\,(f\pm B)t_d}\frac{\sin a}{a}\,da = \frac{1}{2p\,B}Si\left[2p\,(f\pm B)t_d\right]$

Thus $H(f) = \frac{K}{p}e^{-jwt_d}\left\{Si\left[2p\,(f+B)t_d\right] - Si\left[2p\,(f-B)t_d\right]\right\}$

(cont.)

(b) $t_d \gg \dfrac{1}{B}$ $\qquad\qquad$ $t_d = \dfrac{1}{2B}$





3.5-1

(a) $\hat{\delta}(t) = \dfrac{1}{\pi}\displaystyle\int_{-\infty}^{\infty} \frac{\delta(\lambda)}{t-\lambda}\, d\lambda = \frac{1}{\pi}\frac{1}{t-\lambda}\bigg|_{\lambda=0} = \frac{1}{\pi t}$

$\mathbf{F}\left[\hat{\delta}(t)\right] = (-j\,\mathrm{sgn}\, f)\,\mathbf{F}\left[\delta(t)\right] = -j\,\mathrm{sgn}\, f$

Thus, $\mathbf{F}^{-1}\left[-j\,\mathrm{sgn}\, f\right] = \hat{\delta}(t) = \dfrac{1}{\pi t}$

(b) $\hat{\delta}(t) * \dfrac{1}{\pi t} = \hat{\hat{\delta}}(t)$ and $\hat{\delta}(t) * \left(\dfrac{-1}{\pi t}\right) = -\dfrac{1}{\pi t} * \dfrac{1}{\pi t} = -\widehat{\left(\dfrac{1}{\pi t}\right)}$

Thus, $\widehat{\left(\dfrac{1}{\pi t}\right)} = -\delta(t)$

3.5-2

$A\Pi\left(\dfrac{t}{\tau}\right) = x(t+\tau/2)$ where $x(t) = A\left[u(t) - u(t-\tau)\right]$

so $A\hat{\Pi}\left(\dfrac{t}{\tau}\right) = \hat{x}\left(t+\dfrac{\tau}{2}\right) = \dfrac{A}{\pi}\ln\left|\dfrac{t+\tau/2}{t+\tau/2-\tau}\right| = \dfrac{A}{\pi}\ln\left|\dfrac{2t+\tau}{2t-\tau}\right|$

Now let $v(t) = \lim\limits_{\tau\to\infty} A\Pi\left(\dfrac{t}{\tau}\right)$ so $\hat{v}(t) = \lim\limits_{\tau\to\infty} \dfrac{A}{\pi}\ln\left|\dfrac{2t+\tau}{2t-\tau}\right| = \dfrac{A}{\pi}\ln 1 = 0$

3.5-3

$$\mathbf{F}\left[\hat{x}(t)\right] = (-j\,\mathrm{sgn}\, f)\frac{1}{2W}\Pi\left(\frac{f}{2W}\right)$$

$$= \frac{j}{2W}\Pi\left(\frac{f+W/2}{W}\right) - \frac{j}{2W}\Pi\left(\frac{f-W/2}{W}\right)$$

Thus, $\hat{x}(t) = \dfrac{j}{2}\mathrm{sinc}\, Wt\left(e^{-j\pi Wt} - e^{j\pi Wt}\right) = \mathrm{sinc}\, Wt\, \mathrm{sinc}\,\pi Wt = \pi Wt\, \mathrm{sinc}^2\, Wt$

3.5-4

$$x(t) = \cos w_0 t - \frac{1}{3}\cos 3w_0 t + \frac{1}{5}\cos 5w_0 t$$

$$\hat{x}(t) = \sin w_0 t - \frac{1}{3}\sin 3w_0 t + \frac{1}{5}\sin 5w_0 t$$

3.5-5

$$x(t) = 4\cos w_0 t + \frac{4}{9}\cos 3w_0 t + \frac{4}{25}\cos 5w_0 t$$

$$\hat{x}(t) = 4\sin w_0 t + \frac{4}{9}\sin 3w_0 t + \frac{4}{25}\sin 5w_0 t$$

3.5-6

$$x(t) = \text{sinc}\,2Wt \leftrightarrow X(f) = \frac{1}{2W}\Pi\left(\frac{f}{2W}\right) \qquad \left|X(f)\right| = \frac{1}{2W}\Pi\left(\frac{f}{2W}\right)$$

$$\hat{x}(t) = pWt\,\text{sinc}^2 Wt = \text{sinc}\,Wt \sin pWt \leftrightarrow \hat{X}(f) = \frac{1}{2W}\Pi\left(\frac{f-\frac{W}{2}}{W}\right)e^{-jp/2} + \frac{1}{2W}\Pi\left(\frac{f+\frac{W}{2}}{W}\right)e^{jp/2}$$

$$\left|\hat{X}(f)\right| = \left|\frac{1}{2W}\Pi\left(\frac{f-\frac{W}{2}}{W}\right)e^{-jp/2}\right| + \left|\frac{1}{2W}\Pi\left(\frac{f+\frac{W}{2}}{W}\right)e^{+jp/2}\right|$$

Note that the cross term is zero since there is no overlap. From the graph we see that the two rectangle functions form one larger function so

$$\left|\hat{X}(f)\right| = \frac{1}{2W}\Pi\left(\frac{f}{2W}\right) = \left|X(f)\right|$$

3.5-7

$$x(t) = A\cos w_0 t \qquad \hat{x}(t) = A\sin w_0 t$$

$$\int_{-\infty}^{\infty} x(t)\hat{x}(t)\,dt = A^2\int_{-\infty}^{\infty}\cos w_0 t \sin w_0 t\,dt$$

$$= \lim_{T\to\infty}\left[\frac{A^2}{2}\int_{-T}^{T}\sin\left(w_0 - w_0\right)t\,dt + \frac{A^2}{2}\int_{-T}^{T}\sin\left(w_0 + w_0\right)t\,dt\right] = \lim_{T\to\infty}\left[0 + \frac{A^2}{2}\frac{1}{2w_o}\cos 2w_0 t\right]_{-T}^{T}$$

$$= \lim_{T\to\infty}\left[\frac{A^2}{4w_0}\left(\cos 2w_0 T - \cos\left(-2w_0 T\right)\right)\right] = 0$$

3.5-8

$$\mathbf{F}\left[h_e(t)\right] = \int_{-\infty}^{\infty} \frac{1}{2} h\left(|t|\right) e^{-jwt} dt = 2\int_0^{\infty} \frac{1}{2} h\left(|t|\right) \cos wt \, dt$$

$$= \int_0^{\infty} h(t) \cos wt \, dt = \int_{-\infty}^{\infty} h(t) \cos wt \, dt = H_e(f)$$

$$H(f) = \mathbf{F}\left[\left(1+\operatorname{sgn} t\right) h_e(t)\right] = H_e(f) + \frac{1}{j2p\,f} * H_e(f) = H_e(f) - j\left[H_e(f) * \frac{1}{p\,f}\right] = H_e(f) - j\widehat{H}_e(f)$$

Thus, $H_o(f) = -\widehat{H}_e(f)$

3.6-1

$$R_{wv}(t) = \left\langle w(t) v^*(t-t) \right\rangle = \left\langle w^*(t) v(t-t) \right\rangle^*$$

$$= \left\langle v\left[t+(-t)\right] w^*(t) \right\rangle^* = R_{vw}^*(-t)$$

3.6-2

$$R_v\left(t \pm mT_0\right) = \left\langle v\left(t+t \pm mT_0\right) v^*(t) \right\rangle$$

but $v\left(t+t \pm mT_0\right) = v\left(t+t\right)$ so $R_v\left(t \pm mT_0\right) = \left\langle v\left(t+t\right) v^*\left(t\right) \right\rangle = R_v(t)$

3.6-3

$$P_w = \left\langle \left|v(t+t)\right|^2 \right\rangle = \left\langle \left|v(t)\right|^2 \right\rangle = P_v$$

$$\left|R_v(t)\right|^2 = \left|\left\langle v(t) w^*(t) \right\rangle\right|^2 \le P_v P_v = R_v^2(0) \text{ so } \left|R_v(t)\right| \le R_v(0)$$

3.6-4

$x(t) = \cos 2w_0 t$ From Eq. (12) $R_x(t) = \frac{1}{2}\cos 2w_0 t$

$y(t) = \sin 2w_0 t = \cos\left(2w_0 t - 90°\right) \Rightarrow R_y(t) = \frac{1}{2}\cos 2w_0 t$

(Note that the phase delay does not appear in the autocorrelation)

Since $R_y(t) = R_x(t)$ we conclude that $y(t)$ is similar to $x(t)$. This is the expected conclusion since $y(t)$ is just a phase shifted version of $x(t)$.

3.6-5

$$V(f) = AD \operatorname{sinc} fD \, e^{-jwt_d}$$

$$G_v(f) = (AD)^2 \operatorname{sinc}^2 fD \Rightarrow R_v(t) = A^2 D \Lambda\left(t/D\right), \quad E_v = R_v(0) = A^2 D$$

3.6-6

$$V(f)=\left(\frac{A}{4W}\right)\Pi\left(\frac{f}{4W}\right)e^{-jwt_d}$$

$$G_v(f)=\left(\frac{A}{4W}\right)^2\Pi\left(\frac{f}{4W}\right)\Rightarrow R_v(t)=\frac{A^2}{4W}\mathrm{sinc}4Wt\,,\quad E_v=R_v(0)=\frac{A^2}{4W}$$

3.6-7

$$V(f)=\frac{A}{b+j2p\,f}$$

$$G_v(f)=\frac{A^2}{b^2+(2p\,f)^2}\Rightarrow R_v(t)=\frac{A^2}{2b}e^{-b|t|},\quad E_v=R_v(0)=\frac{A^2}{2b}$$

3.6-8

$$v(t)=A_0+\frac{A_1}{2}e^{jf}e^{jw_0t}+\frac{A_1}{2}e^{-jf}e^{-jw_0t}$$

$$G_v(f)=A_0^2d(f)+\frac{A_1^2}{4}\left[d\left(f-f_0\right)+d\left(f+f_0\right)\right]$$

$$R_v(t)=A_0^2+\frac{A_1^2}{2}\cos w_0t\,,\quad P_v=R_v(0)=A_0^2+\frac{A_1^2}{2}$$

3.6-9

$$v(t)=\frac{A_1}{2}\left(e^{jf_1}e^{jw_0t}+e^{-jf_1}e^{-jw_0t}\right)+\frac{A_2}{2}\left(e^{-jp/2}e^{j2w_0t}e^{jf_1}+e^{jp/2}e^{-j2w_0t}e^{-jf_1}\right)$$

$$G_v(f)=\frac{A_1^2}{4}\left[d\left(f-f_0\right)+d\left(f+f_0\right)\right]+\frac{A_2^2}{4}\left[d\left(f-2f_0\right)+d\left(f+2f_0\right)\right]$$

$$R_v(t)=\frac{A_1^2}{2}\cos w_ot+\frac{A_2^2}{2}\cos 2w_0t \qquad P_v=R_v(0)=\frac{A_1^2}{2}+\frac{A_2^2}{2}$$

3.6-10

$$R_v(t)=\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}A^2u(t)u(t-t)\,dt \quad \text{where} \quad u(t)u(t-t)=\begin{cases}0 & t<t\\ 1 & t>t\end{cases}$$

Take $T/2>t>0$, so $\int_{-T/2}^{T/2}u(t)u(t-t)\,dt=\int_{t}^{T/2}dt=\frac{T}{2}-t$

Thus $R_v(t)=\lim_{T\to\infty}\frac{A^2}{T}\left(\frac{T}{2}-t\right)=\frac{A^2}{2}$ for all t

$$P_v=R_v(0)=\frac{A^2}{2} \qquad G_v(f)=\frac{A^2}{2}d(f)$$

3.6-11

$$x(t) = \Pi(10t) = \Pi\left(\frac{t}{1/10}\right) \quad \leftrightarrow \quad X(f) = \frac{1}{10}\text{sinc}\frac{f}{10}$$

$$H(f) = Ke^{-jwt_d}\Pi\left(\frac{f}{2B}\right) = \left|3e^{-jw\,0.05}\Pi\left(\frac{f}{40}\right)\right|$$

$$G_y(f) = |H(f)|^2\, G_x(f) = |H(f)|^2\, |X(f)|^2 \quad \text{since } x(t) \text{ is an energy signal}$$

$$= \left|3e^{-jw\,0.05}\Pi\left(\frac{f}{40}\right)\right|^2 \left|\frac{1}{10}\text{sinc}\frac{f}{10}\right|^2 = \left[9\Pi\left(\frac{f}{40}\right)\right]\left[\frac{1}{100}\text{sinc}^2\frac{f}{10}\right]$$

so $R_y(t) = \int_{-20}^{20} \frac{9}{100}\text{sinc}^2\frac{f}{10}\, e^{j2p\,f\,t}\, df$

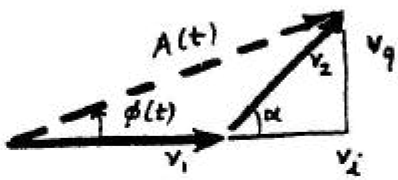
Chapter 4

4.1-1

$$v_i(t) = v_1(t) + v_2(t)\cos\alpha \qquad v_q(t) = v_2(t)\sin\alpha$$

$$A(t) = \sqrt{v_1^2(t) + 2v_1(t)\,v_2(t)\cos\alpha + v_2^2(t)} \approx v_1(t) + v_2(t)\cos\alpha$$

$$\phi(t) = \arctan\frac{v_2(t)\sin\alpha}{v_1(t) + v_2(t)\cos\alpha} \approx \frac{v_2(t)\sin\alpha}{v_1(t)}$$

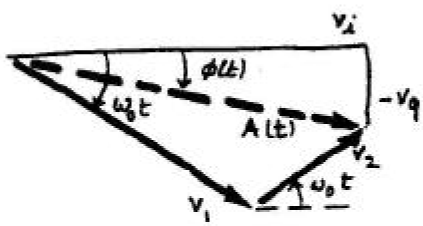


4.1-2

$$v_i(t) = [v_1(t) + v_2(t)]\cos\omega_0 t \qquad v_q(t) = [v_2(t) - v_1(t)]\sin\omega_0 t$$

$$A(t) = \sqrt{v_1^2(t) + 2v_1(t)\,v_2(t)\cos 2\omega_0 t + v_2^2(t)} \approx v_1(t) + v_2(t)\cos 2\omega_0 t$$

$$\phi(t) = \arctan\frac{[v_2(t) - v_1(t)]\sin\omega_0 t}{[v_1(t) + v_2(t)]\cos\omega_0 t} \approx -\omega_0 t$$



4.1-3

(a) $\int_{-\infty}^{\infty} v_{bp}(t)dt = \int_{-\infty}^{\infty} v_i(t)\cos\omega_c t\, dt - \int_{-\infty}^{\infty} v_q(t)\sin\omega_c t\, dt$

$$\int_{-\infty}^{\infty} v_i(t)\cos\omega_c t\, dt = \int_{-\infty}^{\infty} V_i(f)\frac{1}{2}\left[\delta(f - f_c) + \delta(f + f_c)\right]^* df = \frac{1}{2}[V_i(f_c) + V_i(-f_c)] = 0$$

since $f_c > W$ and $V_i(f) = 0$ for $|f| > W$

$$\int_{-\infty}^{\infty} v_q(t)\sin\omega_c t\, dt = \int_{-\infty}^{\infty} V_q(f)\frac{1}{2}\left[e^{-j\pi/2}\delta(f - f_c) + e^{j\pi/2}\delta(f + f_c)\right]^* df$$

$$= \frac{1}{2}\left[V_q(f_c)e^{-j\pi/2} + V_q(-f_c)e^{j\pi/2}\right] = 0$$

Thus, $\int_{-\infty}^{\infty} v_{bp}(t)dt = 0$

(cont).

(b) $E_{bp} = \int_{-\infty}^{\infty} \left(v_i(t) \cos w_c t - v_q \sin w_c t \right)^2 dt$

$$= \frac{1}{2}\left[\int_{-\infty}^{\infty} v_i^2 dt + \int_{-\infty}^{\infty} v_q^2 dt + \int_{-\infty}^{\infty} v_i^2 \cos 2w_c t \, dt + \int_{-\infty}^{\infty} v_q^2 \sin 2w_c t \, dt + \int_{-\infty}^{\infty} v_i v_q \sin 2w_c t \, dt \right]$$

but v_i^2, v_q^2, and $v_i v_q$ are bandlimited in $2W < 2f_c$ so, from the analysis in part (a)

$$\int_{-\infty}^{\infty} v_i^2 \cos 2w_c t \, dt = \int_{-\infty}^{\infty} v_q^2 \sin 2w_c t \, dt = \int_{-\infty}^{\infty} v_i v_q \sin 2w_c t \, dt = 0$$

Hence, $E_{bp} = \frac{1}{2}\left[\int_{-\infty}^{\infty} v_i^2 \, dt + \int_{-\infty}^{\infty} v_q^2 \, dt \right] = \frac{1}{2}\left(E_i + E_q \right)$

4.1-4

$$V_{\ell p}(f) = \Pi\left(\frac{f+100}{400} \right)$$

$$v_{\ell p}(t) = 400 \operatorname{sinc} 400t \; e^{-j2p\,100t}$$

$$= 400 \operatorname{sinc} 400t \left(\cos 2p\,100t + j \sin 2p\,100t \right)$$

$$v_i(t) = 800 \operatorname{sinc} 400t \cos 2p\,100t \qquad v_q(t) = -800 \operatorname{sinc} 400t \sin 2p\,100t$$

4.1-5

$$V_{\ell p}(f) = \frac{1}{2}\Pi\left(\frac{f-75}{100} \right) + \Pi\left(\frac{f+50}{150} \right)$$

$$v_{\ell p}(t) = \frac{150}{2} \operatorname{sinc} 150t \; e^{j2p\,75t} + 100 \operatorname{sinc} 100t \; e^{-j2p\,50t}$$

$$v_i(t) = 2\operatorname{Re}\left[v_{\ell p}(t) \right] = 150 \operatorname{sinc} 150t \cos 2p\,75t + 200 \operatorname{sinc} 100t \cos 2p\,50t$$

$$v_q(t) = 2\operatorname{Im}\left[v_{\ell p}(t) \right] = 150 \operatorname{sinc} 150t \sin 2p\,75t - 200 \operatorname{sinc} 100t \sin 2p\,50t$$

4.1-6

$$v_{bp}(t) = 2z(t)\left[\cos\left(\pm w_0 t + a \right) \cos w_c t - \sin\left(\pm w_0 t + a \right) \sin w_c t \right]$$

so $v_i(t) = 2z(t) \cos\left(\pm w_0 t + a \right) \qquad v_q(t) = 2z(t) \sin\left(\pm w_0 t + a \right)$

$$v_{\ell p}(t) = \frac{1}{2} 2z(t)\left[\cos\left(\pm w_0 t + a \right) + j \sin\left(\pm w_0 t + a \right) \right] = z(t) e^{j(\pm w_0 t + a)}$$

4.1-7

$$\left| H(f) \right|^2 = \left[1 + Q^2 \left(\frac{f}{f_0} - \frac{f_0}{f} \right)^2 \right]^{-1} = \frac{1}{2} \quad \Rightarrow \quad Q\left(\frac{f}{f_0} - \frac{f_0}{f} \right) = \pm 1$$

so $\frac{Q}{f_0} f^2 \pm f = Qf_0 = 0 \quad \Rightarrow \quad f_\ell, f_u = \frac{f_0}{2Q}\left(\sqrt{1+4Q^2} \pm 1 \right)$

$$B = f_\ell - f_u = \frac{f_0}{2Q}\left(\sqrt{1+4Q^2} + 1 \right) - \frac{f_0}{2Q}\left(\sqrt{1+4Q^2} - 1 \right) = \frac{f_0}{Q}$$

4.1-8

$$\frac{f}{f_0} = 1 + d, \quad \frac{f_0}{f} = \left(1+d\right)^{-1} \approx 1 - d, \text{ so}$$

$$H(f) \approx \left\{1 + jQ\left[1+d - \left(1-d\right)\right]\right\}^{-1} = \frac{1}{1 + j2Qd}$$

But $d = \frac{f}{f_0} - 1 = \frac{f - f_0}{f_0}$ so

$$H(f) \approx \frac{1}{1 + j2Q\left(f - f_0\right)/f_0} \quad \text{for} \quad \begin{array}{c} f = f_0\left(1+d\right) > 0 \\ \left|f - f_0\right| = \left|d\right| f_0 \ll f_0 \end{array}$$

4.1-9

$$H(f) \approx \frac{1}{\sqrt{1 + \left(f - f_c + b\right)^2 / b^2}\sqrt{1 + \left(f - f_c - b\right)^2 / b^2}} \quad \text{stagger-tuned}$$

$$\approx \frac{1}{\sqrt{1 + \left(f - f_c\right)^2 / 2b^2}} \quad \text{single tuned}$$

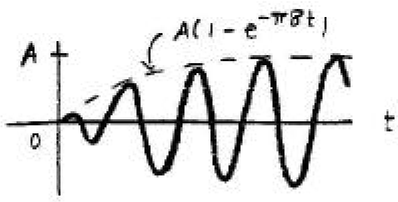
4.1-10

$$H_{\ell p}(f) = \frac{1}{1 + j2f/B} = \frac{pB}{pB + j2pf} \quad \Rightarrow \quad h_{\ell p}(t) = pBe^{-pBt}u(t)$$

$$x_{bp}(t) = 2\mathrm{Re}\left[\frac{A}{2}u(t)e^{jw_c t}\right] \quad \Rightarrow \quad x_{\ell p}(t) = \frac{A}{2}u(t)$$

$$y_{\ell p}(t) = h_{\ell p} * x_{\ell p}(t) = \frac{pBA}{2}\int_0^t e^{-pB(t-l)}dl = \frac{A}{2}\left(1 - e^{-pBt}\right)u(t)$$

$$y_{bp}(t) = 2\mathrm{Re}\left[y_{\ell p}(t)e^{jw_c t}\right] = A\left(1 - e^{-pBt}\right)\cos w_c t \; u(t)$$



4.1-11

$$H_{\ell p}(f)=\Pi\left(\frac{f}{B}\right)e^{-j(w+w_c)t_d} \quad \Rightarrow \quad h_{\ell p}(t)=Be^{-jw_c t_d}\ \text{sinc}\,B(t-t_d)$$

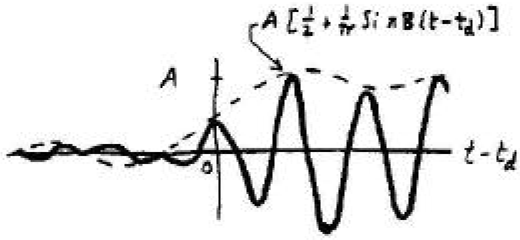
$$x_{bp}(t)=2\,\text{Re}\left[\frac{A}{2}u(t)e^{jw_c t}\right] \quad \Rightarrow \quad x_{\ell p}(t)=\frac{A}{2}u(t)$$

$$y_{\ell p}(t)=h_{\ell p}*x_{\ell p}(t)=\frac{BA}{2}e^{-jw_c t_d}\int_{-\infty}^{t}\text{sinc}\,B(l-t_d)\,dl$$

$$=\frac{A}{2}e^{-jw_c t_d}\left[\int_{-\infty}^{0}\text{sinc}\,m\,dm+\int_{0}^{B(t-t_d)}\text{sinc}\,m\,dm\right]$$

$$=\frac{A}{2}e^{-jw_c t_d}\left[\frac{1}{2}+\frac{1}{p}\text{Si}\,p\,B(t-t_d)\right]$$

$$y_{bp}(t)=2\,\text{Re}\left[y_{\ell p}(t)e^{jw_c t}\right]=A\left[\frac{1}{2}+\frac{1}{p}\text{Si}\,p\,B(t-t_d)\right]\cos w_c(t-t_d)$$



4.1-12

$$x_{\ell p}(t)=2e^{ja}u(t)e^{\pm jw_0 t} \quad \Rightarrow \quad X_{\ell p}(f)=e^{ja}\left[\frac{1}{jp(f\mp f_0)}+d(f\mp f_0)\right]$$

$H_{\ell p}(f)=\Pi\left(\frac{f}{B}\right)$ with $\frac{\text{B}}{2}\ll f_0$ so $d(f\mp f_0)$ falls outside passband.

Thus, $Y_{\ell p}(f)=\frac{e^{ja}}{jp(f\mp f_0)}\Pi\left(\frac{f}{B}\right)\approx\frac{e^{a}}{\mp jp\,f_0}\Pi\left(\frac{f}{B}\right)$ since $f_0\gg f$ for $|f|<\frac{B}{2}$

$$y_{\ell p}(t)\approx\pm j\left(\frac{e^{ja}}{p\,f_0}\right)B\,\text{sinc}\,Bt$$

$$y_{bp}(t)\approx\frac{2B}{p\,f_0}\text{sinc}\,Bt\ \text{Re}\left[\pm je^{ja}e^{jw_c t}\right]=\mp\frac{2B}{p\,f_0}\text{sinc}\,Bt\sin(w_c t+a)$$

4.1-13

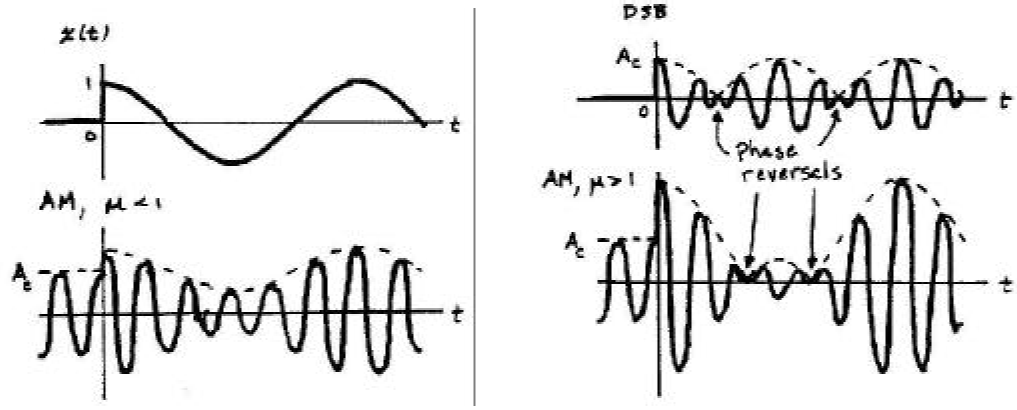
$$H_{\ell p}(f) = e^{jf^2/\beta}\Pi\left(\frac{f}{B}\right) \qquad X_{\ell p}(f) = \frac{1}{2}Z(f) = 0 \quad |f| \le W \le \frac{B}{2}$$

$$Y_{\ell p}(f) = e^{jf^2/\beta}\frac{1}{2}Z(f) \approx \frac{1}{2}\left[1 + j\frac{f^2}{\beta}\right]Z(f) \quad \text{since } \frac{f^2}{\beta} \ll 1 \text{ for } |f| \le W$$

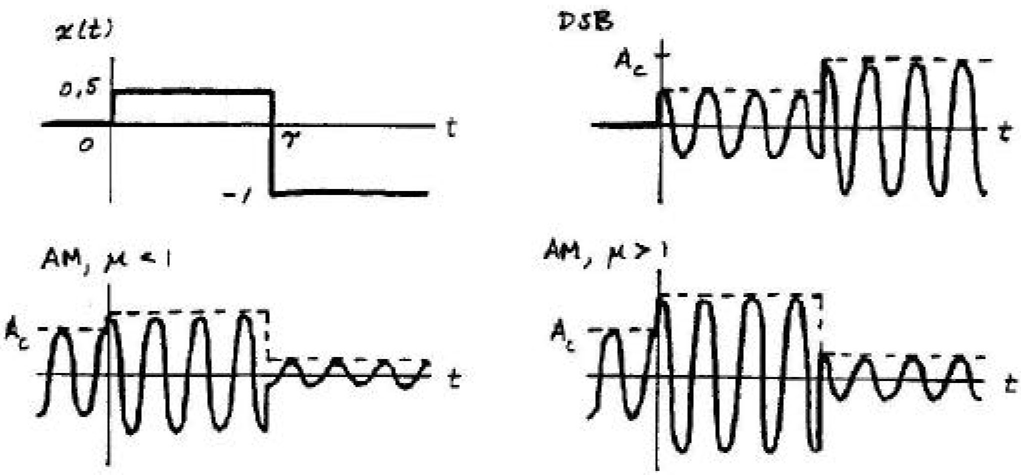
$$\approx \frac{1}{2}\left[Z(f) - \frac{j}{4\pi^2\beta}(j2\pi f)^2 Z(f)\right] \Rightarrow y_{\ell p}(t) \approx \frac{1}{2}\left[z(t) - \frac{j}{4\pi^2\beta}\frac{d^2}{dt^2}z(t)\right]$$

Thus, $y_{bp}(t) \approx z(t)\cos\omega_c t - \frac{1}{4\pi^2\beta}\left[\frac{d^2}{dt^2}z(t)\right]\sin\omega_c t$

4.2-1



4.2-2



4.2-3

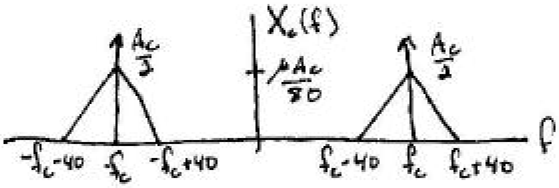
AM: $B_T = 400\text{Hz} \qquad S_T = \frac{1}{2}A_c^2\left(1 + \mu^2 S_x\right) = \frac{100}{2}\left(1 + 0.6^2\right) = 68\text{W}$

DSB: $B_T = 400\text{Hz} \qquad S_T = \frac{1}{2}A_c^2 S_x = \frac{100}{2} = 50\text{W}$

4.2-4

$$\text{sinc}^2\, 40t \quad \leftrightarrow \quad \frac{1}{40}\Lambda\left(\frac{f}{40}\right)$$

$$B_T = 2W = 80 \text{ Hz}$$



4.2-5

$$A_{\max}^2 = \left(2A_c\right)^2 = 32\text{kW} \quad \Rightarrow \quad \text{A}_\text{c}^2 = 8\text{kW}$$

$$m = 1, \quad S_x = \frac{1}{2} \quad \Rightarrow \quad S_T = \frac{1}{2}A_c^2\left(1 + m^2 S_x\right) = 6\text{kW}$$

4.2-6

$$S_x = \frac{1}{2}, \quad S_T = \frac{1}{2}A_c^2\left(1 + \frac{m^2}{2}\right) = 1\text{kW} \quad \Rightarrow \quad \text{A}_\text{c}^2 = \frac{4}{2+m^2}\text{kW}$$

$$\text{A}_{\max}^2 = \left(1+m\right)^2 A_c^2 = 4\frac{\left(1+m\right)^2}{2+m^2}\text{kW} \le 4\text{kW}$$

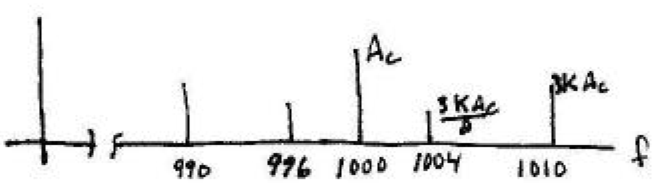
$$\text{so } 1 + 2m + m^2 \le 2 + m^2 \quad \Rightarrow \quad m \le 0.5$$

4.2-7

$$\left|x\right|_{\max} = x(0) = 3K(1+2) \le 1 \quad \Rightarrow \quad K \le 1/9$$

$$P_{sb} = \frac{1}{2}\left(\frac{3}{2}KA_c\right)^2 + \frac{1}{2}\left(3KA_c\right)^2 = \frac{45}{8}K^2A_c^2 = \frac{45}{4}K^2P_c$$

$$\frac{2P_{sb}}{S_T} = \frac{\frac{45}{2}K^2P_c}{P_c + \frac{45}{2}K^2P_c} = \frac{45K^2}{2+45K^2} \le \frac{45}{207} \approx 22\%$$

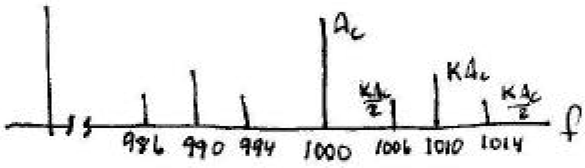


4.2-8

$x(t) = 2K\cos 20\pi t + K\cos 12\pi t + K\cos 28\pi t \qquad |x|_{max} = x(0) = K(2+1+1) \le 1 \Rightarrow K \le 1/4$

$$P_{sb} = \frac{1}{2}(KA_c)^2 + 2\times\frac{1}{2}\left(\frac{1}{2}KA_c\right)^2 = \frac{3}{4}K^2A_c^2 = \frac{3}{2}K^2P_c$$

$$\frac{2P_{sb}}{S_T} = \frac{3K^2P_c}{P_c + 3K^2P_c} = \le \frac{3}{19} \approx 16\%$$



4.2-9

$x(t) = 4\sin\frac{\pi}{2}t = 4\sin 2\pi\frac{1}{4}t \qquad B_T = 2W = \frac{1}{2}$ kHz

$0.01 < \frac{B_T}{f_c} < 0.1 \quad \Rightarrow \quad 10B_T < f_c < 100B_T$

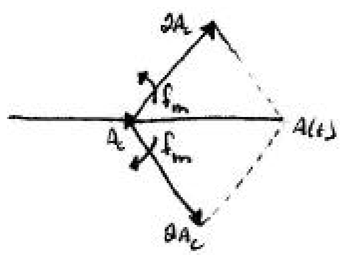
5 kHz $< f_c <$ 50 kHz

4.2-10

$x_c(t) = A_c\left[1 + x(t)\right]\cos\omega_c t \quad \Rightarrow \quad A(t) = A_c\left[1 + x(t)\right] \ge 0$ for no phase reversals to occur

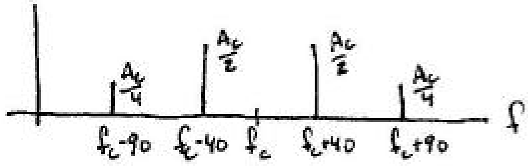
Since $\left. x(t)\right|_{min} = -4$ there is no value of A_c that can keep $A(t)$ from going negative.

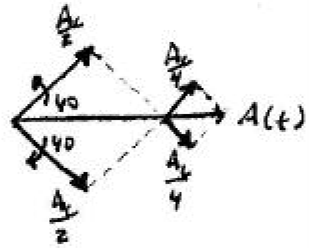
Therefore phase reversals will occur whenever $x(t)$ goes negative.



4.2-11

$$x_c(t) = A_c\left[\cos 2\pi\, 40t + \frac{1}{2}\cos 2\pi\, 90t\right]\cos 2\pi f_c t$$





4.3-1

(a) $v_{out} = a_1 x(t) + a_2 x^2(t) + a_2 \cos^2 w_c t + \underbrace{a_1\left[1 + \frac{2a_2}{a_1} x(t)\right]\cos w_c t}_{\text{desired term}}$

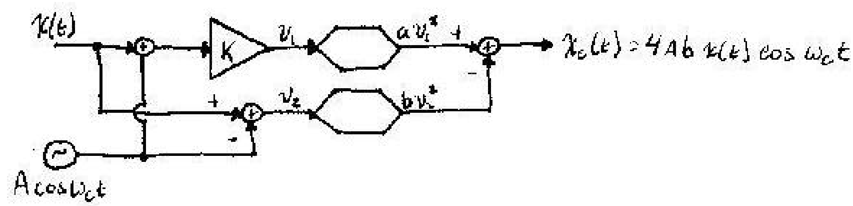
Select a filter centered at f_c=10 kHz with a bandwidth of $2W = 2\times 120 = 240$ Hz.

(b) $a_1\left[1+\frac{2a_2}{a_1}x(t)\right]\cos w_c t = A_c\left[1+mx(t)\right]\cos w_c t = 10\left[1+\frac{1}{2}x(t)\right]\cos w_c t$

$\Rightarrow \quad a_1 = 10 \qquad \frac{2a_2}{a_1} = \frac{1}{2} \quad \Rightarrow \quad a_2 = \frac{5}{2}$

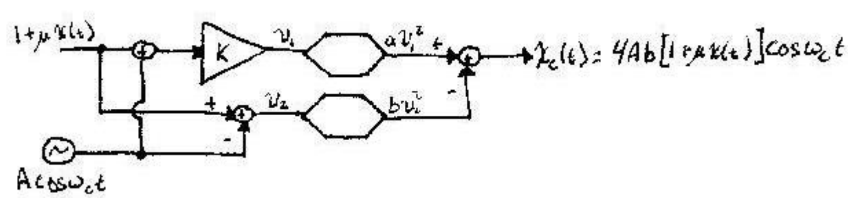
4.3-2

$$x_c(t) = aK^2\left(x + A\cos w_c t\right)^2 - b\left(x - A\cos w_c t\right)^2$$
$$= \left(aK^2 - b\right)\left(x^2 + A^2\cos^2 w_c t\right) + 2A\left(aK^2 + b\right)x\cos w_c t$$
$$= 4Abx(t)\cos w_c t \qquad \text{if } K = \sqrt{\frac{b}{a}}$$



4.3-3

$$x_c(t) = aK^2\left(v + A\cos w_c t\right)^2 - b\left(v - A\cos w_c t\right)^2$$
$$= \left(aK^2 - b\right)\left(v^2 + A^2\cos^2 w_c t\right) + 2A\left(aK^2 + b\right)v\cos w_c t$$
$$= 4Ab\left[1 + mx(t)\right]\cos w_c t \qquad \text{if } K = \sqrt{\frac{b}{a}} \quad \text{and} \quad v(t) = 1 + mx(t)$$

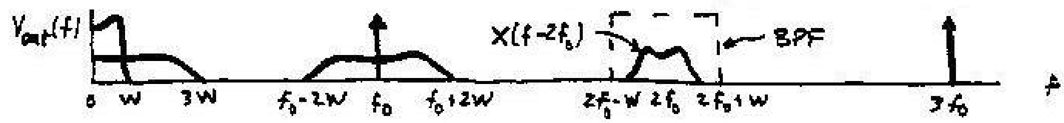


4.3-4

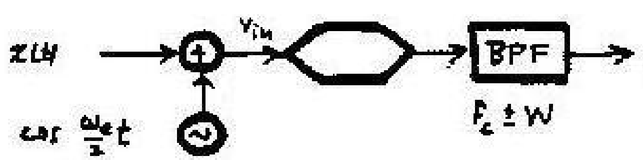
Take $v_{in} = x + \cos w_0 t$ so

$$v_{out} = a_1\left(x + \cos w_0 t\right) + a_3\left(x^3 + 3x^2 \cos w_0 t + 3x\cos^3 w_0 t + \cos^3 w_0 t\right)$$

$$= \left(a_1 + \frac{3}{2}a_3\right)x + a_3 x^3 + \left(a_1 + \frac{3}{4}a_3 + 3a_3 x^2\right)\cos w_0 t + \frac{3}{2}a_3 x \cos 2w_0 t + \frac{1}{4}a_3 \cos 3w_0 t$$



Take $f_c = 2f_0$ where $f_0 + 2W < 2f_0 - W$ so $f_c > 6W$

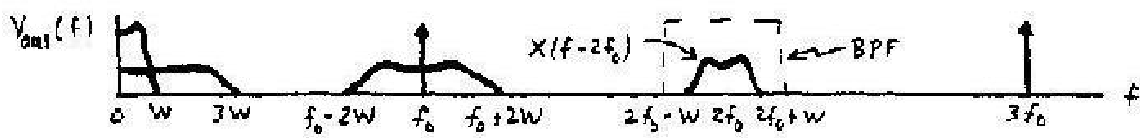


4.3-5

Take $v_{in} = y + \cos w_0 t,$ where $y = Kx(t)$, so

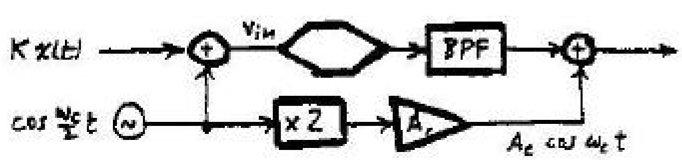
$$v_{out} = a_1\left(y + \cos w_0 t\right) + a_3\left(y^3 + 3y^2 \cos w_0 t + 3y\cos^2 w_0 t + \cos^3 w_0 t\right)$$

$$= \left(a_1 + \frac{3}{2}a_3\right)y + a_3 y^3 + \left(a_1 + \frac{3}{4}a_3 + 3a_3 y^2\right)\cos w_0 t + \frac{3}{2}a_3 y \cos 2w_0 t + \frac{1}{4}a_3 \cos 3w_0 t$$



Take $f_c = 2f_0$ where $f_0 + 2W < 2f_0 - W$ so $f_c > 6W$

$$x_c(t) = \left[\frac{3}{2}a_3 K x(t) + A_c\right]\cos w_c t = A_c\left[1 + \frac{3a_3 K}{2A_c}x(t)\right]\cos w_c t$$



4.3-6

Let $v_{out_+} = a_1\left(A_c \cos w_c t + \frac{1}{2}x\right) + a_2\left(A_c \cos w_c t + \frac{1}{2}x\right)^2 + a_3\left(A_c \cos w_c t + \frac{1}{2}x\right)^3$

$v_{out_-} = b_1\left(A_c \cos w_c t - \frac{1}{2}x\right) + b_2\left(A_c \cos w_c t - \frac{1}{2}x\right)^2 + b_3\left(A_c \cos w_c t - \frac{1}{2}x\right)^3$

Expanding using $\cos^2 w_c t = \frac{1}{2} + \frac{1}{2}\cos 2w_c t, \quad \cos^3 w_c t = \frac{3}{4}\cos w_c t + \frac{1}{4}\cos 3w_c t$

Since BPFs reject components outside $f_c - W < |f| < f_c + W$,

$x_c(t) = v_{out_+}\big|_{BPF} - v_{out_-}\big|_{BPF}$

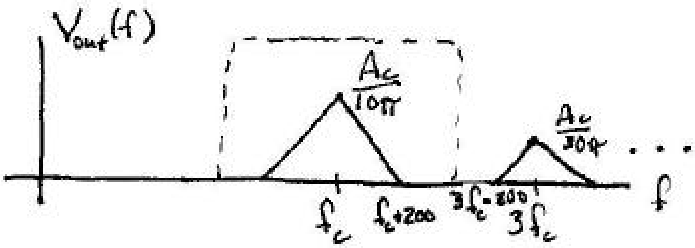
$$= \left(a_1 + \frac{3}{4}a_3 - b_1 - \frac{3}{4}b_3\right)\cos w_c t + 2\left(a_2 + b_2\right)x(t)\cos w_c t + 3\left(a_3 - b_3\right)x^2(t)\cos w_c t$$

so there's unsuppressed carrier and 2nd harmonic distortion

4.3-7

$x(t) = 20\,\mathrm{sinc}^2\, 400t \quad \leftrightarrow \quad X(f) = \frac{20}{400}\Lambda\left(\frac{f}{400}\right) = \frac{1}{20}\Lambda\left(\frac{f}{400}\right)$

$v_{out}(t) = \frac{4}{p}x(t)\cos w_c t - \frac{4}{3p}x(t)\cos 3w_c t + \frac{4}{5p}x(t)\cos 5w_c t - \cdots$



need $f_c + 200 < 3f_c - 200 \quad \Rightarrow \quad f_c > 100$ Hz

But f_c must meet fractional bandwidth requirements as well

so $400 < 0.1 f_c \quad \Rightarrow \quad f_c > 4000$ Hz which meets the earlier requirements as well.

4.4-1

$x_c(t) = 2\,\mathrm{Re}\left\{\frac{1}{4}A_c\left[x(t) \pm j\hat{x}(t)\right]e^{jw_c t}\right\}$

$$= \frac{A_c}{2}\mathrm{Re}\left\{\left[x(t)\cos w_c t \pm (-1)\hat{x}(t)\sin w_c t\right] + j\left[x(t)\sin w_c t \pm \hat{x}(t)\cos w_c t\right]\right\}$$

$$= \frac{A_c}{2}\left[x(t)\cos w_c t \mp \hat{x}(t)\sin w_c t\right]$$

4.4-2

$$x(t)\cos \omega_c t \leftrightarrow \frac{1}{2}X(f-f_c)+\frac{1}{2}X(f+f_c)$$

$$\sin \omega_c t = \frac{1}{j2}\left(e^{j\omega_c t}-e^{-j\omega_c t}\right) \quad \text{and} \quad \hat{X}(f)=(-j\,\mathrm{sgn}\, f)X(f) \ \text{ so}$$

$$\hat{x}(t)\sin \omega_c t \leftrightarrow -\frac{1}{2}\mathrm{sgn}(f-f_c)X(f-f_c)+\frac{1}{2}\mathrm{sgn}(f+f_c)X(f+f_c)$$

Thus, $X_c(f)=\dfrac{A_c}{4}\left\{\left[1\pm\mathrm{sgn}(f-f_c)\right]X(f-f_c)+\left[1\mp\mathrm{sgn}(f+f_c)\right]X(f+f_c)\right\}$

4.4-3

Upper signs for USSB, so

$$1+\mathrm{sgn}(f-f_c)=\begin{cases}2 & f>f_c\\ 0 & f<f_c\end{cases}, \quad 1-\mathrm{sgn}(f+f_c)=\begin{cases}0 & f>-f_c\\ 2 & f<-f_c\end{cases},$$

$$X_c(f)=\begin{cases}\dfrac{A_c}{2}X(f-f_c) & f>f_c\\ 0 & |f|<f_c\\ \dfrac{A_c}{2}X(f+f_c) & f<-f_c\end{cases}$$

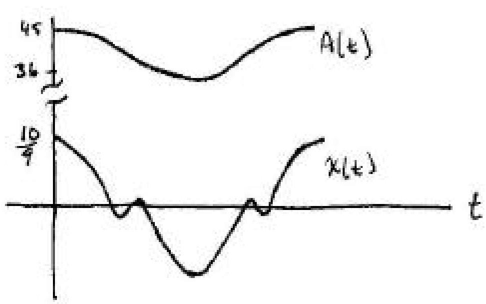
4.4-4

Let $\theta=\omega_m t$ so $\hat{x}(t)=\sin\theta+\frac{1}{9}\sin 3\theta$

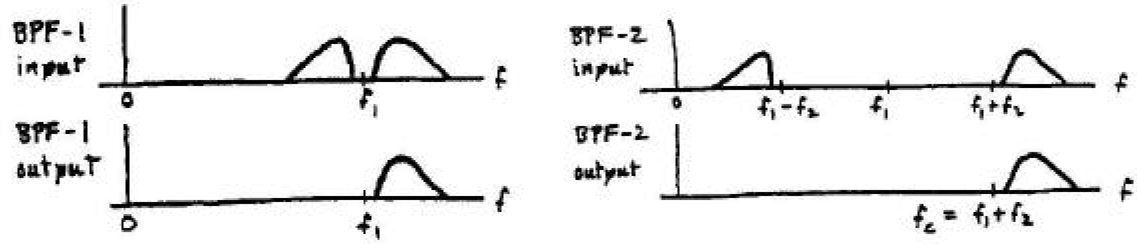
$$x^2+\hat{x}^2=\left(\cos\theta+\frac{1}{9}\cos 3\theta\right)^2+\left(\sin\theta+\frac{1}{9}\sin 3\theta\right)^2$$

$$=1+\frac{1}{81}+\frac{2}{9}\cos\theta\cos 3\theta+\frac{2}{9}\sin\theta\sin 3\theta=\frac{82}{81}+\frac{2}{9}\cos 2\theta$$

$$A(t)=\frac{1}{2}A_c\sqrt{x^2(t)+\hat{x}^2(t)}=\frac{1}{2}\times 81\times\frac{1}{9}\sqrt{82+18\cos 2\theta}=\frac{9}{2}\sqrt{82+18\cos 2\theta}$$



4.4-5



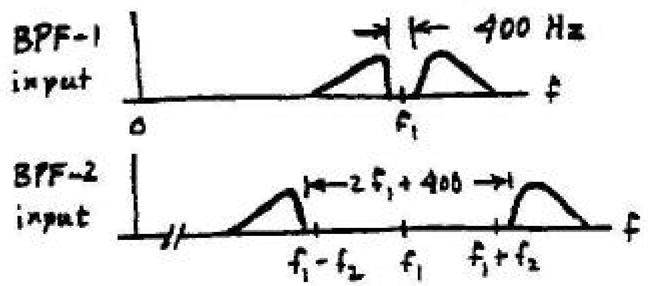
For LSSB, upper cutoffs of BPFs should be f_1 and f_2, respectively.

4.4-6

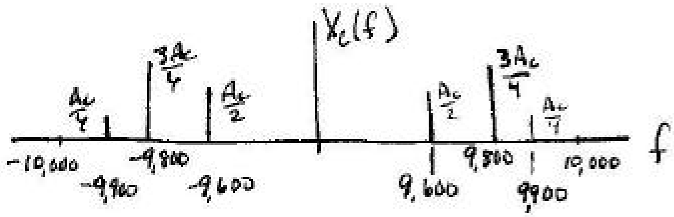
$2b = 400 \geq 0.01 f_1 \Rightarrow f_1 \leq 40\text{kHz}$

$0.01 f_2 \leq 2 f_1 + 400 \leq 80.4\text{kHz}$

$f_2 \leq 8.04\text{MHz}$ and $f_c = f_1 + f_2 \leq 8.08\text{MHz}$



4.4-7

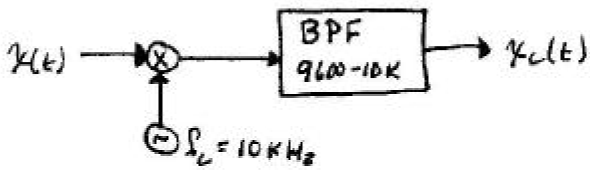


$$S_T = \frac{1}{4} A_c^2 S_x = \frac{1}{4} A_c^2 \left[\frac{1}{2}(1)^2 + \frac{1}{2}(3)^2 + \frac{1}{2}(2)^2 \right] = \frac{7}{4} A_c^2$$

or calculate directly from the line spectrum

$B_T = W = 400$ Hz

4.4-8



Check to make sure BPF meets requirements:

$$0.01 < \frac{B}{f_c} < 0.1 \quad \Rightarrow \quad \frac{B}{f_c} = \frac{10{,}000 - 9600}{10^4} \quad \Rightarrow \quad 0.01 < 0.04 < 0.1 \checkmark$$

Also $f_c < 200\beta = 200 \times 100 = 20\ \text{kHz}\ \checkmark$

Note that a LPF at 10 kHz would have violated the fractional bandwidth requirements so a BPF must be used.

4.4-9

$$\cos(\omega_c t - 90° + \delta) = \sin(\omega_c t + \delta) = \cos\delta \sin\omega_c t + \sin\delta \cos\omega_c t \approx \sin\omega_c t + \delta\cos\omega_c t$$

Thus, $x_c(t) \approx \frac{A_c}{2}\left\{\left[x(t) \mp \delta\,\hat{x}(t)\right]\cos\omega_c t \mp \hat{x}(t)\sin\omega_c t\right\}$

$$A(t) \approx \frac{A}{2}\left[x^2(t) + \hat{x}^2(t) \mp 2\delta\,\hat{x}(t)x(t)\right]^{1/2}$$

4.4-10

$$(1-\varepsilon)\cos(\omega_m t - 90° + \delta) = (1-\varepsilon)\left[\cos\delta \sin\omega_m t + \sin\delta\cos\omega_m t\right] \approx (1-\varepsilon)\sin\omega_m t + \delta\cos\omega_m t$$

$$x_c(t) \approx \frac{A_c}{2}\left[\cos\omega_m \cos\omega_c t - (1-\varepsilon)\sin\omega_m t \sin\omega_c t - \delta\cos\omega_m t \sin\omega_c t\right]$$

$$= \frac{A_c}{4}\left\{2\cos(\omega_c + \omega_m)t + \varepsilon\left[\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t\right]\right.$$
$$\left. -\delta\left[\sin(\omega_c - \omega_m)t + \sin(\omega_c + \omega_m)t\right]\right\}$$

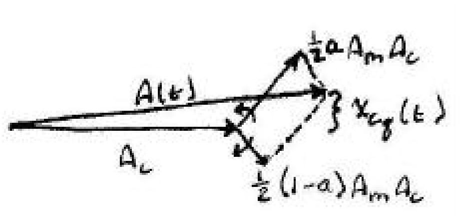
But $\varepsilon\cos\theta - \delta\sin\theta = \sqrt{\varepsilon^2 + \delta^2}\cos\left(\theta + \arctan(\delta/\varepsilon)\right)$

$$(2-\varepsilon)\cos\theta - \delta\sin\theta = \sqrt{(2-\varepsilon)^2 + \delta^2}\cos\left(\theta + \arctan\frac{\delta}{2-\varepsilon}\right)$$

$$\approx 2\sqrt{1-\varepsilon/2}\cos(\theta + \delta/2)$$

Thus $x_c(t) \approx \frac{A_c}{2}\sqrt{1-\varepsilon/2}\cos\left[(\omega_c + \omega_m)t + \delta/2\right] + \frac{A_c}{4}\sqrt{\varepsilon^2 + \delta^2}\cos\left[(\omega_c - \omega_m)t + \arctan\frac{\delta}{\varepsilon}\right]$

4.4-11



The easiest way to find the quadrature component is graphically from the phasor diagram.

$$x_{cq}(t)=\frac{1}{2}aA_mA_c\sin 2\pi f_mt-\frac{1}{2}(1-a)A_mA_c\sin 2\pi f_mt=\left(a-\frac{1}{2}\right)A_mA_c\sin 2\pi f_mt$$

4.4-12

$$x_c(t)=\frac{A_c}{2}\left[(0.5+a)\cos(\omega_c+\omega_m)t+(0.5-a)\cos(\omega_c-\omega_m)t\right]$$

$$=\frac{A_c}{2}\left\{\frac{1}{2}\left[\cos(\omega_c+\omega_m)t+\cos(\omega_c-\omega_m)t\right]+2a\frac{1}{2}\left[\cos(\omega_c+\omega_m)t-\cos(\omega_c-\omega_m)t\right]\right\}$$

$$=\frac{A_c}{2}\left[\cos\omega_mt\cos\omega_ct-2a\sin\omega_mt\sin\omega_ct\right]$$

$$a=0\quad\Rightarrow\quad x_c(t)=\frac{A_c}{2}\cos\omega_mt\cos\omega_ct\quad \text{DSB}$$

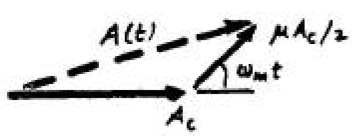
$$a=\pm 0.5\quad\Rightarrow\quad x_c(t)=\frac{A_c}{2}\left[\cos\omega_mt\cos\omega_ct\mp\sin\omega_mt\sin\omega_ct\right]=\frac{A_c}{2}\cos(\omega_c\pm\omega_m)t\quad \text{SSB}$$

4.4-13

$$x_c(t)=A_c\left[\cos\omega_ct+\frac{m}{2}\cos(\omega_c+\omega_m)t\right]$$

$$A(t)=A_c\left[\left(1+\frac{m}{2}\cos\omega_mt\right)^2+\left(\frac{m}{2}\sin\omega_mt\right)^2\right]^{1/2}$$

$$=A_c\left[1+m\cos\omega_mt+\frac{m^2}{4}\right]^{1/2}$$

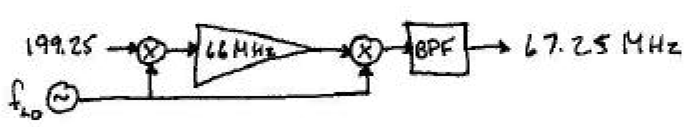


4.5-1

$\left|f_1 \pm 199.25\right| = 66 \text{ MHz} \quad \Rightarrow \quad f_1 = 265.25 \quad \text{or} \quad 133.25$

$\left|f_2 \pm 66\right| = 67.25 \text{ MHz} \quad \Rightarrow \quad f_2 = 133.25 \quad \text{or} \quad 1.25$

Take $f_{LO} = 133.25$ MHz

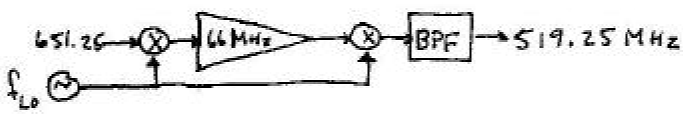


4.5-2

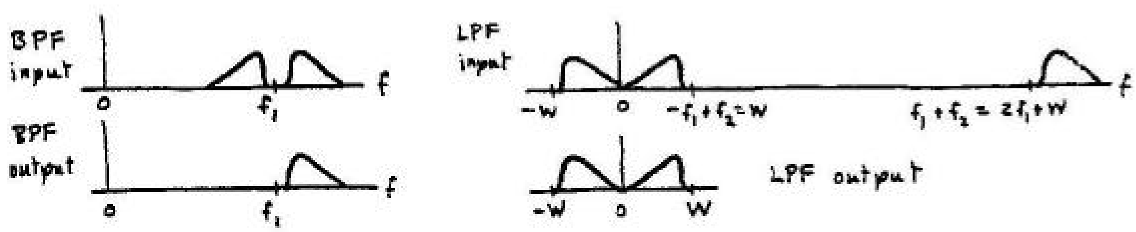
$\left|f_1 \pm 651.25\right| = 66 \text{ MHz} \quad \Rightarrow \quad f_1 = 717.25 \quad \text{or} \quad 585.25$

$\left|f_2 \pm 66\right| = 519.25 \text{ MHz} \quad \Rightarrow \quad f_2 = 585.25 \quad \text{or} \quad 453.25$

Take $f_{LO} = 585.25$ MHz



4.5-3



Output is unintelligible because spectrum is reversed, so low-frequency components become high frequencies, and vice versa.
Output signal can be unscrambled by passing it through a second, identical scrambler which again reverses the spectrum.

4.5-4

$$\text{LPF input } = \left[\left(K_c + K_m x\right)\cos w_c t - K_m x_q \sin w_c t\right]\cos\left(w_c t + f\right)$$

$$= \left(K_c + K_m x\right)\cos f + \left(K_c + K_m x\right)\cos\left(2w_c t + f\right) + K_m x_q \sin f - K_m x_q \sin\left(2w_c t + f\right)$$

$$y_D(t) = \left[K_c + K_m x(t)\right]\cos f + K_m x_q(t)\sin f$$

Modulation	K_c	K_m	$x_q(t)$	$y_D(t)$
AM	A_c	mA_c	0	$A_c\left[1 + mx(t)\right]\cos f$
DSB	0	A_c	0	$A_c x(t)\cos f$
SSB	0	$A_c/2$	$\mp\hat{x}(t)$	$A_c/2\left[x(t)\cos f \mp \hat{x}(t)\sin f\right]$
VSB	0	$A_c/2$	$\hat{x}(t) + x_b(t)$	$A_c/2\left\{x(t)\cos f + \left[\hat{x}(t) + x_b(t)\right]\sin f\right\}$

4.5-5

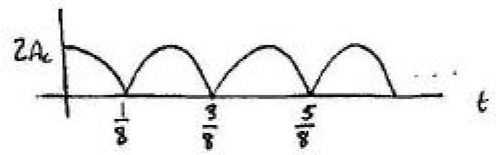
From equation for $x_c(t)$ we see that

$a = \frac{1}{2}$ will produce standard AM with no distortion at the output.

$$\left.\begin{array}{l} a = 1 \text{ will produce USSB + C} \\ a = 0 \text{ will produce LSSB + C} \end{array}\right\} \text{maximum distortion from envelope detector.}$$

4.5-6

Envelope detector follows the shape of the positive amplitude portions of $x_c(t)$.



Envelope detector output is proportional to $|x(t)|$.

4.5-7

A square wave, like any other periodic signal, can be written as a Fourier series of harmonically spaced sinusoids. If the square wave has even symmetry and a fundamental of f_c, it will have terms like $a_1 \cos w_c t + a_3 \cos w_3 t + a_5 \cos w_c t + \cdots$. This will cause signals at $f_c, 3f_c, 5f_c \ldots$ to be shifted to the origin. If f_c is large enough, and our desired signal can be isolated, our synchronous detector will work fine. Otherwise there may be noise or intelligible crosstalk. Note that any phase shift will cause amplitude distortion. For any periodic signal in general, as long as the Fourier series has a term at f_c and our signal can be isolated, this can also serve as our local oscillator signal.

4.5-8

Between peaks $v(t) \approx A_c\left[1+\cos 2\pi W t_1\right] e^{-(t-t_1)/\tau}, \quad t_1 < t < t_1 + 1/f_c$

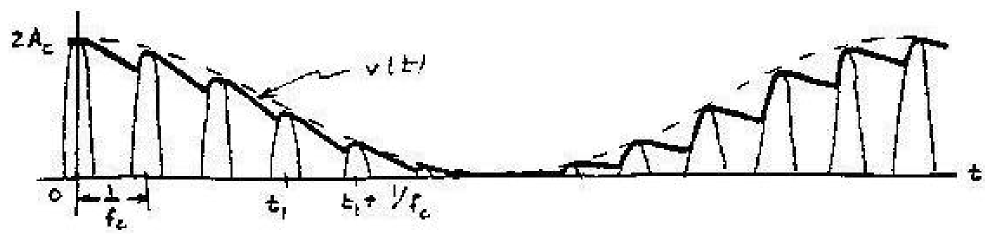
Maximum negative envelope slope occurs at $t_1 = \dfrac{1}{4W}$ and we want

$$v\left(t_1+\frac{1}{f_c}\right) \approx A_c e^{-1/\tau f_c} < A_c\left[1+\cos 2\pi W\left(t_1+\frac{1}{f_c}\right)\right] = A_c\left(1-\sin\frac{2\pi W}{f_c}\right)$$

so $1-\dfrac{1}{\tau f_c} < 1-\dfrac{2\pi W}{f_c}$ if $\tau \,\square\, \dfrac{1}{f_c}$ and $f_c \,\square\, W$

We also want $\tau \,\square\, \dfrac{1}{f_c}$ for linear decay between peaks.

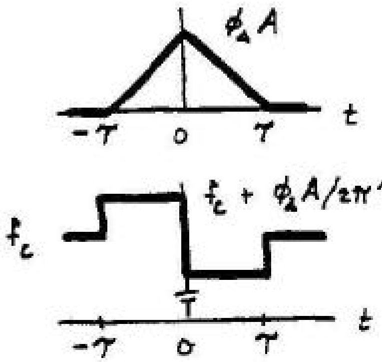
Thus $2\pi W \le R_1C_1 \,\square\, f_c$ and $f_c/W \ge 2\pi \times 10 \approx 60$



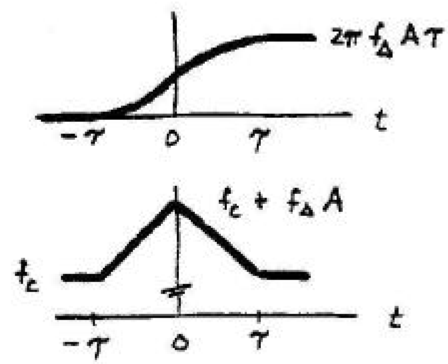
Chapter 5

5.1-1

PM

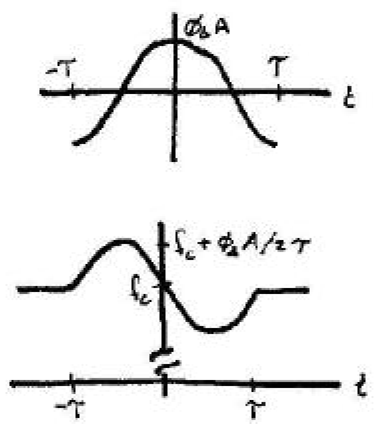


FM

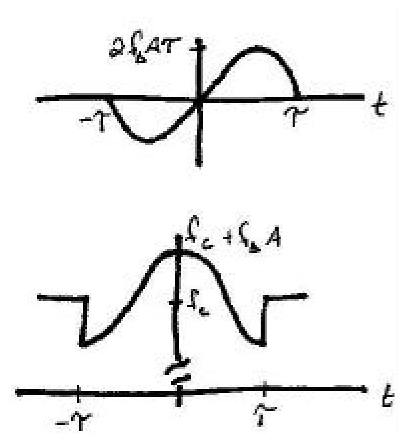


5.1-2

PM



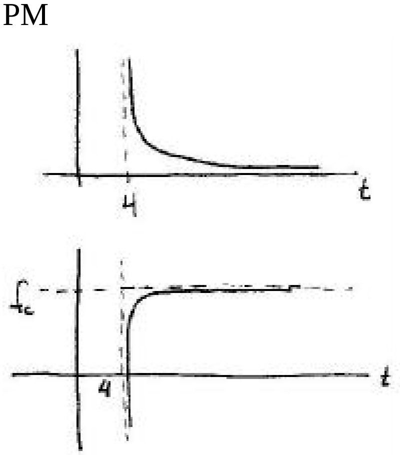
FM



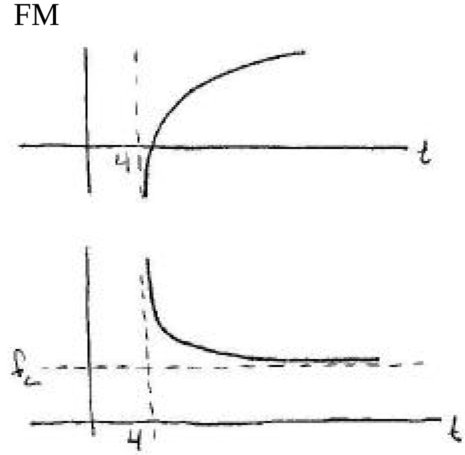
5.1-3

$$\frac{dx(t)}{dt} = -4A\frac{t^2+16}{\left(t^2-16\right)^2} \quad t>4, \qquad \int^t x(l)\,dl = \frac{4A}{2}\log\left(t^2-16\right) \quad t>4$$

PM



FM



5.1-4

$f(t) = a + bt \quad \text{for} \quad 0 < t < T$

$$f(0) = a = f_1, \quad f(T) = a + bT = f_2 \quad \Rightarrow \quad b = \frac{f_2 - f_1}{T}$$

$$q_c(t) = 2p\int_0^t f(l)\,d\,l = 2p\int_0^t \left(f_1 + \frac{f_2 - f_1}{T}\,l \right) d\,l = 2p\left(f_1 t + \frac{f_2 - f_1}{T} t^2 \right)$$

5.1-5

Type	$f(t)$	$f(t)$	$f_{\max}$	$f_{\max}$
Phase-integral	$K\dfrac{dx(t)}{dt}$	$f_c + \dfrac{K}{2p}\dfrac{d^2x(t)}{dt^2}$	$K2p\,f_m$	$f_c + K2p\,f_m^2$
PM	$f_\Delta x(t)$	$f_c + \dfrac{f_\Delta}{2p}\dfrac{dx(t)}{dt}$	f_Δ	$f_c + f_\Delta f_m$
FM	$2p\,f_\Delta\int^t x(l)d\,l$	$f_c + f_\Delta x(t)$	$\dfrac{f_\Delta}{f_m}$	$f_c + f_\Delta$
Phase-accel.	$2p\,K\int^t\left[\int^m x(l)d\,l\right]dm$	$f_c + K\int^t x(l)d\,l$	$\dfrac{K}{2p\,f_m^2}$	$f_c + \dfrac{K}{2p\,f_m}$

5.1-6

$$x_c(t) = A_c\left[\cos\left(b\sin w_m t\right)\cos w_c t - \sin\left(b\sin w_m t\right)\sin w_c t\right]$$

$$= A_c\left[J_0(b)\cos w_c t + \sum_{n\,\text{even}} 2J_n(b)\cos nw_m t\cos w_c t - \sum_{n\,\text{odd}} 2J_n(b)\sin nw_m t\sin w_c t\right]$$

where $\cos nw_m t\cos w_c t = \dfrac{1}{2}\left[\cos\left(w_c - nw_m\right)t + \cos\left(w_c + nw_m\right)t\right]$

$$\sin nw_m t\sin w_c t = \frac{1}{2}\left[\cos\left(w_c - nw_m\right)t - \cos\left(w_c + nw_m\right)t\right]$$

so $x_c(t) = A_c J_0(b)\cos w_c t + \sum_{n\,\text{even}} J_n(b)\left[\cos\left(w_c + nw_m\right)t + \cos\left(w_c - nw_m\right)t\right]$

$$+ \sum_{n\,\text{odd}} J_n(b)\left[\cos\left(w_c + nw_m\right)t - \cos\left(w_c - nw_m\right)t\right]$$

5.1-7

$e^{jb\sin w_m t} = \sum_{-\infty}^{\infty} c_n e^{jnw_m t}$ with period $T_m = 2p/w_m$

so $c_n = \dfrac{1}{T_m}\int_{T_m} e^{jb\sin w_m t}e^{-jnw_m t}dt = \dfrac{1}{2p}\int_{-p}^{p} e^{j(b\sin l - nl)}d\,l = J_n(b)$

Thus, $\cos\left(b\sin w_m t\right) = \mathrm{Re}\left[e^{jb\sin w_m t}\right] = \mathrm{Re}\left[\sum_{n=-\infty}^{\infty} J_n(b)e^{jnw_m t}\right]$

(cont.)

$$= \sum_{n=-\infty}^{\infty} J_n(b)\cos nw_m t = J_0(b) + \sum_{n=1}^{\infty}\left[J_n(b) + J_{-n}(b)\right]\cos nw_m t$$

$$\sin(b\sin w_m t) = \mathrm{Im}\left[e^{jb\sin w_m t}\right] = \mathrm{Im}\left[\sum_{n=-\infty}^{\infty} J_n(b)e^{jnw_m t}\right]$$

$$= \sum_{n=-\infty}^{\infty} J_n(b)\sin nw_m t = 0 + \sum_{n=1}^{\infty}\left[J_n(b) - J_{-n}(b)\right]\sin nw_m t$$

But $J_{-n}(b) = (-1)^n J_n(b)$ so

$$J_n + J_{-n} = \begin{cases} 2J_n & n \text{ even} \\ 0 & n \text{ odd} \end{cases} \qquad J_n - J_{-n} = \begin{cases} 0 & n \text{ even} \\ 2J_n & n \text{ odd} \end{cases}$$

Hence, $\cos(b\sin w_m t) = J_0(b) + \sum_{n\text{ even}}^{\infty}\left[2J_n(b)\right]\cos nw_m t$

$$\sin(b\sin w_m t) = \sum_{n\text{ odd}}^{\infty}\left[2J_n(b)\right]\sin nw_m t$$

5.1-8

$b = f_\Delta A_m$ for PM, $b = A_m f_\Delta / f_m$ for FM

(a) Line spacing remains fixed, while line amplitudes change in the same way since b is proportional to A_m.

(b) Line spacing changes in the same way but FM line amplitudes also change while PM line amplitudes remain fixed.

(c) Line spacing changes in the same way but PM line amplitudes also change while FM line amplitudes remain fixed.

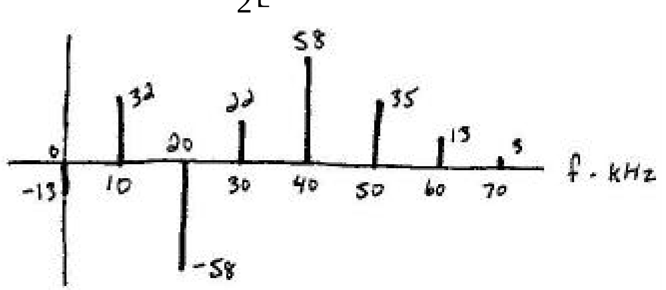
5.1-9

(a) $f(t) = f_c + f_\Delta x(t) = f_c + \dfrac{b\, f_m}{A_m}\cos w_m t$

Assuming $A_m = 1$ $\quad f(t) = 30 + 20\cos w_m t$ kHz

(b) "Folded" component at $|f_c - 4f_m| = 10$ kHz

$$S_T = (-13)^2 + \frac{1}{2}\left[(35-3)^2 + (-58)^2 + 22^2 + 58^2 + 35^2 + 13^2 + 3^2\right] = 4988.5 < \frac{100^2}{2}$$



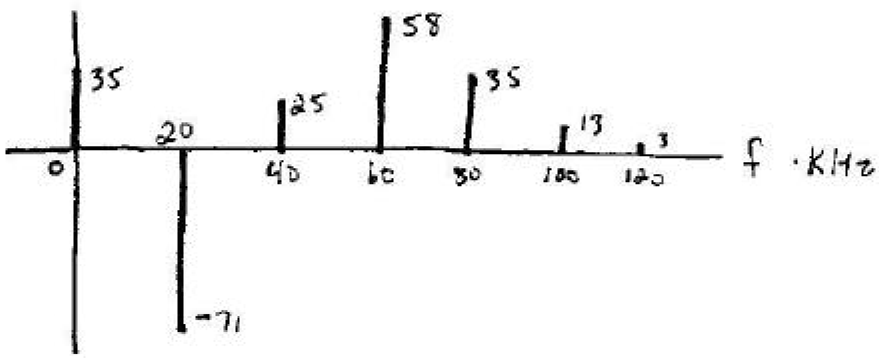
5.1-10

(a) $f(t) = f_c + f_\Delta x(t) = f_c + \frac{b\, f_m}{A_m} \cos w_m t$

Assuming $A_m = 1 \quad f(t) = 40 + 40 \cos w_m t$ kHz

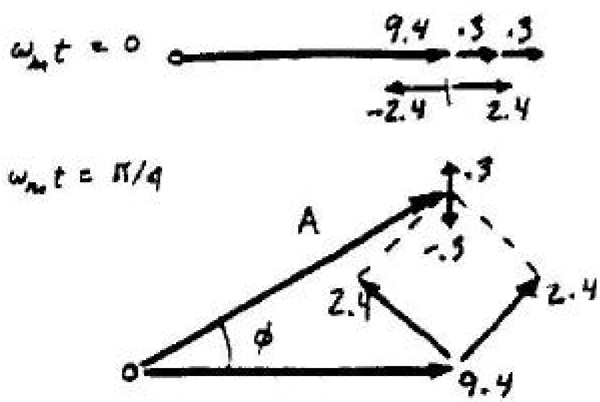
(b) "Folded" components at $|f_c - 3f_m| = 20$ kHz and $|f_c - 4f_m| = 40$ kHz

$$S_T = 35^2 + \frac{1}{2}\left[(-58-13)^2 + (22+3)^2 + 58^2 + 35^2 + 13^2 + 3^2\right] = 6441.5 > \frac{100^2}{2}$$



5.1-11

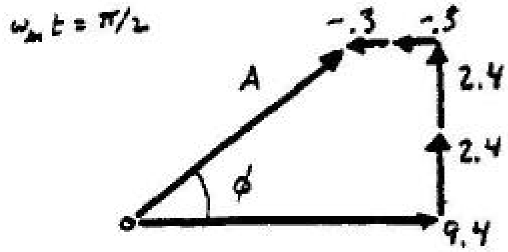
$w_m t = 0 \quad A = 9.4 + 2 \times .3 = 10 \quad f = 0$



$w_m t = \frac{p}{4} \quad A = \sqrt{9.4^2 + \left(2.4\sqrt{2}\right)^2} = 9.99 \quad f = \arctan \frac{2.4\sqrt{2}}{9.4} = 0.347$ rad

$0.5 \text{ rad} \times \sin \frac{p}{4} = 0.356$ rad

(cont.)

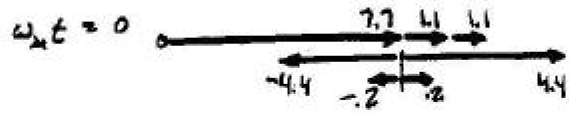


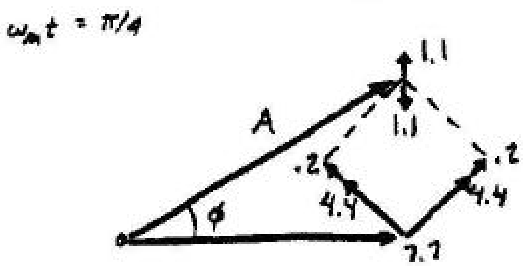
$w_m t = \frac{p}{2} \quad A = \sqrt{(9.4 - .6)^2 + (2 \times 2.4)^2} = 10.02 \quad f = \arctan \frac{2 \times 2.4}{9.4 - .6} = 0.499 \text{ rad}$

$0.5 \text{ rad} \times \sin \frac{p}{2} = 0.5 \text{ rad}$

5.1-12

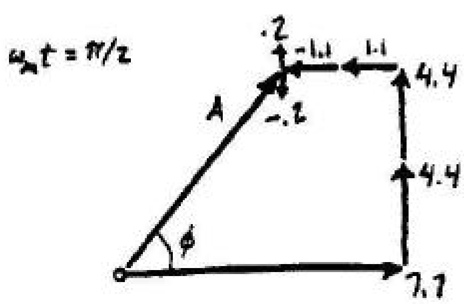
$w_m t = 0 \quad A = 7.7 + 2 \times 1.1 = 9.9 \quad f = 0$





$w_m t = \frac{p}{4} \quad A = \sqrt{7.7^2 + \left(4.6\sqrt{2}\right)^2} = 10.08 \quad f = \arctan \frac{4.6\sqrt{2}}{7.7} = 0.702 \text{ rad}$

$1 \text{ rad} \times \sin \frac{p}{4} = 0.707 \text{ rad}$



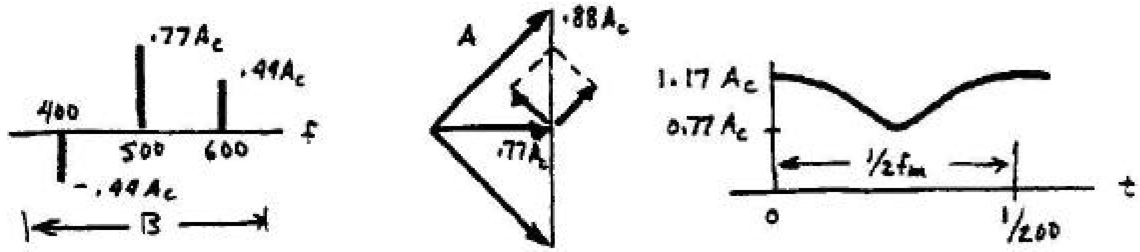
$w_m t = \frac{p}{2} \quad A = \sqrt{(7.7 - 2.2)^2 + (2 \times 4.4)^2} = 10.02 \quad f = \arctan \frac{2 \times 2.4}{7.7 - 2.2} = 1.012 \text{ rad}$

$1 \text{ rad} \times \sin \frac{p}{2} = 1 \text{ rad}$

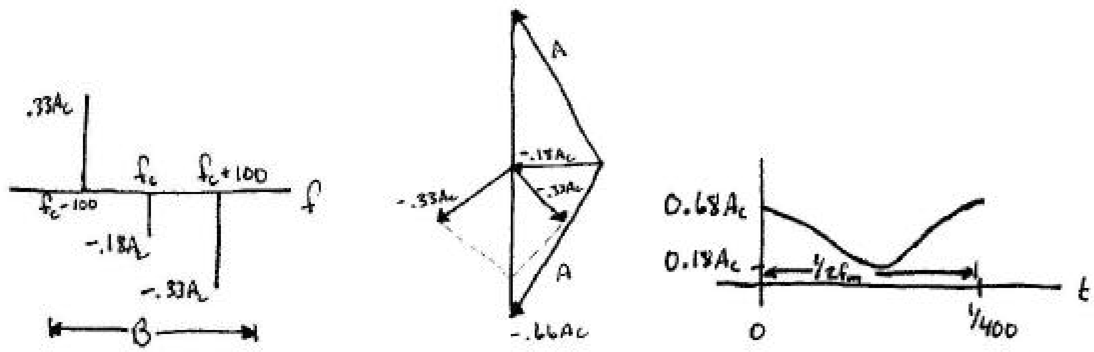
5.1-13

$A_{\min} = 0.77A_c$

$A_{\max} = \sqrt{.77^2 + .88^2}\,A_c = 1.17A_c$



5.1-14



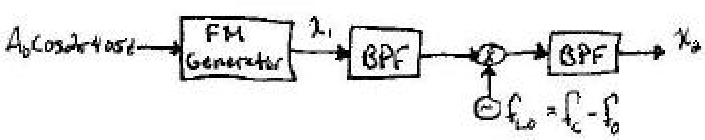
$A_{\min} = 0.18A_c$

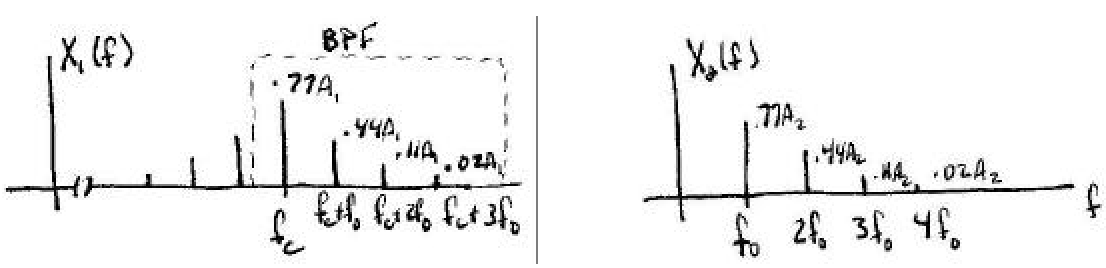
$A_{\max} = \sqrt{.18^2 + .66^2}\,A_c = .68A_c$

5.1-15

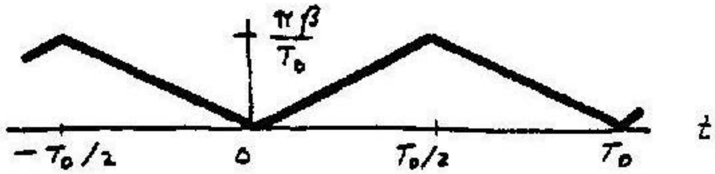
Want f_0 plus 3 harmonics $\Rightarrow$ select $\beta = 1.0$

Generate FM signal with $24{,}300 < f_c < 243{,}000$ to meet fractional bandwidth requirements since $B_T = 6 \times 405 = 2{,}430$ Hz. Apply BPF to select carrier plus 3 sidebands. Use frequency converter at $f_{LO} = f_c - f_0$.





5.1-16



(a) For $0 < t < T/2 \quad f(t) = 2p\, f_\Delta t + f(0) = 2p\, f_\Delta t$

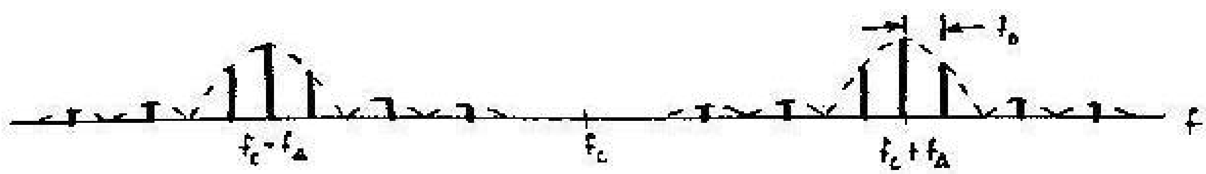
For $|t| < \dfrac{T}{2} \quad f(t) = 2p\, f_\Delta |t| = \dfrac{2pb}{T_0}|t|$

$$c_n = \frac{1}{T_0}\int_{-T_0/2}^{0} e^{-j2p(b+n)t/T_0}dt + \frac{1}{T_0}\int_{0}^{T_0/2} e^{j2p(b-n)t/T_0}dt$$

$$= \frac{\sin p(b+n)/2}{p(b+n)}e^{jp(b+n)/2} + \frac{\sin p(b-n)/2}{p(b-n)}e^{jp(b-n)/2}$$

$$= \frac{1}{2}e^{jpb}\left[\operatorname{sinc}\left(\frac{n+b}{2}\right)e^{jpn/2} + \operatorname{sinc}\left(\frac{n-b}{2}\right)e^{-jpn/2}\right]$$

(b) $b f_0 = f_\Delta \qquad |c_n| \approx \dfrac{1}{2}\left[\left|\operatorname{sinc}\left(\dfrac{n+b}{2}\right)\right| + \left|\operatorname{sinc}\left(\dfrac{n-b}{2}\right)\right|\right]$ when b □ 1



5.2-1

f_Δ, kHz	D	Eq.	B_T, kHz
0.1	0.01	5	(0.01+1)(30) = 30.3
0.5	0.03	5	(0.03+1)(30)= 31
1	0.07	5	(0.07+1)(30) = 32
5	0.33	4	(1.8)(30) = 54
10	0.67	4	(2.2)(30) = 66
50	3.33	6	(3.33+2)(30) ? 160
100	6.67	6	(6.67+2)(30) ? 260
500	33	5 or 6	(33+1)(30) = 1030

5.2-2

f_Δ, kHz	D	Eq.	B_T, kHz
0.1	0.02	5	(0.02+1)(10) = 10.2
0.5	0.1	5	(0.1+1)(10)= 11
1	0.2	5	(0.2+1)(10) = 12
5	1	4	(2.7)(10) = 27
10	2	4	(3.8)(10) = 38
50	10	6	(10+2)(10) = 120
100	20	5 or 6	(20+1)(10) = 210
500	100	5	(100+1)(10) = 1010

5.2-3

	W	D	$M(D)$		B_T
(a)	2 kHz	5	6	(lower curve)	2? 6? 2=24 kHz
(b)	3.2 kHz	3.1	3.1+2	(middle curve)	2? 5.1? 3.2? 33 kHz
(c)	20 kHz	0.5	2.5	(upper curve)	2? 2.5? 20=100kHz

5.2-4

FM:

$$D = \frac{f_\Delta}{W} = \frac{25}{5} = 5 \qquad B_T = 2(4+2)5 = 60 \text{ MHz} \qquad f_c > 10B_T = 600 \text{ MHz}$$

DSB:

$$B_T = 2W = 10 \text{ MHz} \qquad f_c > 10B_T = 100 \text{ MHz}$$

5.2-5

$$W = 15 \qquad f_c = 5\times10^{14}$$

$$0.01 < \frac{B_T}{f_c} < 0.1 \quad \Rightarrow \quad (0.01)\left(5\times10^{14}\right) < B_T < (0.1)\left(5\times10^{14}\right)$$

$$5\times10^{12} < B_T < 5\times10^{13}$$

$$B_T \approx 2(D+1)W = 2\left(f_\Delta + W\right) \approx 2f_\Delta \quad \Rightarrow \quad 2.5\times10^{12} < f_\Delta < 2.5\times10^{13}$$

5.2-6

For CD: $B_T = 2(5+2)15 = 210$ kHz

For talk show: $B_T = 2(5+2)5 = 70$ kHz

Since station must broadcast at CD bandwidth, the fraction of the available bandwidth used during the talk show is

$$\frac{B_{T_{used}}}{B_{T_{available}}} = \frac{B_{T_{talk}}}{B_{T_{CD}}} = \frac{70}{210}\times 100 = 33.3\%$$

5.2-7

$$D = f_\Delta = 30/10 = 3 \quad \Rightarrow \quad B_T = 2M(3)W \approx 100 \text{ kHz}$$

f_m, kHz	b	FM $B = 2M(b)\,f_m$	FM B/B_T	PM $B = 2M(b)\,f_m$	PM B/B_T
0.1	300	600×0.1	60%	1	1%
1.0	30	62×1.0	62%	10	10%
5.0	6	14×5.0	70%	50	50%

5.2-8

Take $x(t) = A_m \cos w_m t$, $\quad b = f_{\max}$, $\quad$ and $\quad B \approx 2(b+1)\,f_m$

	Phase-integral modulation	Phase-acceleration modulation
$f(t)$	$-2p\,KA_m f_m \sin w_m t$	$-\left(KA_m / 2p\, f_m^2\right)\cos w_m t$
b	$2p\,KA_m f_m$	$KA_m / 2p\, f_m^2$
B	$2\left(2p\,KA_m f_m^2 + f_m\right)$	$2\left(KA_m / 2p\, f_m + f_m\right)$
B_T	$\begin{cases} 4p\,KW^2 & 2p\,KW \square 1 \\ 2W & 2p\,KW \square 1 \end{cases}$	$\begin{cases} K/p\, f_{m_{\min}} & K/2p \square f_{m_{\min}} W \\ 2W & K/2p \square f_{m_{\min}} W \end{cases}$

In both cases, spectral lines are spaced by f_m and B increases with A_m. However, in phase-integral modulation, tones at $f_m \square W$ occupy much less than B_T if $2p\,KW \square 1$. In phase-acceleration modulation, mid-frequency tones may occupy the most bandwidth and will determine B_T when $K/2p \approx f_{m_{\min}} W$.

5.2-9

$$x_{\ell p}(t) = \frac{1}{2} A_c e^{jf(t)} \approx \frac{1}{2} A_c\left[1 + jf(t)\right], \quad f(t) = f_\Delta x(t)$$

$$Y_{\ell p}(f) = \frac{1}{1 + j2Qf/f_c}\frac{1}{2}A_c\left[d(f) + jf_\Delta X(f)\right] = \frac{1}{2}A_c\left[d(f) + jf_\Delta \frac{p\,f_c}{Q}\frac{1}{\frac{p\,f_c}{Q} + j2p\,f}X(f)\right]$$

$$y_{\ell p}(t) = \frac{1}{2}A_c\left[1 + jf_\Delta \frac{p\,f_c}{Q}\tilde{x}(t)\right] \text{ where } \tilde{x}(t) = \left[e^{-p\,f_c t/Q}u(t)\right] * x(t)$$

$$y_c(t) = A_c \operatorname{Re}\left\{e^{jw_c t} + jf_\Delta \frac{p\,f_c}{Q}\tilde{x}(t)\left[\cos w_c t + j\sin w_c t\right]\right\}$$

$$= A_c\left[\cos w_c t - f_\Delta \frac{p\,f_c}{Q}\tilde{x}(t)\sin w_c t\right]$$

so $f((t) = \arctan\left[f_\Delta \frac{p\,f_c}{Q}\tilde{x}(t)\right]$

5.2-10

$$Y_{\ell p}(f) = \left[K_0 - K_2 f^2\right] X_{\ell p}(f) = K_0 X_{\ell p}(f) - \frac{K_2}{(j2\pi f)^2}(j2\pi f)^2 X_{\ell p}(f)$$

$$y_{\ell p}(t) = K_0 x_{\ell p}(t) + \frac{K_2}{4\pi^2}\ddot{x}_{\ell p}(t)$$

where $x_{\ell p}(t) = \frac{1}{2}A_c e^{j\phi(t)}$ and $\ddot{x}(t) = \frac{j}{2}A_c\left[\ddot{\phi}(t)e^{j\phi(t)} + j\dot{\phi}^2(t)e^{j\phi(t)}\right]$

with $\dot{\phi}(t) = 2\pi f_\Delta x(t)$, $\ddot{\phi}(t) = 2\pi f_\Delta \dot{x}(t)$

$$y_c(t) = A_c \operatorname{Re}\left\{K_0 e^{j[\omega_c t + \phi(t)]} + \frac{K_2}{4\pi^2}\left[j\ddot{\phi}(t) - \dot{\phi}^2(t)\right]e^{j[\omega_c t + \phi(t)]}\right\}$$

$$= A_c\left\{\left[K_0 - \frac{K_2}{4\pi^2}4\pi^2 f_\Delta^2 x^2(t)\right]\cos\left[\omega_c t + \phi(t)\right] - \frac{K_2}{4\pi^2}2\pi f_\Delta \dot{x}(t)\sin\left[\omega_c t + \phi(t)\right]\right\}$$

so $A(t) = A_c\left\{\left[K_0 - K_2 f_\Delta^2 x^2(t)\right]^2 + \left[\frac{K_2 f_\Delta}{2\pi}\dot{x}(t)\right]^2\right\}$

5.2-11

$y_c(t) = A_c \cos\left[\omega_c t + \phi_y(t)\right]$ with $\phi_y(t) = \phi(t) + \arg H\left[f(t)\right]$

$f(t) - f_c = f_\Delta x(t) \quad \Rightarrow \quad \arg H\left[f(t)\right] = \alpha_1 f_\Delta x(t) + \alpha_3 f_\Delta^3 x^3(t)$

$$f_y(t) = f_c + \frac{1}{2\pi}\dot{\phi}_y(t) = f_c + f_\Delta x(t) + \frac{\alpha_1 f_\Delta}{2\pi}\dot{x}(t) + \frac{3\alpha_3 f_\Delta^3}{2\pi}x^2(t)\dot{x}(t)$$

5.2-12

$$H\left[f(t)\right] = \left[1 + j\frac{2Qf_\Delta x(t)}{f_c}\right]^{-1} = \left[1 + j\alpha x(t)\right]^{-1}, \quad \alpha^2 \ll 1$$

$$\left|H\left[f(t)\right]\right| = \left[1 + \alpha^2 x^2(t)\right]^{-1/2} \approx 1 - \frac{1}{2}\alpha^2 x^2(t), \quad \arg H\left[f(t)\right] = -\arctan \alpha x(t) \approx -\alpha x(t)$$

Thus, $y_c(t) \approx A_c\left[1 - \frac{1}{2}\alpha^2 x^2(t)\right]\cos\left[\omega_c t + 2\pi f_\Delta \int^t x(\lambda)\,d\lambda - \alpha x(t)\right]$

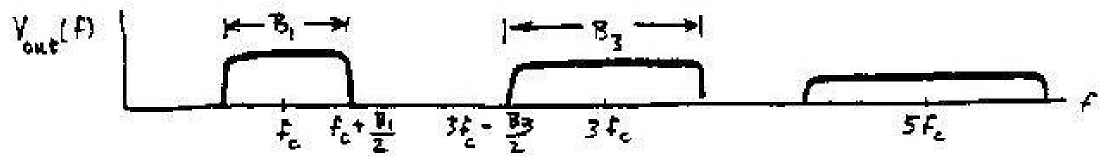
$$f_y(t) = f_c + \frac{1}{2\pi}\dot{\phi}_y(t) = f_c + f_\Delta x(t) - \frac{\alpha}{2\pi}\dot{x}(t)$$

5.2-13

$B_1 \approx 2(D+1)W\,,\quad B_3 \approx 2(3D+1)W$ since $3f(t) = 2p\,(3f_\Delta)\int^t x(l)\,dl$

We want $3f_c - \frac{B_3}{2} > f_c + \frac{B_1}{2} \quad \Rightarrow \quad 2f_c > (D+1)W + (3D+1)W = 4f_\Delta + 2W$

hence $f_\Delta < \frac{(f_c - W)}{2}$



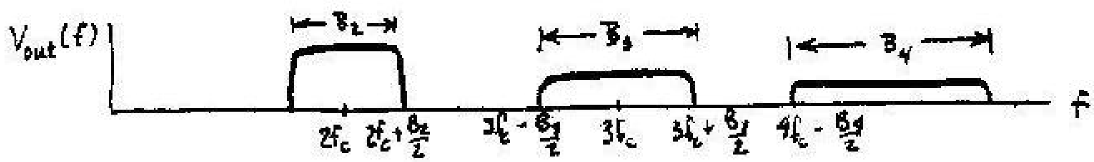
5.2-14

$nf(t) = 2p\,(nf_\Delta)\int^t x(l)\,dl \quad \Rightarrow \quad B_n \approx 2(nD+1)W$

We want $2f_c + \frac{B_2}{2} < 3f_c + \frac{B_3}{2}$ and $3f_c + \frac{B_3}{2} < 4f_c + \frac{B_4}{2}$

so $f_c > \frac{1}{2}(B_3 + B_4) = (3D+1+4D+1)W = 7f_\Delta + 2W$

and $f_\Delta < \frac{f_c - 2W}{7}$



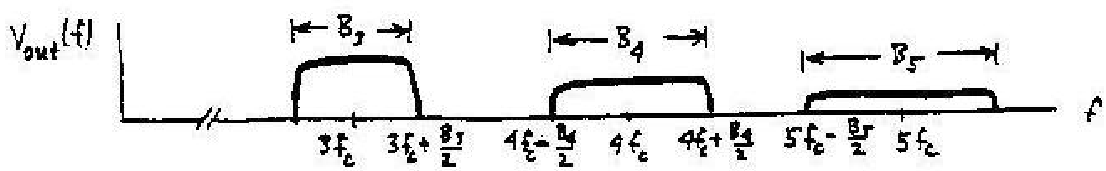
5.2-15

$nf(t) = 2p\,(nf_\Delta)\int^t x(l)\,dl \quad \Rightarrow \quad B_n \approx 2(nD+1)W$

We want $3f_c + \frac{B_3}{2} < 4f_c - \frac{B_4}{2}$ and $4f_c + \frac{B_4}{2} < 5f_c - \frac{B_5}{2}$

so $f_c > \frac{1}{2}(B_4 + B_5) = (4D+1+5D+1)W = 9f_\Delta + 2W$

and $f_\Delta < \frac{(f_c - 2W)}{9}$



5.3-1

Let $\alpha = 1/NV_B \ll 1$

$$c(t) = c_1 + \frac{c_2}{\sqrt{V_B}}(1+\alpha x)^{-1/2} = c_1 + \frac{c_2}{\sqrt{V_B}}\left(1 - \frac{1}{2}\alpha x + \frac{3}{8}\alpha^2 x^2 + \cdots\right)$$

Since $|x| \le 1$, we want $\frac{3}{8}\alpha^2 \le \frac{1}{100}\frac{\alpha}{2} \quad \Rightarrow \quad NV_B \ge \frac{300}{4}$

Then $c(t) \approx c_0 - cx(t)$ with

$$c_0 = c_1 + \frac{c_2}{\sqrt{V_B}} \qquad c = \frac{c_2}{\sqrt{V_B}}\frac{\alpha}{2} = \frac{c_2}{2NV_B\sqrt{V_B}}$$

Thus, $c \le \frac{c_2}{150\sqrt{V_B}}, \quad c_0 > \frac{c_2}{\sqrt{V_B}}$

so $\frac{f_\Delta}{f_c} = \frac{c}{2c_0} < \frac{1}{300}$

5.3-2

$f_\Delta = 150 \text{ kHz} \qquad B_T \approx 2(f_\Delta + 2W)$

$0.01 < \frac{B_T}{f_c} < 0.1 \quad \Rightarrow \quad 0.01 f_c < 2f_\Delta + 4W < 0.1 f_c$

If $f_c = 10$ MHz $\qquad \frac{(0.01\times 10 \text{ MHz}) - (2\times 150 \text{ kHz})}{4} < W < \frac{(0.1\times 10 \text{ MHz}) - (2\times 150 \text{ kHz})}{4}$

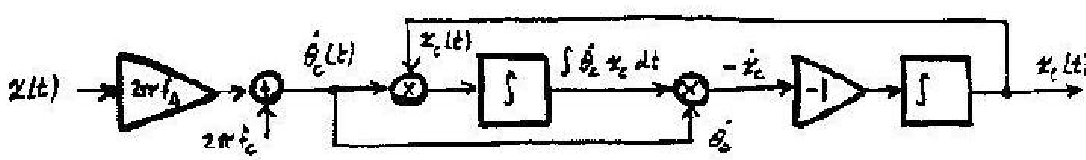
Since W cannot be negative, $W < 175$ kHz

5.3-3

$\dot{x}_c(t) = -A_c\dot{\theta}_c(t)\sin\theta_c(t), \quad \int \dot{\theta}_c(t)\, x_c(t)\, dt = A_c \int \cos\theta_c(t)\frac{d\theta_c}{dt}\, dt = A_c \sin\theta_c(t)$

Thus, $-\dot{\theta}_c(t)\int \dot{\theta}_c(t)\, x_c(t)\, dt = -\dot{\theta}_c(t) A_c \sin\theta_c(t) = \dot{x}_c(t)$

For FM, we want $\dot{\theta}_c(t) = 2\pi f(t) = 2\pi\left[f_c + f_\Delta x(t)\right]$



5.3-4

The frequency modulation index is proportional to the message amplitude and inversely proportional to the message frequency, whereas the phase modulation index is proportional to amplitude only. Therefore the output of an FM detector tends to boost higher frequencies, resulting in the higher frequencies in the output message signal being boosted relative to the original message signal.

5.3-5

The lower frequencies would have much more phase deviation than a PM modulator would have given them. Since the output from a PM demodulator is proportional to the phase deviation, the lower frequencies in the output message signal would be boosted relative to the original message signal.

5.3-6

$$D = \frac{f_\Delta}{W} \quad \Rightarrow \quad f_\Delta = DW = 5 \times 15 = 75 \text{ kHz}$$

$$f_\Delta = n\left(\frac{f_\Delta}{2pT}\right) \text{ so } 75{,}000 < n \times 20 \quad \Rightarrow \quad n > 3750$$

Since we are using triplers we need $n = 3^m > 3750$

For $m = \begin{cases} 7 & 3^7 = 2187 \\ 8 & 3^8 = 6561 \end{cases}$ therefore $m = 8$ triplers are needed.

If the local oscillator is placed at the end, $6581 f_{c_1} - f_{LO} = 915 \times 10^6$

Thus, $f_{LO} = 6581 \times \left(500 \times 10^3\right) - 915 \times 10^6 = 2.37 \times 10^9$ Hz

5.3-7

$$f_\Delta = DW = 25 \text{ kHz}, \quad n > \frac{25 \text{ kHz}}{20 \text{ Hz}} = 1250$$

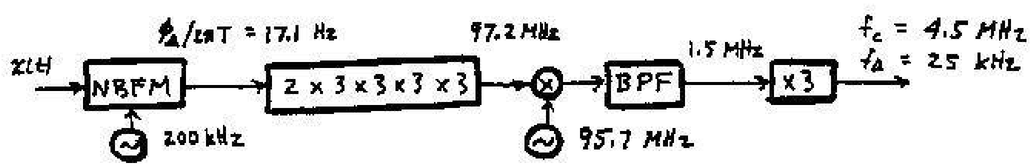
One doubler and 6 triplers yield $n = 2 \times 3^6 = 1458$

$$\text{so } \frac{f_\Delta}{2pT} = \frac{25 \text{ kHz}}{1458} = 17.1 \text{ Hz}$$

$200 \text{ kHz} \times 1458 = 291.6 \text{ MHz} > 100 \text{ MHz}$

Use down-converter before last tripler, where $291.6/3 = 97.2$ MHz

so $f_{LO} = 97.2 - (4.5/3) = 95.7$ MHz



5.3-8

$$f_\Delta = n\frac{f_\Delta}{2pT} \quad \Rightarrow \quad n = \frac{f_\Delta}{\dfrac{f_\Delta}{2pT}} > 120$$

Using doublers only $2^7 = 128 > 120 \quad \Rightarrow \quad$ 7 doublers

$nf_{c_1} = 128 \times 10 \text{ kHz} = 1.28 \text{ MHz}$

Since this doesn't exceed 10 MHz, the down converter can be located at any point.

$\Rightarrow$ Choose to place it after the last doubler.

$f_n(t) = nf_{c_1} + n\dfrac{f_\Delta}{2pT}x(t)$ at the end of the last doubler

$f_c = \left| nf_{c_1} \pm f_{LO} \right| \quad \Rightarrow \quad 1 \text{ MHz} = \left| 128 \times 10 \text{ kHz} \pm f_{LO} \right| \quad \Rightarrow \quad f_{LO} = 280 \text{ kHz}$



5.3-9

(a) $\dfrac{f_\Delta}{T}\int A_m \cos w_m t \, dt = b \sin w_m t$

NBFM output $= A_c \cos w_{c_1} t - A_c b \sin w_m t \sin w_{c_1} t = A(t) \cos\left[w_{c_1} t + \arctan\left(b \sin w_m t \right) \right]$

$$f_1(t) = f_{c_1} + \frac{1}{2p}\frac{d}{dt}\left[\arctan\left(b \sin w_m t \right) \right] = f_{c_1} + b\, f_m \frac{\cos w_m t}{1 + b^2 \sin^2 w_m t}$$

$$\frac{\cos w_m t}{1 + b^2 \sin^2 w_m t} = \cos w_m t \left[1 - b^2 \sin^2 w_m t + b^4 \sin^4 w_m t + \cdots \right]$$

$$\approx \cos w_m t \left[1 - b^2 \left(\frac{1}{2} - \frac{1}{2}\cos 2w_m t \right) \right], \quad b \ll 1$$

$$= \left(1 - \frac{b^2}{2} \right) \cos w_m t + \frac{b^2}{4}\left(\cos w_m t + \cos 3w_m t \right)$$

Thus, $f_1(t) \approx f_{c_1} + b\, f_m \left[\left(1 - \dfrac{b^2}{4} \right) \cos w_m t + \dfrac{b^2}{4} \cos 3w_m t \right]$

$$\approx f_{c_1} + b\, f_m \left[\cos w_m t + \left(\frac{b}{2} \right)^2 \cos 3w_m t \right]$$

(cont.)

(b) 3rd harmonic distortion $=\left(\frac{\beta}{2}\right)^2\times 100 = 25\left(\frac{f_\Delta}{2\pi T}\right)^2\frac{A_m^2}{f_m^2}\le 1\%$

Worst case occurs with A_m maximum and f_m minimum, so

$$5\left(\frac{f_\Delta}{2\pi T}\right)\frac{1}{30\text{ Hz}}\le 1 \quad\Rightarrow\quad \frac{f_\Delta}{2\pi T}\le 6\text{Hz}$$

5.3-10

$$f(t)=f_c+f_\Delta x(t)=f_0-b+f_\Delta x(t)$$

$$\left|H[f(t)]\right|=\left[1+\left(\frac{2Q}{f_0}\right)^2(b-f_\Delta x)^2\right]^{-1/2}=\left[1+\alpha^2\left(1-\frac{f_\Delta x}{b}\right)^2\right]^{-1/2}\quad\text{where}\quad \alpha=\frac{2Qb}{f_0}$$

$$\approx 1-\frac{1}{2}\alpha^2\left(1-\frac{f_\Delta x}{b}\right)^2\quad\text{for}\quad \alpha^2\left(1-\frac{f_\Delta x}{b}\right)^2\ll 1$$

$$A(t)=A_c\left|H[f(t)]\right|\approx A_c\left(1-\frac{\alpha^2}{2}\right)+A_c\left(\frac{\alpha^2 f_\Delta}{b}\right)x(t)$$

so $y_D(t)\approx K_D f_\Delta x(t)$ where $K_D=A_c\dfrac{\alpha^2}{b}=A_c\left(\dfrac{2Q}{f_0}\right)^2 b$

5.3-11

$f(t)=f_c+f_\Delta x(t)$ and $H(f)=\left[1+j\left(\dfrac{f}{f_c}\right)\right]^{-1}$

Let $\alpha=f_\Delta/f_c$ so $|\alpha x|\ll 1$

$$\left|H[f(t)]\right|=\left[1+(1+\alpha x)^2\right]^{-1/2}=\frac{1}{\sqrt{2}}\left(1+\alpha x+\frac{\alpha^2}{2}x^2\right)^{-1/2}$$

$$=\frac{1}{\sqrt{2}}\left[1-\frac{1}{2}\left(\alpha x+\frac{\alpha^2x^2}{2}\right)+\frac{3}{8}\left(\alpha x+\frac{\alpha^2x^2}{2}\right)^2+\cdots\right]$$

$$\approx\frac{1}{\sqrt{2}}\left(1-\frac{1}{2}\alpha x+\frac{1}{8}\alpha^2x^2\right)$$

$$A(t)=A_c\left|H[f(t)]\right|\approx\frac{A_c}{\sqrt{2}}\left[1-\frac{f_\Delta}{2f_c}x(t)+\frac{1}{8}\left(\frac{f_\Delta}{f_c}\right)^2x^2(t)\right]$$

so $y_D(t)\approx -K_1 f_\Delta x(t)+K_2 f_\Delta^2 x^2(t)$ with $K_1=\dfrac{A_c}{2\sqrt{2}f_c}$, $\quad K_2=\dfrac{A_c}{8\sqrt{2}f_c^2}$

(cont.)

If $x(t) = \cos w_m t$, then $x^2(t) = \frac{1}{2} + \frac{1}{2}\cos 2w_m t$

So 2nd harmonic distortion $= \dfrac{K_2 f_\Delta^2/2}{K_1 f_\Delta - K_2 f_\Delta^2/2} \times 100 \approx 100\dfrac{K_2}{2K_1} f_\Delta$

$$= 100\frac{f_\Delta}{8 f_c} < 1\% \quad \Rightarrow \quad \frac{f_\Delta}{f_c} < 0.08$$

5.3-12

$y_D(t) = A_c\left\{\left|H_u\left[f(t)\right]\right| - \left|H_\ell\left[f(t)\right]\right|\right\} \qquad f(t) = f_c + f_\Delta x(t)$

$$= A_c\left\{\left[1 + \frac{a^2}{b^2}\left(f_c + f_\Delta x - f_c - b\right)^2\right]^{-1/2} - \left[1 + \frac{a^2}{b^2}\left(f_c + f_\Delta x - f_c + b\right)^2\right]^{-1/2}\right\}$$

$$\frac{y_D(t)}{A_c} = \left[1 + a^2\left(1 - \frac{f_\Delta x}{b}\right)^2\right]^{-1/2} - \left[1 + a^2\left(1 + \frac{f_\Delta x}{b}\right)^2\right]^{-1/2}$$

$$= \left[1 - \frac{a^2}{2}\left(1 - \frac{f_\Delta x}{b}\right)^2 + \frac{3}{8}a^4\left(1 - \frac{f_\Delta x}{b}\right)^4 + \cdots\right] - \left[1 - \frac{a^2}{2}\left(1 + \frac{f_\Delta x}{b}\right)^2 + \frac{3}{8}a^4\left(1 + \frac{f_\Delta x}{b}\right)^4 + \cdots\right]$$

$$\approx \frac{1}{2}\frac{a^2 4 f_\Delta x}{b} - \frac{3}{8}a^4\left[8\left(\frac{f_\Delta x}{b}\right) + 8\left(\frac{f_\Delta x}{b}\right)^3\right] \text{ when } \quad a^2\left(1 \pm \frac{f_\Delta x}{b}\right)^2 \ll 1$$

so $y_D(t) \approx A_c\left[\left(2a^2 - 3a^4\right)\dfrac{f_\Delta x}{b} - 3a^4\left(\dfrac{f_\Delta x}{b}\right)^3\right] = K_1 x(t) - K_3 x^3(t)$

where $K_1 = A_c\dfrac{\left(2a^2 - 3a^4\right) f_\Delta}{b} \approx A_c\dfrac{2a^2}{b} f_\Delta$ and $K_3 = A_c\dfrac{3a^4}{b^3} f_\Delta^3$

$\dfrac{K_3}{K_1} = \dfrac{3}{2}\left(\dfrac{a f_\Delta}{b}\right)^2 = \dfrac{2}{3}a^2\left(\dfrac{a f_\Delta}{b}\right)^2 \ll 1$ since $a^2 \ll 1$ and $b \le f_\Delta$

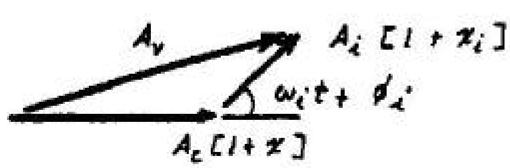
5.4-1

$v(t) = A_c\left[1 + x(t)\right]\cos w_c t + A_i\left[1 + x_i(t)\right]\cos\left[\left(w_c + w_i\right)t + f_i\right]$ where $\quad A_i \ll A_c$

$A_v(t) \approx A_c\left[1 + x\right] + A_i\left[1 + x_i\right]\cos\left(w_i t + f_i\right) = A_c\left\{1 + x(t) + r\left[1 + x_i(t)\right]\cos\left(w_i t + f_i\right)\right\}$

$y_D(t) \approx K_D\left[x(t) + r\cos\left(w_i t + f_i\right) + r\, x_i(t)\cos\left(w_i t + f_i\right)\right]$

$x_i(t)$ will be unintelligible if $w_i \neq 0$

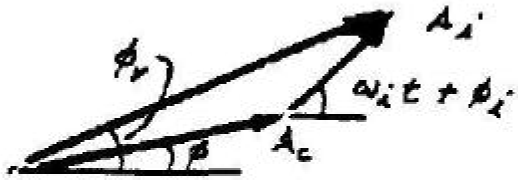


5.4-2

$v(t) = A_c \cos[w_c t + f(t)] + A_i \cos\left[\left(w_c + w_i\right)t + f_i(t)\right]$ where $f(t) = f_\Delta x(t) \square 1, \quad A_i \square A_c$

$$f_v(t) = \arctan\frac{A_c \sin f + A_i \sin\left(w_i t + f_i\right)}{A_c \cos f + A_i \cos\left(w_i t + f_i\right)} \approx \arctan\left[f + \frac{A_i}{A_c}\sin\left(w_i t + f_i\right)\right]$$

$$\approx f_\Delta x(t) + r \sin\left[w_i t + f_i(t)\right] \qquad f_i(t) \text{ will be unintelligible if } w_i \neq 0.$$



5.4-3

$v(t) = A_c\left[1 + mx(t)\right]\cos w_c t + a A_c\left[1 + mx\left(t - t_d\right)\right]\cos\left(w_c t - w_c t_d\right)$

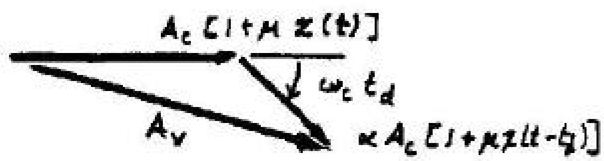
Envelope detection: $y_D(t) = K_D\left[A_v(t) - \left\langle A_v(t)\right\rangle\right]$

where $A_v(t) = A_c\left(\left\{1 + mx(t) + a\left[1 + mx\left(t - t_d\right)\right]\cos w_c t_d\right\}^2 + \left\{a\left[1 + mx\left(t - t_c\right)\right]\sin w_c t_d\right\}^2\right)^{1/2}$

Synchronous detection: $y_D(t) = K_D\left[v_i(t) - \left\langle v_i(t)\right\rangle\right]$

where $v_i(t) = A_c\left\{1 + mx(t) + a\left[1 + mx\left(t - t_d\right)\right]\cos w_c t_d\right\}$

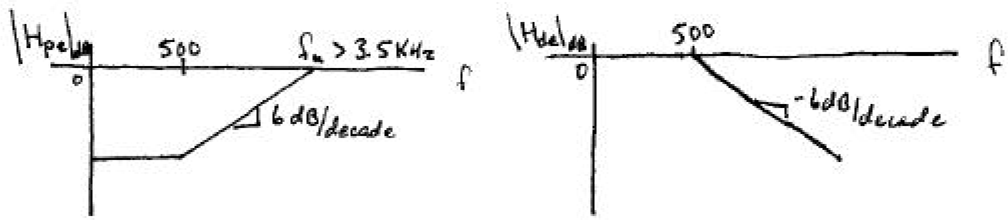
Thus, $A_v(t)$ always has the same or more distortion than $v_i(t)$. If $w_c t_d \approx p/2$ then $\cos w_c t_d = 0$ and $v_i(t)$ is distortionless. If $w_c t_d \approx p$ then $\sin w_c t_d = 0$ and $A_v(t) \approx v_i(t)$.



5.4-4

Motorized electric appliances generate electromagnetic waves that can interfere with the amplitude of the AM signals. FM signals do not suffer in quality when the amplitude of the transmitted signal is corrupted. In addition, most FM cordless phones are above the frequencies of these interfering signals and other household remote-controlled devices such as garage door openers.

5.4-5



Preemphasis increases the energy above 500 Hz so S_x will increase.

$S_T = P_c + 2P_{sb}$ for AM $\qquad S_T = 2P_{sb}$ for DSB

but $\frac{P_{sb}}{A_{max}^2} = \begin{cases} \frac{S_x}{4} & \text{DSB} \\ \frac{S_x}{16} & \text{AM} \end{cases}$

Assuming the peak envelope power allowed by the system is the same for both AM and DSB

$S_T = P_c + 2\frac{S_x}{16}A_{max}^2$ for AM $\qquad S_T = 2\frac{S_x}{4}A_{max}^2$ for DSB

Thus, the transmitted power for DSB is increased much more than it is for AM.

5.4-6

Transmitted power is the same in both cases since it depends only on the carrier amplitude.

Transmitted bandwidth is greater if preemphasis is done prior to transmission since the frequency deviation is increased by a factor of W / B_{de}. However, since speech has very little energy at high frequencies, the bandwidth is driven by the higher amplitude lower frequencies that are not affected by the preemphasis.

Preemphasis after transmission will amplify any noise or interference signals along with the signal of interest. Therefore preemphasis prior to transmission is less susceptible to interference.

Overall, the greater difference is in susceptibility to interference since B_T is not much larger with preemphasis before transmission. Therefore preemphasis at the microphone end is better than at the receiver end.

5.4-7

$$G_{x_{pe}}(f) = |H_{pe}(f)|^2 G_x(f) \approx \begin{cases} G_x(f) & |f| < B_{de} \\ \left(\dfrac{f}{B_{de}}\right)^2 G_x(f) & |f| > B_{de} \end{cases}$$

Thus, for $|f| < B_{de}$, $G_{x_{pe}}(f) \le G_x(f)\big|_{\max} = G_{\max}$

while for $|f| > B_{de}$, $G_{x_{pe}}(f) = \left(\dfrac{f}{B_{de}}\right)^2 G_x(f) \le G_{\max}$ if $G_x(f) \le \left(\dfrac{B_{de}}{f}\right)^2 G_{\max}$

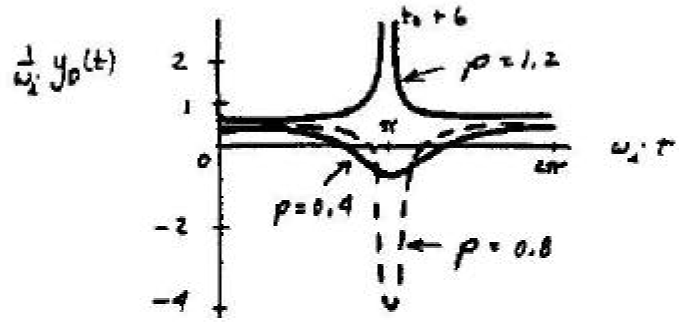
Since B_T is essentially determined by the combination of maximum amplitude and maximum-frequency sinusoidal components in the modulating signal, B_T is not increased if $G_{x_{pe}}(f) \le G_{\max}$.

5.4-8

$y_D(t) = a(r, w_i t) w_i$

where

$$a = \begin{cases} \dfrac{r}{1+r} & w_i t = 0 \\ \dfrac{r^2}{1+r^2} & w_i t = \dfrac{p}{2} \\ \dfrac{-r}{1-r} & w_i t = p \end{cases}$$



5.4-9

$$a(1 \pm e, p) = \frac{(1 \pm e)^2 - (1 \pm e)}{1 + (1 \pm e)^2 - 2(1 \pm e)} = \frac{e^2 \pm e}{e^2} = 1 \pm \frac{1}{e}$$

Thus, $a(1 \pm e, p) \to \pm\infty$ as $e \to 0$

5.4-10

$$v(t) = A_c \cos[w_c t + f(t)] + r A_c [w_c + q_i(t)]$$

$$\text{so} \quad f_v(t) = \arctan \frac{\sin f + r \sin q_i}{\cos f + r \cos q_i}$$

$$y_D(t) = \frac{1}{2p} \dot{f}_v(t) = \frac{1}{2p} \left[1 + \left(\frac{\sin f + r \sin q_i}{\cos f + r \cos q_i} \right)^2 \right]^{-1} \frac{d}{dt} \left[\frac{\sin f + r \sin q_i}{\cos f + r \cos q_i} \right]$$

$$= \frac{1}{2p} \frac{(\cos f + r \cos q_i)(\dot{f} \cos f + r \dot{q}_i \cos q_i) - (\sin f + r \sin q_i)(-\dot{f} \sin f - r \dot{q}_i \sin q_i)}{(\cos f + r \cos q_i)^2 + (\sin f + r \sin q_i)^2}$$

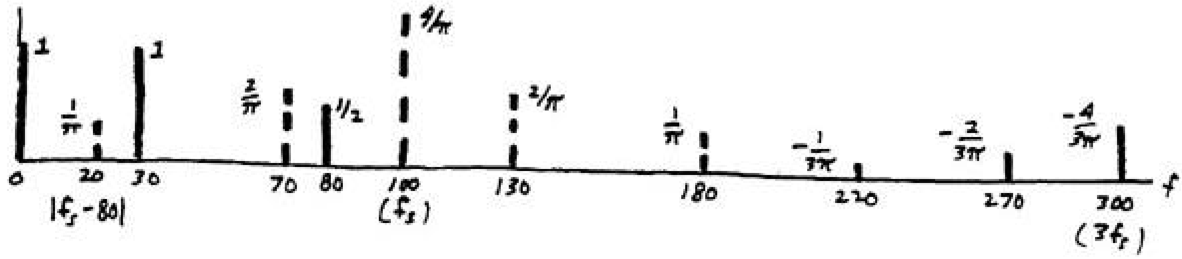
$$= \frac{\{1 + r \cos[f(t) - q(t)]\} \dot{f}(t)/2p + \{r + \cos[f(t) - q_i(t)]\} r f_i}{1 + r^2 + 2r \cos[f(t) - q_i(t)]}$$

Chapter 6

6.1-1

$$c_n = \frac{1}{2}\operatorname{sinc}\frac{n}{2} \quad \Rightarrow \quad c_0 = 1/2,\;\; 2c_1 = 2/\pi,\;\; 2c_2 = 0,\;\; 2c_3 = -2/3\pi$$

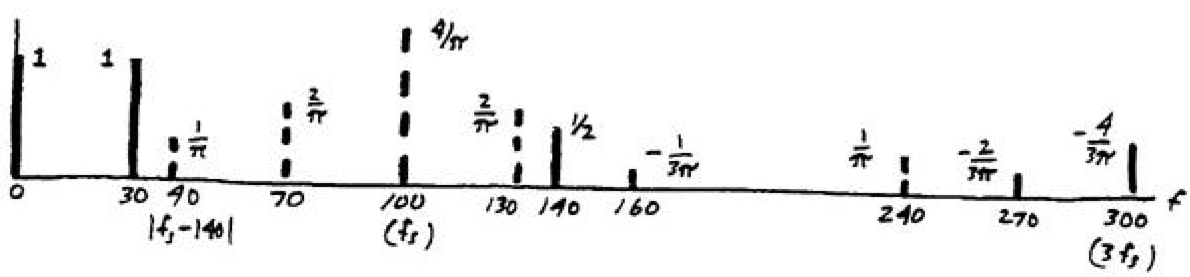
$$y(t) = 1 + \frac{1}{\pi}\cos 2\pi 20t + \cos 2\pi 30t + \frac{2}{\pi}\cos 2\pi 70t$$



6.1-2

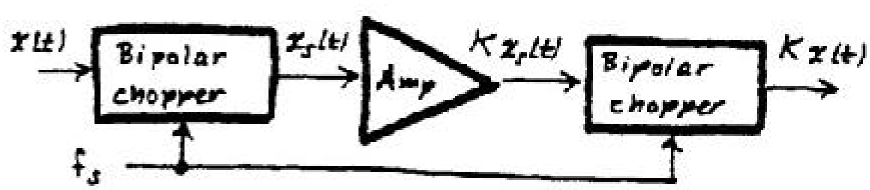
$$c_n = \frac{1}{2}\operatorname{sinc}\frac{n}{2} \quad \Rightarrow \quad c_0 = 1/2,\;\; 2c_1 = 2/\pi,\;\; 2c_2 = 0,\;\; 2c_3 = -2/3\pi$$

$$y(t) = 1 + \cos 2\pi 30t + \frac{1}{\pi}\cos 2\pi 40t + \frac{2}{\pi}\cos 2\pi 70t$$

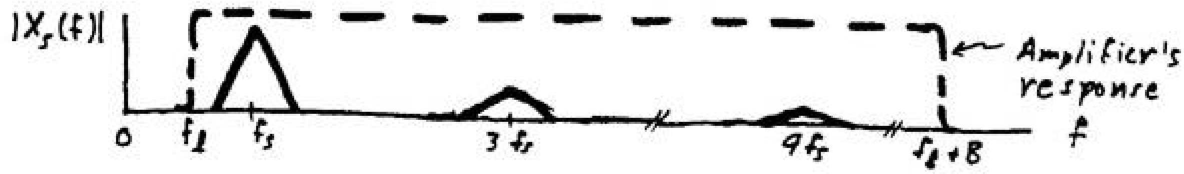


6.1-3

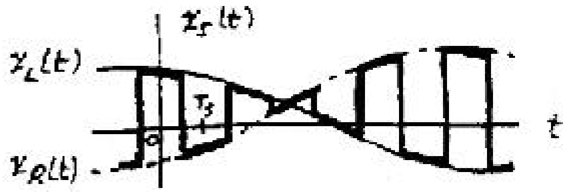
Take $f_s = f_l + W + \varepsilon$. Amplifier then passes $x_s(t)$ since $f_l > f_s - W$ and $f_l + B >> f_s$.
Second chopper with synchronization yields $Kx_s(t)s(t) = Kx(t)s^2(t) = Kx(t)$ since $s^2(t) = 1$.

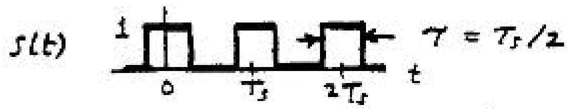


6.1-3 continued



6.1-4





$$x_s(t) = x_L(t)s(t) + x_R(t)[1 - s(t)]$$

$$c_n = \frac{1}{2}\text{sinc}\frac{n}{2} = \frac{1}{\pi n}\sin\frac{\pi n}{2} \quad \Rightarrow \quad s(t) = \frac{1}{2} + \frac{2}{\pi}\cos\omega_s t - \frac{2}{3\pi}\cos 3\omega_s t + \ldots$$

$$x_s(t) = x_L(t) + \left[\frac{1}{2} + \frac{2}{\pi}\cos\omega_s t - \frac{2}{3\pi}\cos 3\omega_s t + \ldots\right] + x_R(t)\left[\frac{1}{2} - \frac{2}{\pi}\cos\omega_s t + \frac{2}{3\pi}\cos 3\omega_s t + \ldots\right]$$

Since LPF rejects $|f| \geq 99$ kHz and $3f_s - W = 3 \text{ x } 38 - 15 = 99$ kHz,

$$x_b(t) = \frac{K_1}{2}[x_L(t) + x_R(t)] + \frac{2K_2}{\pi}[x_L(t) - x_R(t)] + A\cos\frac{\omega_s}{2}t$$

so we want $K_1 = 2$ and $K_2 = \pi/2$

6.1-5

$$c_n = \frac{1}{2}\text{sinc}\frac{n}{2} = \frac{1}{\pi n}\sin\frac{\pi n}{2} \quad \Rightarrow \quad s(t) = \frac{1}{2} + \frac{2}{\pi}\cos\omega_s t - \frac{2}{3\pi}\cos 3\omega_s t + \ldots$$

$$x_1(t) = \frac{1}{2}[x_L(t) + x_R(t)] + \frac{1}{2} \text{ x } \frac{2}{\pi}[x_L(t) - x_R(t)] + \text{ high-frequency terms}$$

$$x_2(t) = \frac{1}{2}[x_L(t) + x_R(t)] + \frac{1}{2} \text{ x } \left(-\frac{2}{\pi}\right)[x_L(t) - x_R(t)] + \text{ high-frequency terms}$$

6.1-5 continued

(a) $$\mathrm{v}_L(t)=\left[\left(\frac{1}{2}+\frac{1}{\pi}\right)-K\left(\frac{1}{2}-\frac{1}{\pi}\right)\right]x_L(t)+\left[\left(\frac{1}{2}-\frac{1}{\pi}\right)-K\left(\frac{1}{2}+\frac{1}{\pi}\right)\right]x_R(t)+...$$

$$\mathrm{v}_R(t)=\left[\left(\frac{1}{2}-\frac{1}{\pi}\right)-K\left(\frac{1}{2}+\frac{1}{\pi}\right)\right]x_L(t)+\left[\left(\frac{1}{2}+\frac{1}{\pi}\right)-K\left(\frac{1}{2}-\frac{1}{\pi}\right)\right]x_R(t)+..$$

we want $\left(\frac{1}{2}-\frac{1}{\pi}\right)-K\left(\frac{1}{2}+\frac{1}{\pi}\right)=0 \quad\Rightarrow\quad K=\frac{\pi-2}{\pi+2}=0.222$

so lowpass filtering yields $v_L^{'}(t)=0.778x_L(t),\quad v_R^{'}(t)=0.778x_R(t)$

(b) If $K=0$, then lowpass filtering yields

$$v_L^{'}(t)=0.818x_L(t)\ +\ 0.182x_R(t),\quad v_R^{'}(t)=0.182x_L(t)\ +\ 0.818x_R(t)$$

So there's incomplete separation of left and right channels at output.

6.1-6

Let $v(t)=s_\delta(t)=\sum_k \delta(t-kT_s)$ with period $T_s=1/f_s$ so

$$c(nf_0)=\frac{1}{T_s}\int_{-T_s/2}^{T_s/2} s_\delta(t)dt=f_s\int_{-T_s/2}^{T_s/2}\delta(t)dt=f_s$$

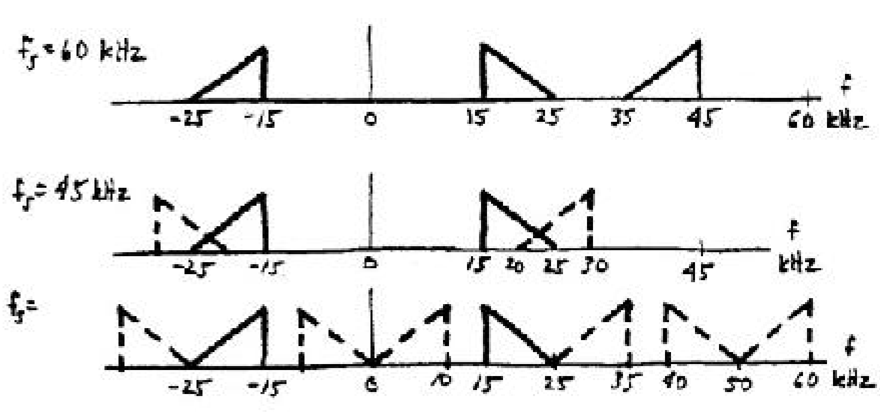
Thus $S_\delta(f)=V(f)\ =\sum_n f_s\delta(f-nf_s)=f_s\sum_n\delta(f-nf_s)$

6.1-7

$f_s=$ 60 kHz Recover using LPF $25\le B\le 35$ kHz

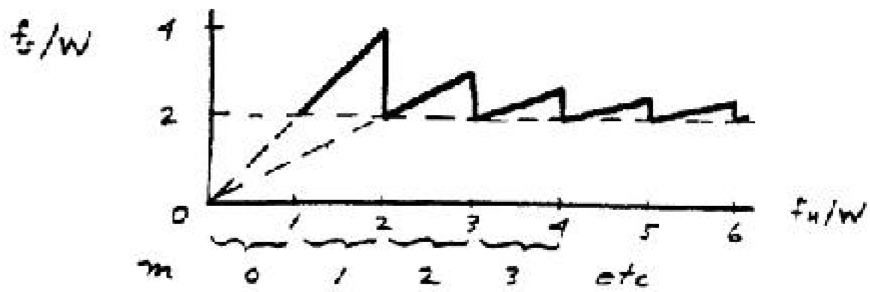
$f_s=$ 45 kHz Can't recover by filtering

$f_s=$ 25 kHz Recover using BPF over $f_l\le|f|\le 25$ kHz with $10< f_l <15$ kHz

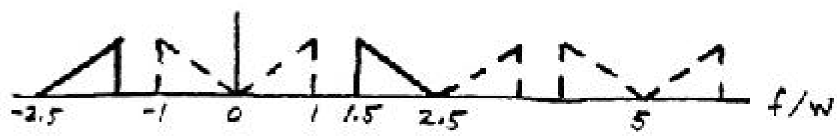


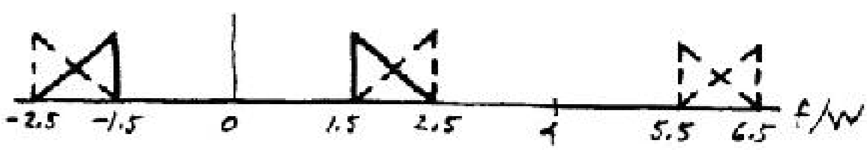
6.1-8

(a) $\frac{f_s}{W} = \frac{1}{M} 2(f_u / W)$



(b) $m = 2,\ f_s = \frac{1}{2}$ x 2 x 2.5 $W = 2.5W$ and $f_s = 4W$





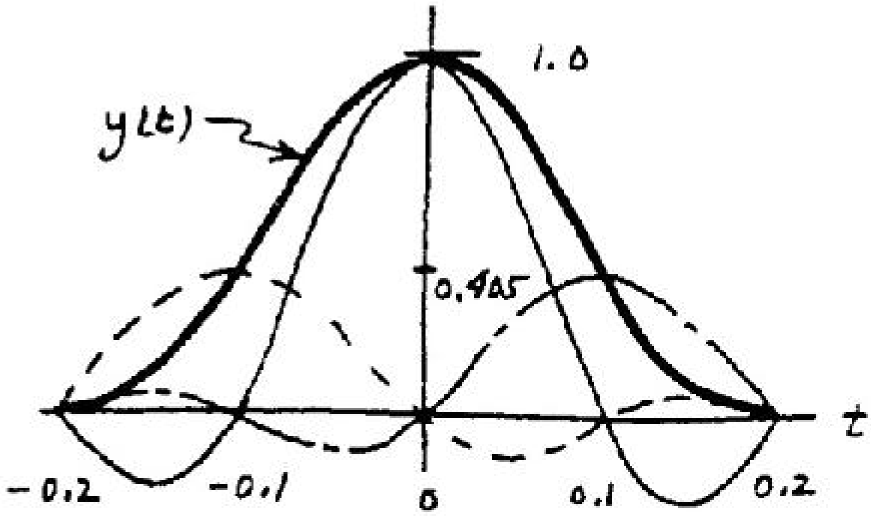
6.1-9

$x(kT_s) = \text{sinc}^2 5(0.1k) = \text{sinc}^2 0.5k$

since $\text{sinc}^2 0.5k \ll 1$ for $|k| \geq 2$,

$y(t) \approx 0.405\ \text{sinc}10(t+0.1) + \text{sinc}10t + 0.405\text{sinc}10(t - 0.1) \approx \text{sinc}^2 5t$

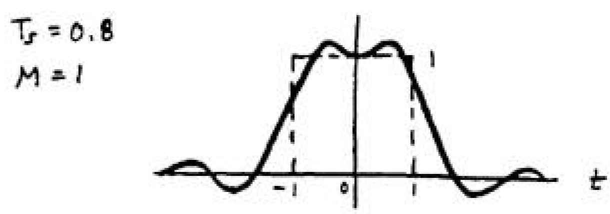
6.1-9 continued

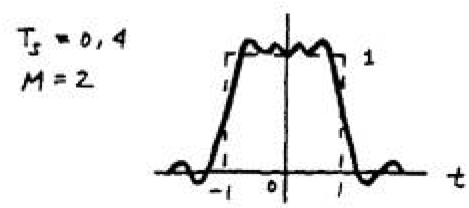


6.1-10

$$x(kT_s) = \Pi\left(\frac{kT_s}{2}\right) = \begin{cases} 1, & |kT_s| \le 1 \\ 0, & |kT_s| > 1 \end{cases}$$

Take $K = \frac{1}{f_s}$ and $t_d = 0$ so $y(t) = \sum_{k=-M}^{M} \text{sinc}\, f_s(t - kT_s)$ where $\frac{1}{T_s} - 1 < M \le \frac{1}{T_s}$



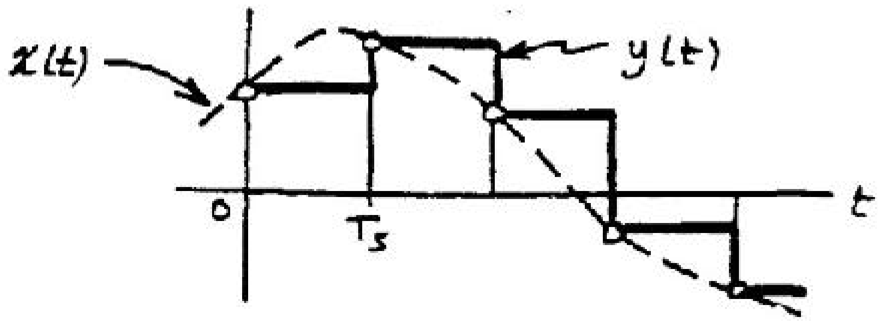


6.1-11

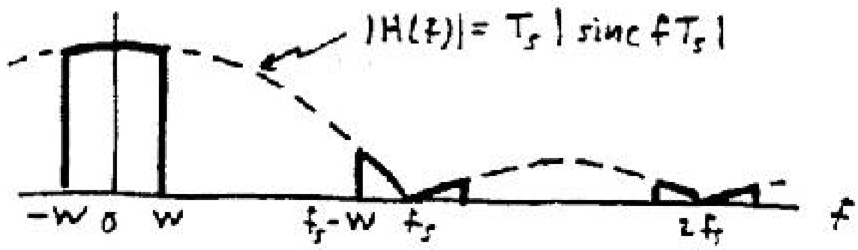
(a) $h(t) = u(t) - u(t - T_s)$

$$y(t) = h(t) * x_\delta(t) = \sum_k x(kT_s)[u(t - kT_s) - u(t - kT_s - T_s)]$$

6.1-11 continued



(b)



$|H(f)| = T_s\,|\text{sinc}\, fT_s|$

since $W \ll f_s$, $|Y(f)| \approx T_s\,|X(f)|$ for $|f| \le W$

so $x(t)$ can be recovered using a simple LPF to remove $|f| \ge f_s - W$

6.1-12

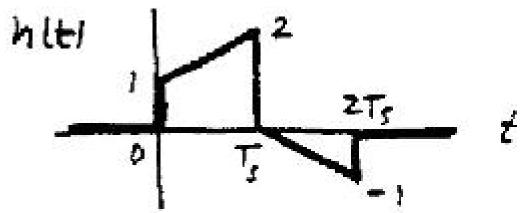
(a) Let $h_z(t) =$ impulse response of a ZOH = $u(t) - u(t - T_s)$

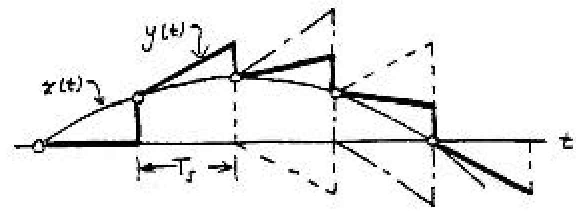
Then $h(t) = \dfrac{1}{T_s} h_z(t) * h_z(t) + h_z(t) - h_z(t - T_s)$

where $\dfrac{1}{T_s} h_z(t) * h_z(t) = \Lambda\left(\dfrac{t - T_s}{T_s}\right)$

$y(t)$ is a linear piecewise approximation obtained by extrapolating forward from the two previoius values.

6.1-12 continued





(b) Let $H_Z(f) = \Im\left[h_z(t)\right] = T_s \operatorname{sinc} fT_s e^{-j\omega T_s/2}$

$$H(f) = \frac{1}{T_s} H_z^2(f) + H_z(f) - H_z(f) e^{-j\omega T_s}$$

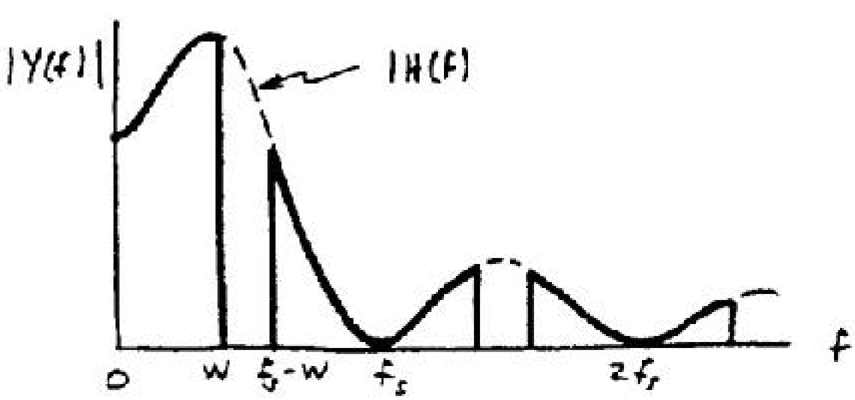
$$= T_s \operatorname{sinc}^2 fT_s e^{-j\omega T_s} + T_s \operatorname{sinc} fT_s \left(e^{j\omega T_s/2} - e^{-j\omega T_s}\right) e^{-j\omega T_s}$$

$$= T_s \left(1 + \frac{1}{\operatorname{sinc} fT_s} j2\sin \pi f T_s\right) \operatorname{sinc}^2 fT_s e^{-j\omega T_s}$$

$$= T_s \left(1 + j2\pi f T_s\right) \operatorname{sinc}^2 fT_s e^{-j\omega T_s}$$

$$\left|H(f)\right| = T_s \sqrt{\left(1 + (j2\pi f T_s)^2\right.} \operatorname{sinc}^2 fT_s$$

Note that high frequency components of $X(f)$ are accentuated.



6.1-13

$$X(f) = \Im\left[\sum_k x\left(\frac{k}{2W}\right)\text{sinc } 2Wt\left(t - \frac{k}{2W}\right)\right] = \sum_{k=-\infty}^{\infty} x\left(\frac{k}{2W}\right)\frac{1}{2W}\Pi\frac{f}{2W}e^{-j\omega k/2W}$$

$$E = \int_{-\infty}^{\infty} X(f)X^*(f)df = \int_{-W}^{+W}\left[\sum_k x\left(\frac{k}{2W}\right)\frac{1}{2W}e^{-j\omega k/2W}\right]\left[\sum_m x^*\left(\frac{m}{2W}\right)\frac{1}{2W}e^{+j\omega m/2W}\right]df$$

$$= \frac{1}{2W}\sum_k\sum_m x\left(\frac{k}{2W}\right)x\left(\frac{m}{2W}\right)\frac{1}{2W}\int_{-W}^{W} e^{+j\pi(m-k)f/W}df$$

$$= \frac{1}{2W}\sum_k\sum_m x\left(\frac{k}{2W}\right)x\left(\frac{m}{2W}\right)\text{sinc}(m-k)$$

$$= \frac{1}{2W}\sum_{k=-\infty}^{\infty}\left|x\left(\frac{k}{2W}\right)\right|^2 \qquad \text{since sinc}(m-k) = \begin{cases}1 & m=k\\ 0 & m\neq k\end{cases}$$

6.1-14

$$v(t) = \sum_{n=-\infty}^{\infty} c_v(nf_0)e^{jn\omega_0 t} \quad\Rightarrow\quad V(f) = \sum_{n=-\infty}^{\infty} c_v(nf_0)\delta(f-nf_0)$$

where

$$c_v(nf_0) = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} v(t)e^{-jn\omega_0 t}dt = \frac{1}{T_0}\int_{-T}^{T} x(t)e^{-jn\omega_0 t}dt = \frac{1}{T_0}\int_{-\infty}^{\infty} x(t)e^{-jn\omega_0 t}dt = f_0X(nf_0)$$

But $x(t) = v(t)\Pi(t/2T) \quad\Rightarrow\quad X(f) = V(f) * (2T \text{ sinc } 2Tf)$ so

$$X(f) = \left[\sum_n f_0X(nf_0)\delta(f-nf_0)\right] * (2T \text{ sinc } 2Tf)$$

$$= 2Tf_0\sum_{n=-\infty}^{\infty} X(nf_0) \text{ sinc } 2T(f-nf_0)$$

Hence, $X(f)$ is completely determined by the sample values of $X(nf_0)$.

6.1-15

$B = f_s/2 \quad\Rightarrow\quad f_s \leq 2B = 12$ MHz

$f_x = 1/T_x = 12.5$ MHz, $\quad 1-\alpha = f_s/f_x = 0.96 \quad\Rightarrow\quad \alpha = 0.04$

$2m+1 < 1/\alpha = 25 \quad\Rightarrow\quad m_{max} = 11$

Presampling bandwidth $\leq$ 11 x 12.5 = 137.5 MHz

6.1-16

$B = f_s/2 \quad \Rightarrow \quad f_s \le 2B$

$1-\alpha = f_s/f_x \quad \le \quad 2BT_x < 2/3 \quad \Rightarrow \quad \alpha = 1/3$

$2m+1 \quad < 1/\alpha \; < \; 3 \quad \Rightarrow \quad m_{\max} = 0$

so only the dc component could be displayed

6.1-17

$W = 15$ kHz, $f_s =$ 150 kHz

with

$$|H_{ZOH}(f)| = |T_s \,\text{sinc}(fT_s)| \text{ and } |H_{FOH}(f)| = T_s\sqrt{1+(2\pi f T_s)^2}\,\text{sinc}^2(fT_s)$$

(a) For a ZOH, the maximum aperature error in the signal passband occurs at $f = 15$ kHz and thus:

$$|H_{ZOH}(f)|_{f=15\text{ kHz}} = 0.9836 \text{ and } |H_{ZOH}(f)|_{f=0\text{ kHz}} = 1$$

$$\Rightarrow \; \%\, aperatureerror \; = \; \frac{1-0.9836}{1} \text{ x } 100\% = 1.640\%$$

(b) For a FOH, the maximum aperature error in the signal passband occurs at $f = 15$ kHz and thus:

$$|H_{FOH}(f)|_{f=15\text{kHz}} = 1.1427 \text{ and } |H_{FOH}(f)|_{f=0\text{kHz}} = 1$$

$$\Rightarrow \; \%\, aperatureerror \; = \; \frac{1-1.1427}{1} \text{ x } 100\% = -14.27\%$$

6.1-18

$W = 15$ kHz, $f_s =$ 150 kHz , $Error\% = \dfrac{1/0.707}{\sqrt{1+(f_a/B)^2}}$ x 100% and $B = W$

$\Rightarrow \; f_a \; = \;$ 150-15=135 kHz, $\Rightarrow \; Error\% = \dfrac{1/0.707}{\sqrt{1+(135/15)^2}}$ x 100% = 15.61%

6.1-19

If $x(t)$ is a sinusoid with period $2T_0$ with its zero crossings occuring at $t = T_0$ and the sampling function has period $T_s = 2T_0$. It is possible for the sampler to sample $x(t)$ at $t = T_0$. Therefore, the output of the sampler is always = 0.

6.1-20

(a) $\text{sinc}(100t) = \text{sinc}(2 \text{ x } 50t) \Leftrightarrow \dfrac{1}{100}\Pi(\dfrac{f}{100}) \Rightarrow$ to sample, $f_s \geq 100$

(b) $\text{sinc}^2(100t) = \text{sinc}^2(2 \text{ x } 50t) \Leftrightarrow \dfrac{1}{100}\Lambda(\dfrac{f}{100}) \Rightarrow$ to sample, $f_s \geq 200$

(c) $10\cos^3 2\pi\text{x}10^5 t = \dfrac{10}{4}(3\cos 2\pi\text{x}10^5 t + \cos 2\pi\text{x}3\text{x}10^5 t)$

Its bandwidth = (3 -1) x 10^5 = 2 x 10^5 Hz $\Rightarrow f_s >$ 4 x 10^5 Hz.

6.1-21

At f =159 kHz the signal level is down -3 dB and we want aliased components down -40 dB.

$\Rightarrow$ at f =159 kHz, aliased components should be down -43 dB = 5 x 10^{-5}.

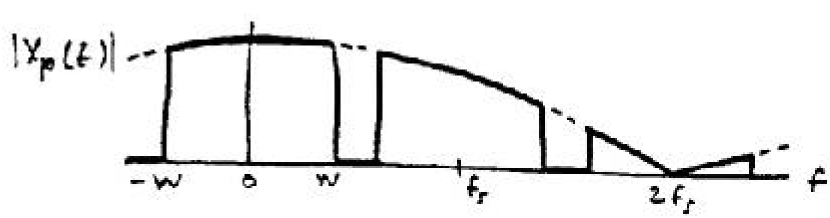
$$\left|H(f)\right| = \frac{1}{\sqrt{1+(f/B)^2}} \Rightarrow 5 \text{ x } 10^{-5} = \frac{1}{\sqrt{1+(f/159)^2}} \Rightarrow f = 3172 \text{ MHz.}$$

6.2-1

$$P(f) = \tau \text{sinc} f\tau = \frac{T_s}{2}\text{sinc } \frac{f}{2f_s}$$

$$H_{eq}(f) = \frac{K}{\text{sinc }(f/2f_s)} = \frac{K}{\text{sinc }(f/5fW)} \quad \text{for } \left|f\right| \leq W$$

$H_{eq}(0) = K$, $H_{eq}(W) = 1.07K$, so equalization is not essential



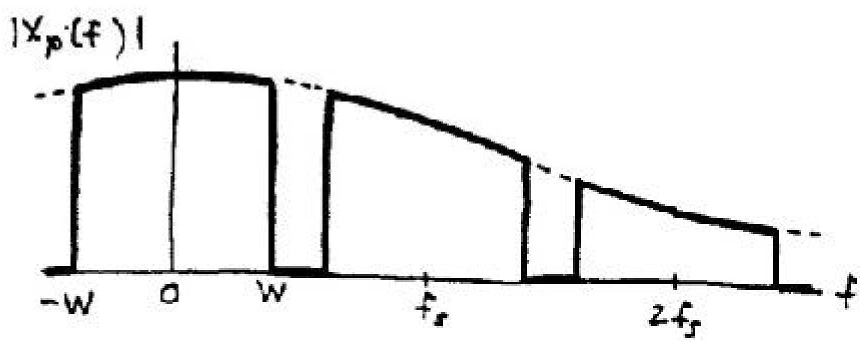
6.2-2

$$P(f) = 2\int_0^{\tau/2} \cos\frac{\pi t}{\tau}\cos 2\pi f t\ dt$$

$$= \frac{\tau}{2}\left[\operatorname{sinc}\left(f\tau - \frac{1}{2}\right) + \operatorname{sinc}\left(f\tau + \frac{1}{2}\right)\right] = \frac{T_s}{4}\left[\operatorname{sinc}\left(\frac{f - f_s}{2f_s}\right) + \operatorname{sinc}\left(\frac{f + f_s}{2f_s}\right)\right]$$

$$H_{eq}(f) = K\left[\operatorname{sinc}\left(\frac{f - 2.5W}{5W}\right) + \operatorname{sinc}\left(\frac{f + 2.5W}{5W}\right)\right]^{-1} \quad \text{for } |f| \le W$$

$H_{eq}(0) = 0.785K,\ \ H_{eq}(W) = 0.816K$ so equalization is not essential.



6.2-3

(a) Let $\bar{x}(t) = \frac{1}{\tau}\int_{t-\tau}^{t} x(\lambda)d\lambda = x(t) * h(t)$ where

$$h(t) = \frac{1}{\tau}\int_{t-\tau}^{t} \delta(\lambda)d\lambda = \frac{1}{\tau}\left[u(t) - u(t-\tau)\right] \ \Rightarrow\ H(f) = \operatorname{sinc} f\tau\ e^{-j\omega\tau/2}$$

Averaging filter $\bar{X}(f)$ $\bar{X}_\delta(f)$ $X_p(f)$

$X(f) \ \rightarrow\ H(f) = \operatorname{sinc} f\tau\ e^{-j\omega\tau/2} \ \rightarrow$ Ideal sampler $\rightarrow\ P(f)\ \rightarrow$

$X_p(f) = P(f)\bar{X}_\delta(f)$

where $\bar{X}_\delta(f) = f_s\sum_n \bar{X}(f - nf_s) = f_s\sum_n H(f - nf_s)X(f - nf_s)$

(b) $X_p(f) = P(f)f_s H(f)X(f)$ for $|f| \le W$, where $P(f) = \tau \sin c\, f\tau$

Thus, $H_{eq}(f) = Ke^{-j\omega(t_d - \tau/2)}/\operatorname{sinc}^2 f\tau,\ \ |f| \le W$

6.2-4

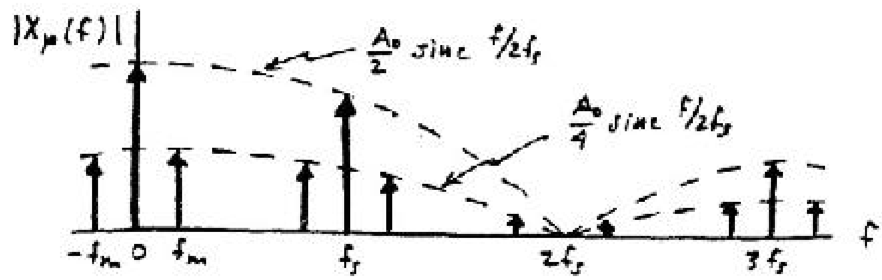
(a) Let $v(t) = A_0[1+\mu x(t)] \quad \leftrightarrow \quad V(f) = A_0[\delta(f)+\mu X(f)]$

$$x_p(t) = \sum_k v(kT_s)\, p(t-kT_s) = p(t) * v_\delta(t)$$

$$X_p(f) = P(f)V_\delta(f) = A_0 f_s P(f)\left\{\sum_n [\delta(f-nf_s)+\mu X(f-nf_s)]\right\}$$

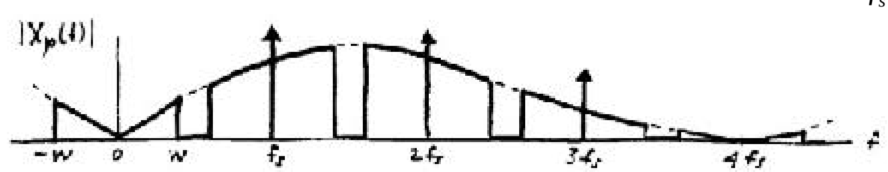
(b) $P(f) = \tau \operatorname{sinc} f\tau = \dfrac{1}{2f_s}\operatorname{sinc} \dfrac{f}{2f_s}$

$$\mu X(f) = \frac{1}{2}[\delta(f-f_m)+\delta(f+f_m)]$$



6.2-5

(a) $P(f) = \tau \operatorname{sinc} f\tau\left(e^{-j\omega\tau/2} - e^{j\omega\tau/2}\right) = \tau \operatorname{sinc} f\tau\left(-2j\sin \pi f\tau\right) = -\dfrac{j\pi f}{8f_s^2}\operatorname{sinc}^2 \dfrac{f}{4f_s}$



(b) $X_p(f) = -A_0 \dfrac{j\pi f}{8f_s}\operatorname{sinc}^2 \dfrac{f}{4f_s}\mu X(f)$ for $|f| \le W$

$$H_{eq}(f) = \frac{Ke^{-j\omega t_d}}{-jf\ \operatorname{sinc}^2(f/4f_s)} \qquad f_l \le |f| \le W$$

If $f_l \to 0$ then $|H_{eq}(0)| \to \infty$ and equalization is not possible.

6.2-6

The spectrum of a PAM signal is like that of the chopper sampled signal of Fig. 6.1-4 and can be written as $X_s(f) = c_0 X(f) + c_1[X(f - f_s) + X(f + f_{s)}] + ...$

With product detection $\Rightarrow x_s(t)$ x $\cos 2\pi f_s t$ giving a frequency domain expression of

$$\frac{c_0}{2} X(f - f_s) + \frac{c_0}{2} X(f + f_s)$$

$$+\frac{c_1}{2} X(f - f_s + f_s) + \frac{c_1}{2} X(f - f_s - f_s) + \frac{c_1}{2} X(f + f_s + f_s) + \frac{c_1}{2} X(f + f_s - f_s)$$

Combining terms and using a LPF the output spectra from the product detector gives

$$\frac{c_1}{2} X(f) + \frac{c_1}{2} X(f) = c_1 X(f)$$

6.3-1

$$\tau_{\min} = \frac{T_s}{5}(1 - 0.8) = \frac{1}{25 f_s} \quad \Rightarrow \quad t_r \le \frac{1}{100 f_s}$$

so $B_T \ge 1/2t_r \ge 50 f_s = 400$ kHz

6.3-2

$$\tau_{\min} = \tau_0(1 - 0.8) \ge 3t_r \quad \text{and} \quad t_r \ge 1/2B_T \quad \Rightarrow \quad 0.2\tau_0 \ge 3/2B_T \quad \Rightarrow \quad \tau_0 \ge 15\ \mu\text{s}$$

$$\tau_{\max} = \tau_0(1 + 0.8) \le T_s/3 \quad \Rightarrow \quad \tau_0 \le \frac{1}{1.8 \text{ x } 3 f_s} = 23.1\ \mu\text{s}$$

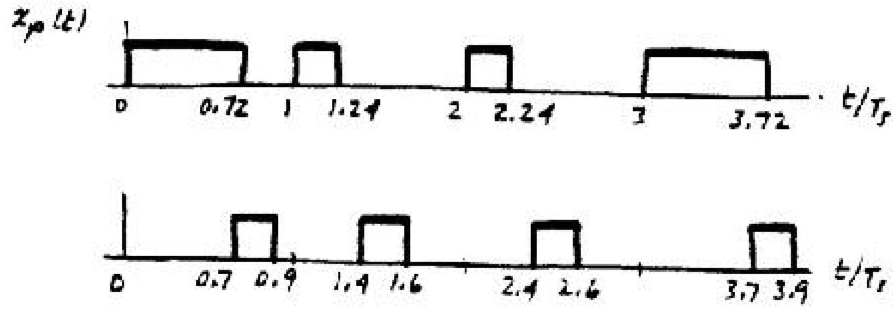
Thus, $15 \le \tau_0 \le 23.1\ \mu$s

6.3-3

(a) $\tau_k = 0.4T_s\left[1 + 0.8\cos\left(\frac{2\pi k}{3}\right)\right]$

(b) $t_k = T_s\left[k + 0.5 + 0.2\cos\left(\frac{2\pi k}{3}\right)\right]$

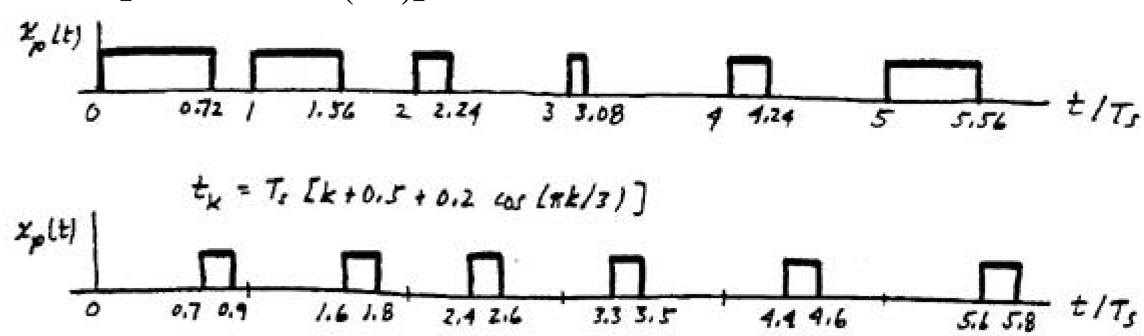
6.3-3 continued



6.3-4

(a) $\tau_k = 0.4T_s\left[1+0.8\cos\left(\frac{\pi k}{3}\right)\right]$

(b) $t_k = T_s\left[k+0.5+0.2\cos\left(\frac{\pi k}{3}\right)\right]$



6.3-5

Take $t_0 \ll T_s$ so that $|t_0\dot{x}(t)|<1$. Apply -$x(t)$ to PPM generator to get

$$x_p(t) = A(t)\left\{1+\sum_n 2\cos\left[n\omega_s t + n\omega_s t_0 x(t)\right]\right\}$$

where $A(t) = Af_s\left[1+t_0\dot{x}(t)\right] > 0$

6.3-5 continued

$$-x(t) \rightarrow \text{PPM} \xrightarrow{x_p(t)} \underset{f_0 = mf_s}{\text{BPF}} \xrightarrow{v(t)} \text{Lim} \rightarrow \underset{f_0 = mf_s}{\text{BPF}} \rightarrow x_c(\text{t})$$

First BPF yields $v(t) = 2A(t)\cos\left[\omega_c t + \phi_\Delta x(t)\right]$

with $f_c = mf_s, \quad \phi_\Delta = 2\pi m f_s t_0 \approx \pi$ if $m \approx \dfrac{T_s}{2t_0}$

Limiter and second BPF give $x_c(t) = A_c \cos\left[\omega_c t + \phi_\Delta x(t)\right]$

6.3-6

(a) $s_\delta(t) = \Im^{-1}\left[S_\delta(f)\right] = f_s \sum_n e^{-j2\pi n f_s t}$ and $s_\delta(t) = \sum_k \delta(t - kT_s)$

Thus, $\sum_{n=-\infty}^{\infty} e^{\pm j2\pi nt/L} = L \sum_{k=-\infty}^{\infty} \delta(t - kL)$ where $L = 1/f_s = T_s$

(b) $S_\delta(f) = \Im\left[s_\delta(t)\right] = \sum_k e^{-j2\pi f k T_s}$ and $S_\delta(f) = f_s \sum_n \delta\left(f - nf_s\right)$

Thus, $\sum_{k=-\infty}^{\infty} e^{\pm j2\pi kf/L} = L \sum_{n=-\infty}^{\infty} \delta\left(f - nL\right)$ where $L = 1/T_s = f_s$

6.3-7

$g(t) = v \quad \Rightarrow \quad t = g^{-1}(v)$ and $\lambda = g^{-1}(0)$

$$\dot{g}(t) = \frac{dv}{dt} \quad \Rightarrow \quad dt = \frac{1}{\dot{g}(t)} dv = \frac{1}{\dot{g}\left[g^{-1}(v)\right]} dv$$

Thus, $\displaystyle\int_a^b d[g(t)]dt = \int_{v=g(a)}^{v=g(b)} \frac{\delta(v)}{\dot{g}\left[g^{-1}(v)\right]} dv$

If $\dot{g}(\lambda) > 0$, then $g(b) > 0 > g(a)$ so

$$\int_{g(a)}^{g(b)} \frac{\delta(v)}{\dot{g}\left[g^{-1}(v)\right]} dv = \frac{1}{\dot{g}\left[g^{-1}(0)\right]} = \frac{1}{\dot{g}(\lambda)}$$

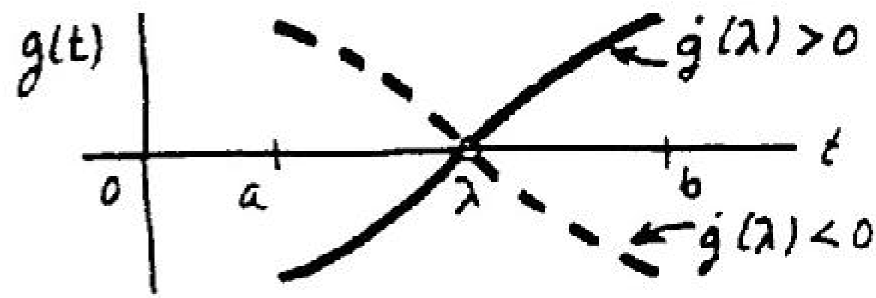
6.3-7 continued

If $\dot{g}(\lambda) < 0$, then $g(b) < 0 < g(a)$ so

$$\int_{g(a)}^{g(b)} \frac{\delta(v)}{\dot{g}\left[g^{-1}(v)\right]} dv = -\int_{g(b)}^{g(a)} \frac{\delta(v)}{\dot{g}\left[g^{-1}(v)\right]} dv = -\frac{1}{\dot{g}(\lambda)}$$

Hence $\int_a^b \delta[g(t)]\, dt = \frac{1}{|\dot{g}(\lambda)|} = \int_a^b \frac{1}{|\dot{g}(t)|} \delta(t-\lambda)\, dt$

so $\delta[g(t)] = \delta(t-\lambda)/|\dot{g}(t)|$



Chapter 7

7.1-1

$$f_c' = f_c + 2f_{IF} \geq 1600 + \frac{10}{2} \text{ kHz} \Rightarrow f_{IF} \geq (1605 - 540)/2 = 532.5 \text{ kHz}$$
$$f_{LO} = f_c + f_{IF} = 1072.5 \text{ to } 2132.5 \text{ kHz}, \; B_T = 10 \text{ kHz} < B_{RF} < 2f_{IF} = 1065 \text{ kHz}$$

7.1-2

$$f_c' = f_c - 2f_{IF} \leq 88{,}100 - \frac{250}{2} \text{ kHz} \Rightarrow f_{IF} \geq (107.9 - 87.975)/2 = 9.9625 \text{ MHz}$$
$$f_{LO} = f_c - f_{IF} = 78.1375 \text{ to } 97.9375 \text{ MHz}, \; B_T = 250 \text{ kHz} < B_{RF} < 2f_{IF} = 19.925 \text{ kHz}$$

7.1-3

$$C = 1/4\pi^2 L f_{lo}^2 = 2.533 \text{ x } 10^4 / f_{lo}^2$$
$$f_{lo} = f_c + f_{IF} = 995 - 2055 \text{ kHz} \Rightarrow C = 6.0 \text{ - } 25.6 \text{ nF}$$
$$f_{lo} = f_c - f_{IF} = 85 - 1145 \text{ kHz} \Rightarrow C = 19.3 \text{ - } 3{,}506 \text{ nF}$$

7.1-4

$$f_c = 1/2\pi\sqrt{LC} \Rightarrow C = 1/4\pi^2 L f_c^2 = 9.9 \text{ - } 86.9 \text{ nF}$$
$$Q = R\sqrt{\frac{C}{L}} = \frac{f_c}{B_{RF}} = \frac{1}{2\pi\sqrt{LC}B_{RF}} \Rightarrow R = \frac{1}{2\pi B_{RF} C}$$
$$B_{RF} > B_T \Rightarrow R < \frac{1}{2\pi \text{ x } 10 \text{ kHz x } 9.9 \text{ nF}} = 1.6 \text{ k}\Omega,$$
$$B_{RF} > 2f_{IF} \Rightarrow R > \frac{1}{2\pi \text{ x } 910 \text{ kHz x } 86.9 \text{ nF}} = 2.0\,\Omega$$

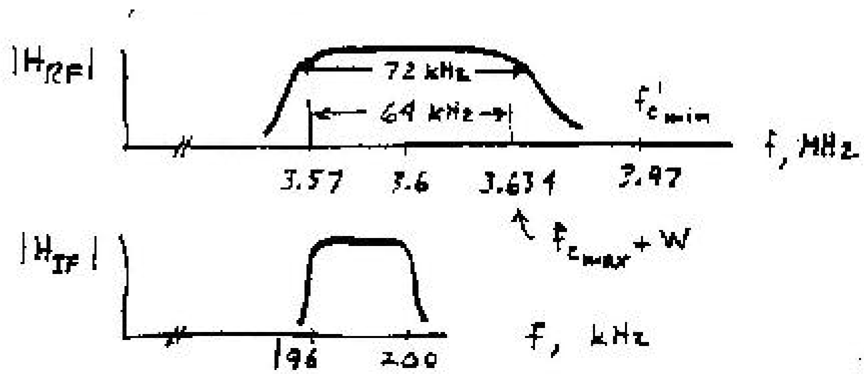
7.1-5

$$f_{IF} \approx B_T/0.02 = 200 \text{ kHz since } B_T = W$$
$$f_{LO} = f_c + f_{IF} = 3.77 \text{ - } 3.83 \text{ MHz}, \; f_c' = f_{LO} + f_{IF} = 3.97 \text{ - } 4.03 \text{ MHz}$$

Take $B_{RF} \approx$ 0.02 x 3.6 MHz = 72 kHz centered at 3.6 MHz

IF must pass $f_{IF} - W \leq f \leq f_{IF}$

7.1-5 continued



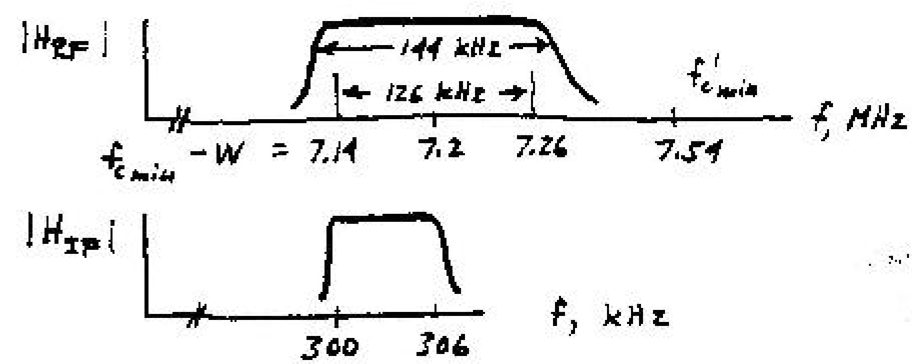
7.1-6

$f_{IF} \approx B_T / 0.02 = 300$ kHz since $B_T = W$

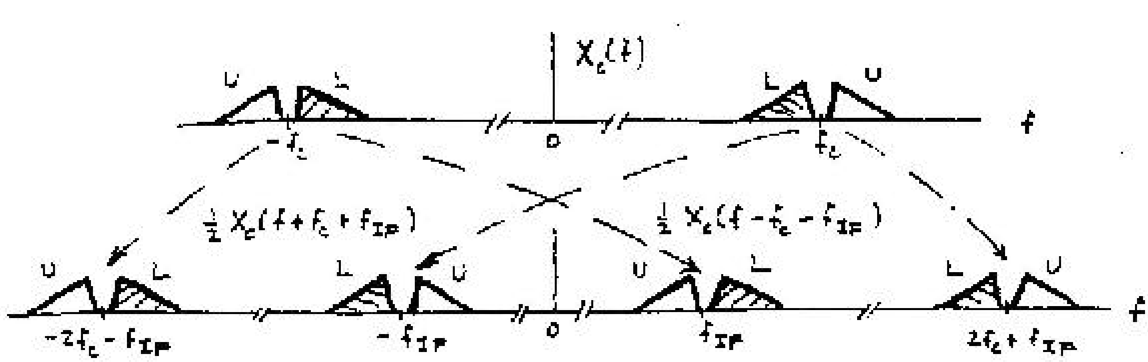
$f_{LO} = f_c + f_{IF} = 7.34$ - 7.46 MHz, $f_c^{'} = f_{LO} + f_{IF} = 7.54$ - 7.66 MHz

Take $B_{RF} \approx 0.02$ x 7.2 MHz = 144 kHz centered at 7.2 MHz

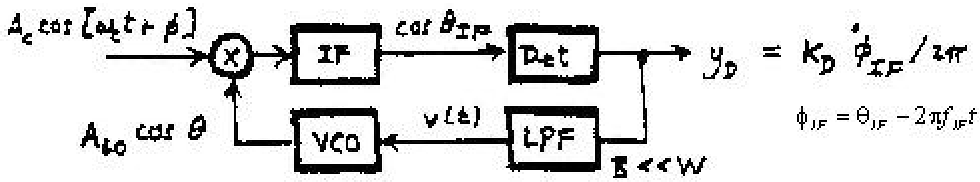
IF must pass $f_{IF} \leq f \leq f_{IF} + W$



7.1-7



7.1-8



$\theta_{IF} = \omega_c t + \phi - \theta$ where $\phi = 2\pi f_\Delta x(t)$, $\dot{\theta} = 2\pi\left[f_c - f_{IF} + kv(t) + \varepsilon(t)\right]$

$$y_D = \frac{K_D}{2\pi}\left(\dot{\theta}_{IF} - 2\pi f_{IF}\right) = K_D\left[f_c + f_\Delta x - f_c + f_{IF} - Kv - \varepsilon - f_{IF}\right]$$

$$= K_D\left[f_\Delta x(t) - Kv(t) - \varepsilon(t)\right]$$

so $v(t) = K_D\left[-Kv(t) - \varepsilon(t)\right]$

Thus, $v(t) = -K_D \varepsilon(t) / (1 + K_D K)$

and $y_D(t) = K_D\left[f_\Delta x(t) - \dfrac{-K_D K \varepsilon(t)}{1 + K_D K} - \varepsilon(t)\right] = K_D\left[f_\Delta x(t) - \dfrac{1}{1 + K_D K}\varepsilon(t)\right]$

$\approx K_D f_\Delta x(t)$ if $K_D K \gg 1$

7.1-9

(a) With $f_c = 50 \to 54$ MHz and $f_{IF} = 455$ kHz $\Rightarrow f_{LO} = 50.455 \to 54.455$ MHz.
$\Rightarrow f_c' = f_c + 2 f_{IF} = 50.910 \to 54.910$ MHz.

(b) With $f_c = 50 \to 54$ MHz and $f_{IF} = 7$ MHz $\Rightarrow f_{LO} = 57 \to 61$ MHz.
$\Rightarrow f_c' = f_c + 2 f_{IF} = 64 \to 68$ MHz.

7.1-10

If W = signal bandwidth, then the incomming signal is $50 + W \to 54 + W$ MHz.
With f_{IF} =100 MHz, to avoid sideband reversal use $f_{LO} = 150 \to 154$ MHz.
At the product detector stage, use an oscillator frequency of 100 MHz.

The image frequency is $f_c' = f_c + 2 f_{IF}$ and its range is thus $250 \to 254$ MHz.

7.1-11

Image frequency = $f_c' = f_c + 2f_{IF} = 2 + 2$ x 455 = 2.91 kHz.

For a BPF with center frequency of $f_0 = f_c \Rightarrow |H(f)| = \dfrac{1}{\sqrt{1+Q^2(\frac{f}{f_0}-\frac{f_0}{f})^2}}$

and $Q = f_0$/B. We use the BPF to reject images and thus we have

$$|H(f)|_{f=2.9\text{ MHz}} = \frac{1}{\sqrt{1+4^2(\frac{2.91}{2}-\frac{2}{2.91})^2}} = 0.3123 \Rightarrow 20\log(0.3123) = -10\text{ dB.}$$

Images are rejected by -10 dB.

7.1-12

(a) With $f_{LO} = 2.455$ MHz and $f_{IF} = 455$ KHz, then $f_c = 2$ MHz, and $f_c' = 2.910$ MHz (image).
With $f_{LO} = 2.455$ x 2 = 4.910 MHz ⇒ Input frequencies accepted are:
$f_c'' = 4.910 - 0.455 = 4.455$ MHz, and $f_c''' = 4.455 + 2\text{x } 0.455 = 5.365$ MHz.
Given the RCL BPF with B= 0.5 MHz $\Rightarrow Q = 2/0.5 = 4$

$$|H(f)|_{f=2.9\text{ MHz}} = \frac{1}{\sqrt{1+4^2(\frac{2.91}{2}-\frac{2}{2.91})^2}} = 0.3123 \Rightarrow 20\log(0.3123) = -10\text{ dB}$$

We repeat the above calculation for the spurious frequencies of 4.455 and 5.360 MHz.
But because the LO oscillator harmonic is 1/2 that of the fundamental we multiply the result by 1/2. Hence,

$$|H(f)|_{f=4.455\text{ MHz}} = \frac{1}{\sqrt{1+4^2(\frac{4.455}{2}-\frac{2}{4.455})^2}} = 0.139\text{ x } 1/2 = 0.070 \Rightarrow 20\log(0.070) = -23.1\text{ dB}$$

and

$$|H(f)|_{f=5.365\text{ MHz}} = \frac{1}{\sqrt{1+4^2(\frac{5.365}{2}-\frac{2}{5.365})^2}} = 0.108\text{ x } 1/2 = 0.054 \Rightarrow 20\log(0.054) = -25.4\text{ dB}$$

7.1-12 continued

(b) To reduce spurious inputs: (1) use a more selective BPF, (2) Use filter to reject the LO second harmonic, (3) use a higher f_{IF}.

7.1-13

If $f_{c_1} = 50 \rightarrow 51$ MHz and $f_{IF_1} = 7 \rightarrow 8$ MHz.

We could choose a fixed frequency output LO with f_{LO} = 43 MHz.

(a) With f_{c_1} = 50 MHz and f_{IF_1} = 7 MHz, and f_{LO} =43 MHz, the image frequency is

$f_{c_1}^{'} = f_{c_1} - 2 \text{ x } f_{IF_1} = 50 - 2 \text{ x } 7 = 36$ MHz

But, the original 7 MHz receiver also suffers from images, so if the incomming signal is supposed to be 7.0 MHz, it could also be 7 + 2 x 0.455 = 7.910 MHz $\Rightarrow$ $f_{IF_1}^{'}$ = 7.910 MHz.

$\Rightarrow$ $f_{c_1}^{'}$ = 43 + 7.910 = 50.910 MHz will also be heard.

(b) Use a more selective BPF at the output of the first mixer and/or at the input of the 7 MHz receiver.

7.1-14

With $f_c = 50 \rightarrow 54$ MHz, let's use $f_c = f_0 = 52$ MHz. Assume $f_{LO} = f_c + f_{IF}$

(a) With f_{IF} = 20 MHz $\Rightarrow$ f_{LO} = 72 MHz and $\Rightarrow$ $f_c^{'} = 52 + 2$ x 20 =92 MHz.

$Q = f_0 / B = 52/4 = 13$

$$\left|H(f)\right|_{f=92\text{ MHz}} = \frac{1}{\sqrt{1+13^2\left(\frac{92}{52}-\frac{52}{92}\right)^2}} = 0.064 \Rightarrow 20\log(0.064) = -23.9 \text{ dB}$$

(b) With f_{IF} =100 MHz $\Rightarrow$ f_{LO} =152 MHz and $\Rightarrow$ $f_c^{'} = 52 + 2$ x 100 =252 MHz.

$$\left|H(f)\right|_{f=152\text{ MHz}} = \frac{1}{\sqrt{1+13^2\left(\frac{252}{52}-\frac{52}{252}\right)^2}} = 0.017 \Rightarrow 20\log(0.017) = -35.6 \text{ dB}$$

7.1-15

Given $f_{c_1} = 850$ MHz and $f_{c_2} = 1950$ MHz, let's pick a common 500 MHz IF $\Rightarrow$ f_{IF} = 500 MHz.
For f_{c_1} = 850 MHz, select $f_{LO_1} = f_{c_1} + f_{IF} \Rightarrow f_{LO}$ = 1350 MHz
and
for $f_{c_2} = 1950$ MHz, select $f_{LO_2} = f_{c_2} - f_{IF} \Rightarrow f_{LO} = 1450$ MHz.
$\Rightarrow f_{LO} = 1350 \rightarrow 1450$ MHz.

Image frequencies:
$f_c = 850$ MHz $\Rightarrow f_c^{'} = 850 + 2$ x 500 = 1850 MHz
and
$f_c = 1950$ MHz $\Rightarrow f_c^{'} = 1950$ - 2 x 500 = 950 MHz.

7.1-16

$B_{IF_2} = 2W, \quad f_{IF_2} \approx 2W/0.02 = 1$ MHz
From Exercise 7.1-2, $f_{IF_1} \approx 9.5 f_c =$ 38 MHz so $B_{IF_1} \approx$ 0.02 x 38 = 760 kHz
$f_{LO_1} = f_c + f_{IF_1} = 42$ MHz, $\quad f_{LO_2} = f_{IF_1} \pm f_{IF_2} = 37$ or 39 MHz

7.1-17

$f_{LO_1} = f_c + f_{IF_1} = 330$ MHz $\Rightarrow$ $f_c^{'} = 330 + 30 = 360$ MHz
$f_{LO_2} = f_{IF_1} + f_{IF_2} = 33$ MHz, so image frequency at input of 2nd mixer is
$f_{LO_2} + f_{IF_2} = 36$ MHz produced by
$\left| f_c^{'} - f_{LO_1} \right| = 36$ MHz $\Rightarrow$ $f_c^{'} = 294$ and 366 MHz

7.1-18

$f_{LO_1} = f_c - f_{IF_1} = 270$ MHz $\Rightarrow$ $f_c^{'} = 270 - 30 = 240$ MHz
$f_{LO_2} = f_{IF_1} - f_{IF_2} = 27$ MHz, so image frequency at input of 2nd mixer is
$f_{LO_2} - f_{IF_2} = 24$ MHz produced by
$\left| f_c^{'} - f_{LO_1} \right| = 24$ MHz $\Rightarrow$ $f_c^{'} = 246$ and 294 MHz

7.1-19

$1/T_0 = 20\ \text{Hz}$, so take $B < 20$ Hz to resolve lines

$$f_1 = 0,\ f_2 = 10/T_0 = 200\ \text{Hz},\ T \ge \frac{f_2 - f_1}{B^2} > \frac{200\ \text{Hz}}{(20\ \text{Hz})^2} = 0.5\ \text{sec}$$

7.1-20

Take $B < f_m = 1$ kHz to resolve lines, $\beta = 5 \Rightarrow$ 8 pairs of sideband lines.

$$f_1 = f_c - 8f_m = 92\ \text{kHz},\ f_2 = f_c + 8f_m = 108\ \text{kHz}$$

$$T \ge \frac{f_2 - f_1}{B^2} > \frac{16\ \text{kHz}}{(1\ \text{kHz})^2} = 16\ \text{ms}$$

7.1-21

$$h_{bp}(t) = \cos\alpha t^2 \cos\omega_c t - \sin\alpha t^2 \sin\omega_c t \quad \text{so}$$

$$h_{lp}(t) = \frac{1}{2}\left(\cos\alpha t^2 + j\sin\alpha t^2\right) = \frac{1}{2}e^{j\alpha t^2}$$

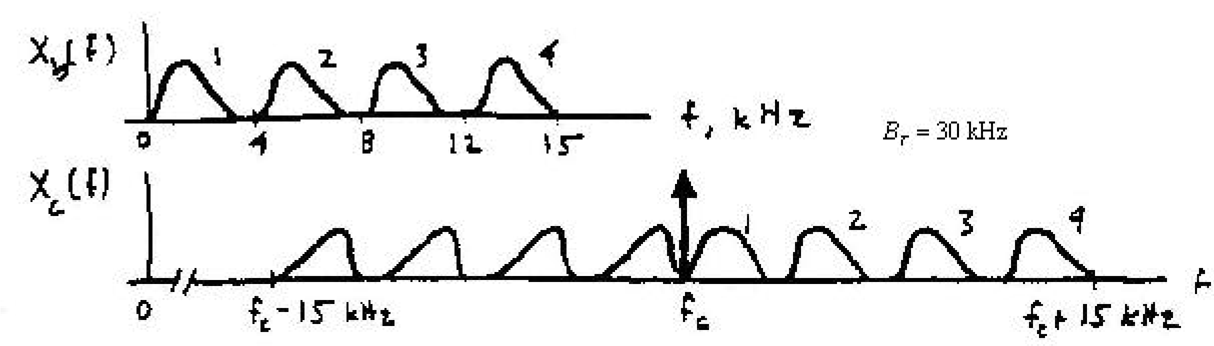
$$x_{bp}(t) = v(t)\cos\alpha t^2 \cos\omega_c t - \left[-v(t)\sin\alpha t^2\right]\sin\omega_c t \quad \text{so}$$

$$x_{lp}(t) = \frac{1}{2}\left[v(t)\cos\alpha t^2 - jv(t)\sin\alpha t^2\right] = \frac{1}{2}v(t)e^{-j\alpha t^2}$$

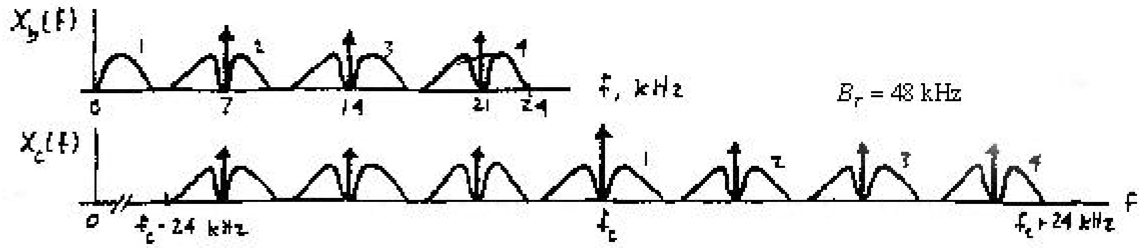
$$y_{lp}(t) = x_{lp} * h_{lp} = \frac{1}{4}\int_{-\infty}^{\infty} v(\lambda)e^{-j\alpha\lambda^2}e^{j\alpha(t-\lambda)^2}d\lambda = \frac{1}{4}e^{j\alpha t^2}\int_{-\infty}^{\infty} v(\lambda)e^{-j2\alpha t\lambda}d\lambda$$

$$A_y(t) = \left|y_{lp}(t)\right| = \left|\frac{1}{4}\int_{-\infty}^{\infty} v(\lambda)e^{-j2\pi(\alpha t/\pi)\lambda}d\lambda\right| = \frac{1}{4}V(f)\Big|_{f=\alpha t/\pi}$$

7.2-1

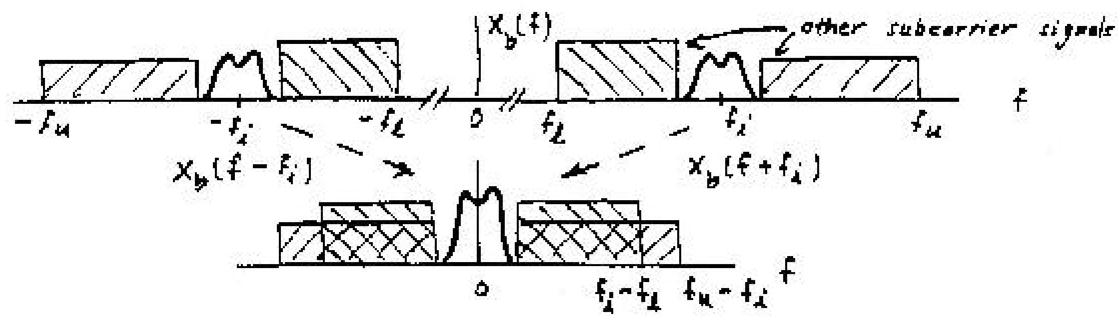


7.2-2

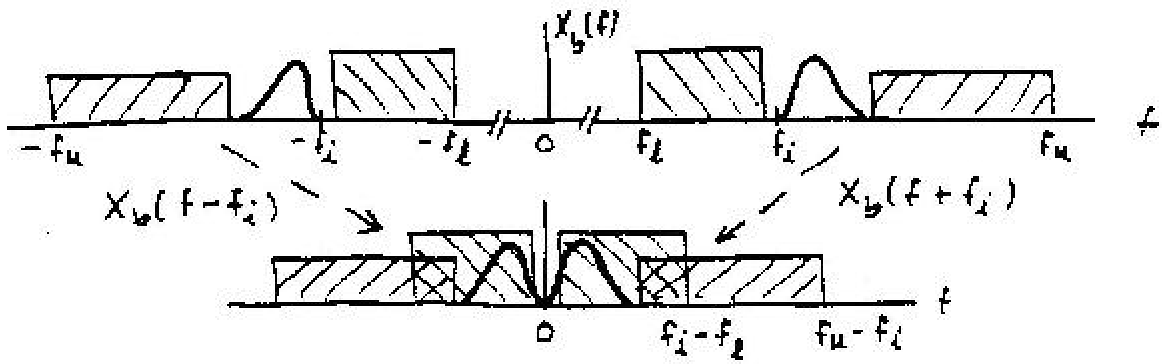


7.2-3

DSB



SSB



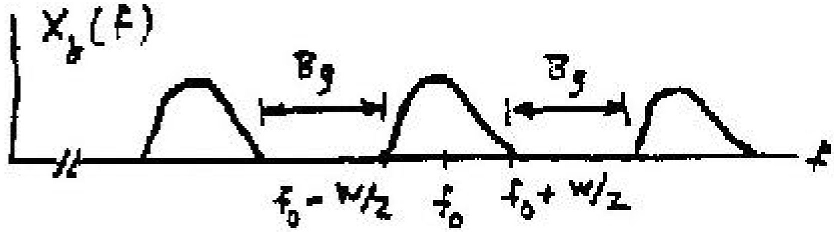
7.2-4

We want $|H(f)| \le 0.1$ for $|f - f_0| \ge \frac{W}{2} + B_g$

so $\frac{W/2 + B_g}{W} \ge \frac{1}{1.2}\sqrt{\ln(1/0.1)} \approx 1.26 \quad \Rightarrow \quad B_g \ge 0.76W$

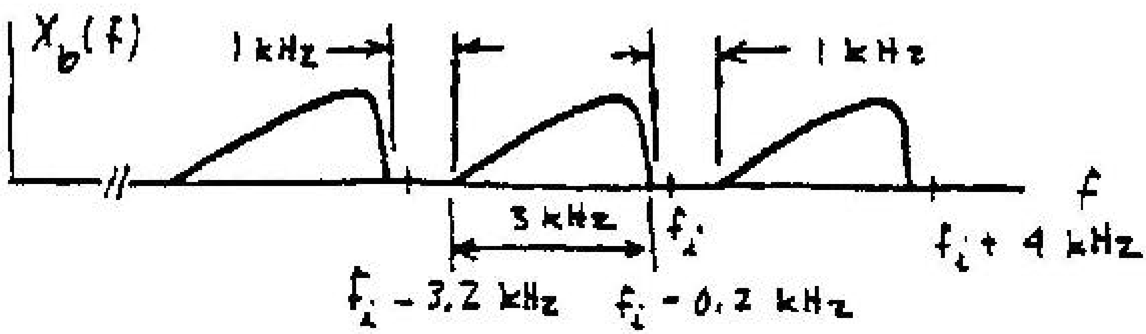
Thus, $B_T = 10W + 9B_g \ge 17W$

7.2-4 continued



7.2-5

Let $f_i = i^{th}$ subcarrier, take $B = 3$ kHz

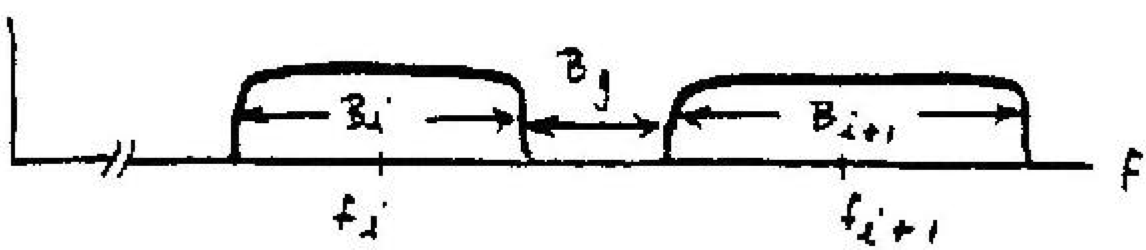


$$f_0 = f_i - 0.2 \text{ kHz } - \frac{B}{2} = f_i - 1.7\text{kHz}$$

We want $|H(f)|^2 \le 0.01$ for $|f - f_0| \ge B/2 + 1 \text{ kHz} = 2.5 \text{ kHz}$

Thus, $1 + \left(\dfrac{2 \text{ x } 2.5}{3}\right)^{2n} \ge 100 \;\Rightarrow\; n \ge \dfrac{1}{2}\dfrac{\ln 99}{\ln(5/3)} \approx 4.5 \;\Rightarrow\; n = 5$

7.2-6



(a) $B_i = 2M(D)W_i = 2\alpha M(D) f_i$. $f_i + B_i/2 + B_g = f_{i+1} - B_{i+1}/2$

Thus $f_{i+1} = \dfrac{[1+\alpha M(D)] f_i + B_g}{1 - \alpha M(D)}$

7.2-6 continued

(b) $\alpha M(D) = \frac{1}{2} B_1 / f_1 = 0.2 \quad \Rightarrow \quad f_{i+1} = (1.2 f_i + 400)/0.8$

so $f_2 = 3.5$ kHz, $f_3 = 5.75$ kHz, $f_4 = 9.125$ kHz

7.2-7

$x_c(t) = x_1(t)\cos\omega_c t + x_2(t)\cos(\omega_c t \pm 90^0)$ taking $A_c = 1$

$$2x_c(t)\cos(\omega_c t + \phi') = x_1(t)\left[\cos\phi' + \cos(2\omega_c t + \phi')\right] + x_2(t)\left[\cos(\phi' \pm 90^0) + \cos(2\omega_c t + \phi' \pm 90^0)\right]$$

$$2x_c(t)\cos(\omega_c t + \phi' \pm 90^0) = x_1(t)\left[\cos(\phi' \pm 90^0) + \cos(2\omega_c t + \phi' \pm 180^0)\right] + x_2(t)\left[\cos\phi' + \cos(2\omega_c t + \phi' \pm 180^0)\right]$$

Thus, LPF outputs are

$y_1(t) = K\left[x_1(t)\cos\phi' \mp x_2(t)\sin\phi'\right]$

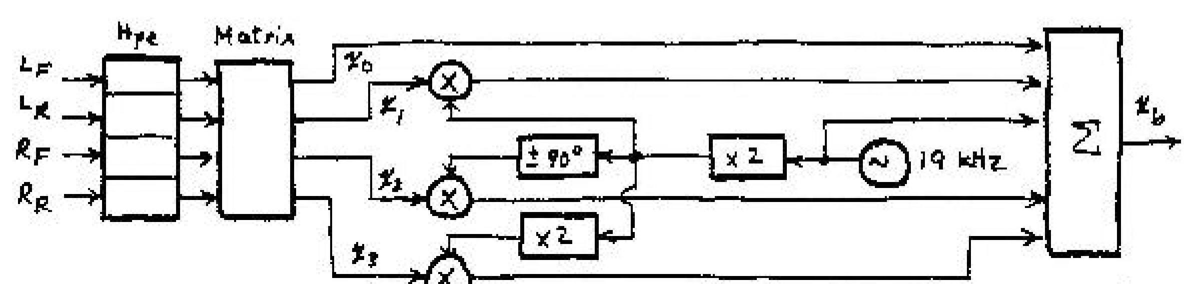
$y_2(t) = K\left[\mp x_1(t)\sin\phi' + x_2(t)\cos\phi'\right]$

7.2-8

We want $\left.\begin{aligned} x_0 + x_1 &= 2(L_F + L_R) \\ x_0 - x_1 &= 2(R_F + R_R) \end{aligned}\right\} \Rightarrow x_1(t) = L_F + L_R - (R_F + R_R)$

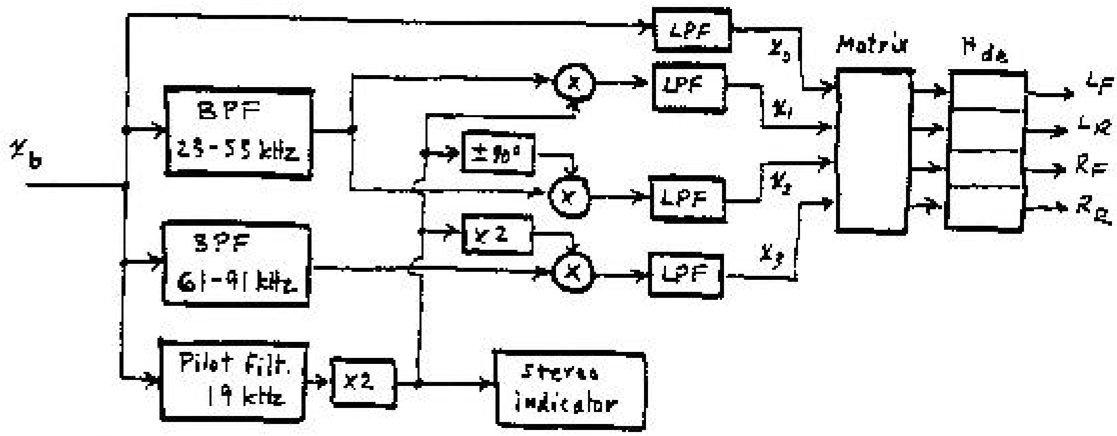
Take $x_2(t) = L_F - L_R - R_F + R_R$ so that

$x_0 + x_1 + x_3 = 3L_F + L_R + R_F - R_R$	$x_0 + x_1 + x_2 + x_3 = 4L_F$
$x_0 + x_1 - x_3 = L_F + 3L_R - R_F + R_R$	$x_0 + x_1 - x_2 - x_3 = 4L_R$
$x_0 - x_1 + x_3 = L_F - L_R + 3R_F + R_R$	$x_0 - x_1 - x_2 + x_3 = 4R_F$
$x_0 - x_1 - x_3 = -L_F + L_R + R_F + 3R_R$	$x_0 - x_1 + x_2 - x_3 = 4R_R$



(cont.)

7.2-8 continued



7.2-9

$$\Im\left[x_2(t)\sin\omega_c t\right] = -\frac{j}{2}\left[X_2(f-f_c) - X_2(f+f_c)\right] \text{ so}$$

$$X_c(f) = \frac{A_c}{2}\left[X_1(f-f_c) + X_1(f+f_c) \mp jX_2(f-f_c) \pm jX_2(f+f_c)\right]$$

$$Y_c(f) = H_C(f)X_c(f)$$

$$\Im\left[y_c(t)\cos\omega_c t\right] = \frac{1}{2}\left[H_C(f-f_c)X_c(f-f_c) + H_C(f+f_c)X_c(f+f_c)\right]$$

$$= \frac{A_c}{4}\left\{H_C(f-f_c)\left[X_1(f-2f_c) + X_1(f) \mp jX_2(f-2f_c) \pm jX_2(f)\right]\right.$$

$$\left. + H_C(f+f_c)\left[X_1(f) + X_1(f+2f_c) \mp jX_2(f) \pm jX_2(f+2f_c)\right]\right\}$$

The output of the lower LPF is

$$Y_1(f) = \frac{A_c}{4}\left\{\left[H_C(f-f_c) + H_C(f+f_c)\right]X_1(f) \pm j\left[H_C(f-f_c) - H_C(f+f_c)\right]X_2(f)\right\}$$

To remove cross talk from $X_2(f)$, we must have $H_C(f-f_c) - H_C(f+f_c) = 0$ for $|f| < W$

Then, $Y_1(f) = \frac{A_c}{4}\left[H_C(f-f_c)\right]X_1(f)$ so $Y_1(f)H_{eq}(\mathrm{f}) = KX_1(f)e^{-j\omega t_d}$

where the equalizer has $H_{eq}(f) = \dfrac{4K/A_c}{H_C(f-f_c)}e^{-j\omega t_d}$

7.2-10

$$r = (24+1) \text{ x } 8 \text{ kHz} = 200 \text{ kHz}, \;\; \tau = 0.5\frac{T_s}{24+1} = 2.5 \;\; \mu\text{s}, \;\; B_T \geq \frac{1}{2\tau} = 200 \;\; \text{kHz}$$

7.2-11

$$r = (24+1) \text{ x } 6 \text{ kHz} = 150 \text{ kHz}, \;\; \tau = 0.3\frac{T_s}{24+1} = 2 \;\; \mu\text{s}, \;\; B_T \geq \frac{1}{2\tau} = 250 \;\; \text{kHz}$$

7.2-12

$$f_s = 2W + B_g = \;\; 10 \;\; \text{kHz}$$

(a) $\tau = 0.25\dfrac{T_s}{20} = \; 1.25 \; \mu\text{s} \quad \Rightarrow B_T \geq 1/\tau \; = \; 800 \text{ kHz}$

(b) $B_T \; = B_b \; = \; \dfrac{1}{2} \text{ x } 20 \, f_s \; = 100 \text{ kHz}$

7.2-13

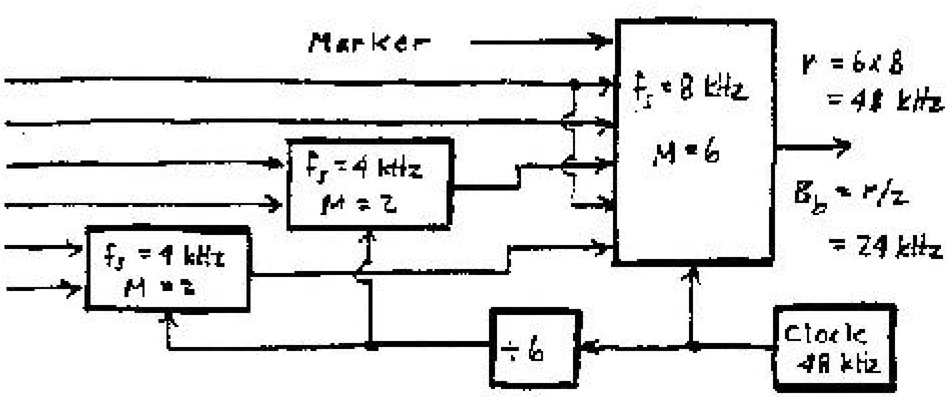
$$f_s = 2W + B_g \; = \;\; 5 \;\; \text{kHz}$$

(a) $\tau = 0.2\dfrac{T_s}{10} = \; 4\,\mu\text{s} \quad \Rightarrow B_T \geq 1/\tau \; = \; 250 \text{ kHz}$

(b) $B_b \; = \; \dfrac{1}{2} \text{ x } 10 \; f_s \; = 25 \text{ kHz}, \;\; D = f_\Delta / B_b = 3, \;\; B_T \approx 2(3+2)B_b \; = 250 \text{ kHz}$

7.2-14

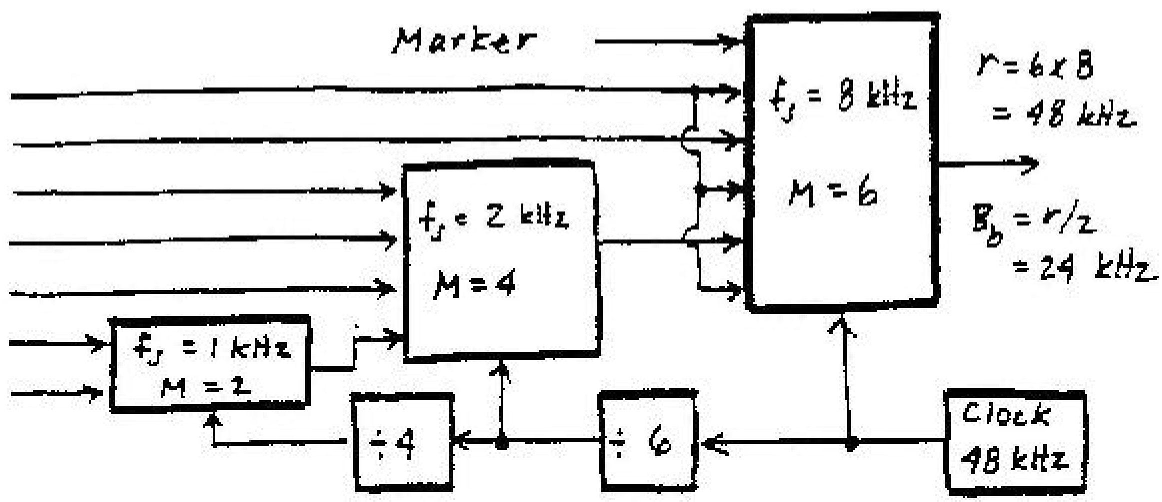
Sampling rate (kHz)	Minimum	Actual
	16	2 x 8
	7	8
	4	4
	3.6	4
	3	4
	2.4	4



FDM - SSB: $B_T \geq \sum_i W_i = 18 \;\; \text{kHz}$

7.2-15

Sampling rate (kHz)	Minimum	Actual
	24	3 x 8
	8	8
	2	2
	1.8	2
	1.6	2
	1.0	1
	0.6	1



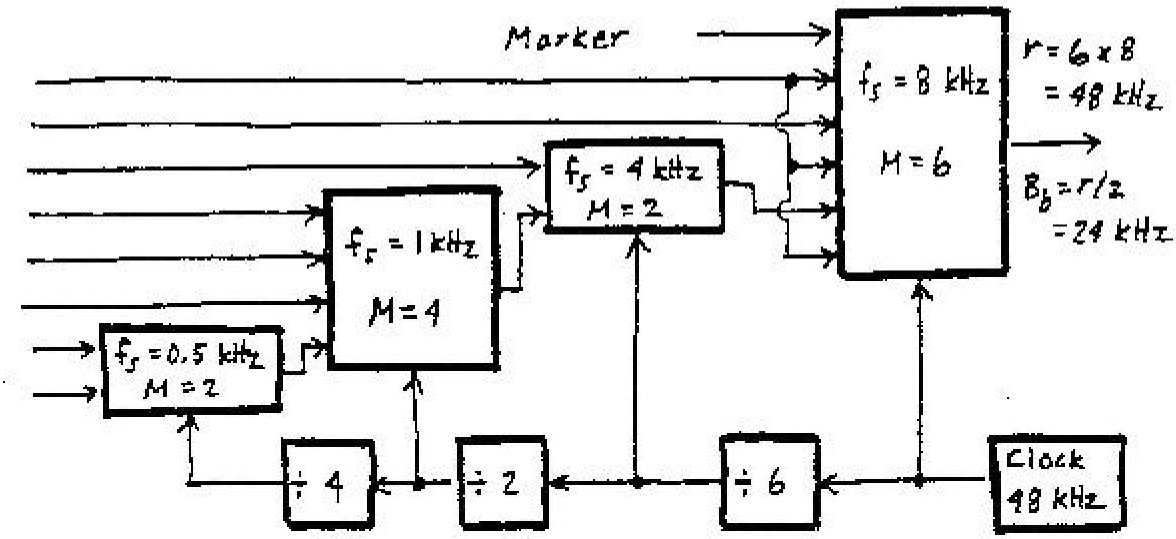
$$\text{FDM - SSB: } B_T \geq \sum_i W_i = 19.5 \text{ kHz}$$

7.2-16

Sampling rate (kHz)

Sampling rate (kHz)	Minimum	Actual
	24	3 x 8
	7	8
	4	4
	1	1
	0.8	1
	0.6	1
	0.4	0.5
	0.2	0.5

7.2-16 continued



FDM - SSB: $B_T \geq \sum_i W_i = 19$ kHz

7.2-17

$$-54.5BT_g \leq -40 \quad \Rightarrow \quad T_g \geq 0.734/B$$

$$\frac{T_s}{M} = T_g + 2t_0 + \tau \geq \frac{0.734}{B} + 3t_0, \quad t_0 = \tau = \frac{0.2}{25 \text{ x } 8 \text{ kHz}} = 1 \ \mu\text{s}$$

$$\frac{0.734}{B} \leq \frac{1}{Mf_s} - 3t_0 = 2 \ \mu\text{s} \quad \Rightarrow \quad B \geq 367 \ \text{kHz}$$

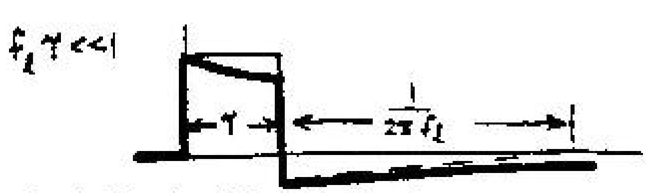
7.2-18

$$-54.5BT_g \leq -30 \quad \Rightarrow \quad T_g \geq 0.55/B$$

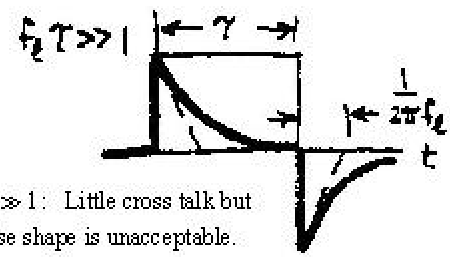
$$\frac{T_s}{M} = T_g + 2t_0 + \tau = T_g + \ 3 \text{ x } \frac{0.25T_s}{M} > \frac{0.55}{B} + \frac{3}{4}\frac{T_s}{M}$$

Thus, $\frac{1}{4}\frac{T_s}{M} > \frac{0.55}{B} \quad \Rightarrow \quad M < \frac{1}{4 \text{ x } 0.55}\frac{B}{f_s} = 28.4 \quad \Rightarrow \quad M = 28$

7.2-19



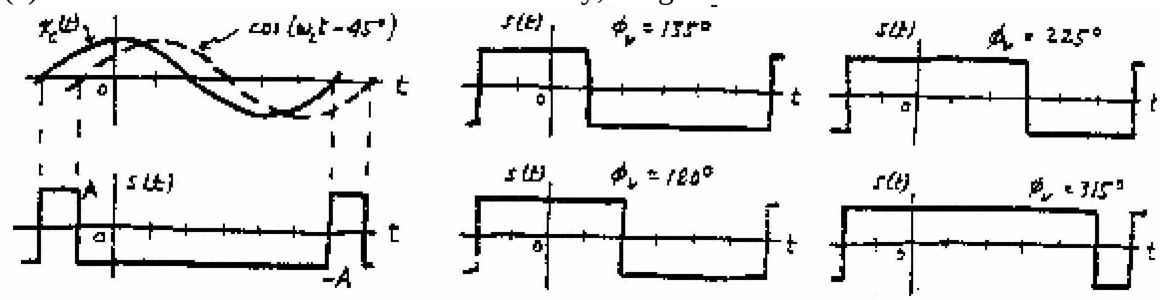
$f_\ell\tau \ll 1$: Negative tail decays slowly and may produce cross talk over several pulse slots.



$f_\ell\tau \gg 1$: Little cross talk but pulse shape is unacceptable.

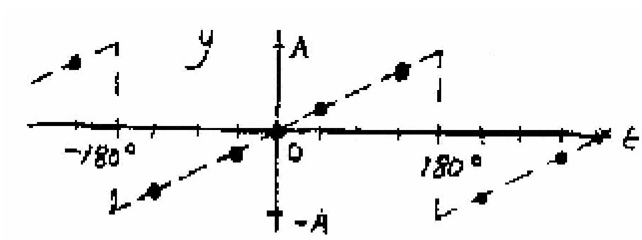
7.3-1

(a) similarly, we get



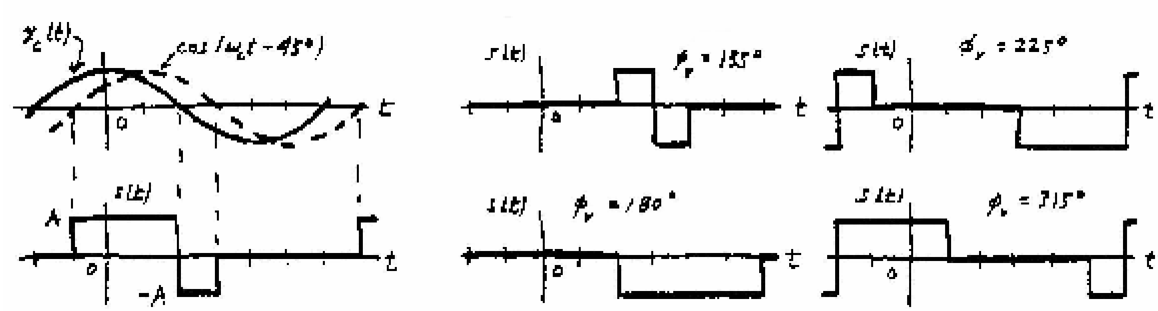
(b)

ϕ_V	45^0	135^0	180^0	225^0	315^0
y/A	-6/8	-2/8	0	2/8	6/8



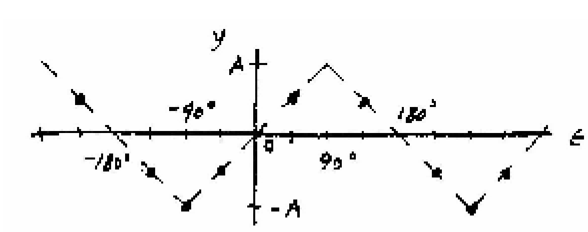
7.3-2

similarly, we get



(b)

ϕ_V	45^0	135^0	180^0	225^0	315^0
y/A	2/8	0	-4/8	-2/8	2/8



7.3-3

$t < 0, \quad \varepsilon_{ss} = \Delta f / K$

$t > 0, \quad \phi' = 2\pi f_1$ and $\dfrac{\Delta f + f_1}{K} \ll 1$ so assume $|\varepsilon| \ll 1$ and $\sin\varepsilon \approx \varepsilon$

Thus, $\dot{\varepsilon} + 2\pi K\varepsilon = 2\pi\left(\Delta f + f_1\right) \quad \Rightarrow \quad$ trial solution $\varepsilon = A + Be^{st}$

Then $Bse^{st} + 2\pi KA + 2\pi KBe^{st} = 2\pi\left(\Delta f + f_1\right)$

$$\text{so} \quad \left.\begin{matrix} 2\pi KA = 2\pi\left(\Delta f + f_1\right) \\ (s + 2\pi K)Be^{st} = 0 \end{matrix}\right\} \quad \Rightarrow \quad \begin{cases} A = \dfrac{\Delta f + f_1}{K} \\ s = -2\pi K \end{cases}$$

and $\varepsilon(t) = \dfrac{\Delta f + f_1}{K} + Be^{-2\pi Kt}, \quad t > 0,$

Since $\varepsilon(t)$ can make a step change at $t = 0$,

$$\varepsilon(0^+) = \frac{\Delta f + f_1}{K} + B = \varepsilon(0^-) = \frac{\Delta f}{K} \quad \Rightarrow \quad B = -\frac{f_1}{K}$$

Hence,

$$\varepsilon(t) = \begin{cases} \dfrac{\Delta f}{K} & t < 0 \\ \dfrac{\Delta f}{K} + \dfrac{f_1}{K}(1 - e^{-2\pi Kt}) & t > 0 \end{cases}$$



7.3-4

$$x_c(t) = \frac{1}{2}A_c\left[x(t)\cos\omega_c t - x_q(t)\sin\omega_c t\right] \quad \text{where } x_q(t) = \pm\tilde{x}(t) \text{ for SSB}$$

$$= A(t)\cos\left[\omega_c t + \phi(t)\right]$$

$$\text{with } A(t) = \frac{1}{2}A_c\sqrt{x^2(t) + x_q^2(t)}, \quad \phi(t) = \arctan\frac{x_q(t)}{x(t)}$$

If loop locks to $\phi(t)$ and $\varepsilon_{ss} \approx 0$, then the output is proportional to A(t).
Otherwise, $\phi(t)$, may be too rapid for loop to lock.

7.3-5

$$\cos\theta_v(t) \text{ x } \cos(\omega_1 t+\phi_1)=\frac{1}{2}\cos\left[\theta_v(t)-(\omega_1 t+\phi_1)\right]+ \text{ high frequency term}$$

Thus, $\cos\left[\theta_v(t)-(\omega_1 t+\phi_1)\right]=\cos\left(\omega_c t+\phi_0+90^0-\varepsilon_{ss}\right)$

so $\cos\theta_v(t)=\cos\left[(\omega_c+\omega_1)t+\phi_0+\phi_1+90^0-\varepsilon_{ss}\right]$

7.3-6

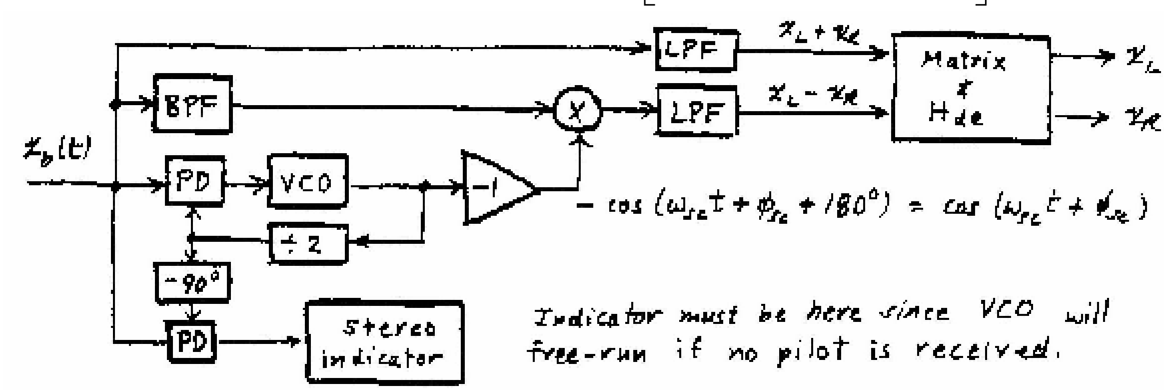
$$\cos\left[\theta_v(t)/n\right]=\cos(\omega_c t+\phi_0+90^0-\varepsilon_{ss})$$

so $\cos\theta_v(t)=\cos(n\omega_c t+n\phi_0+n90^0-n\varepsilon_{ss})$

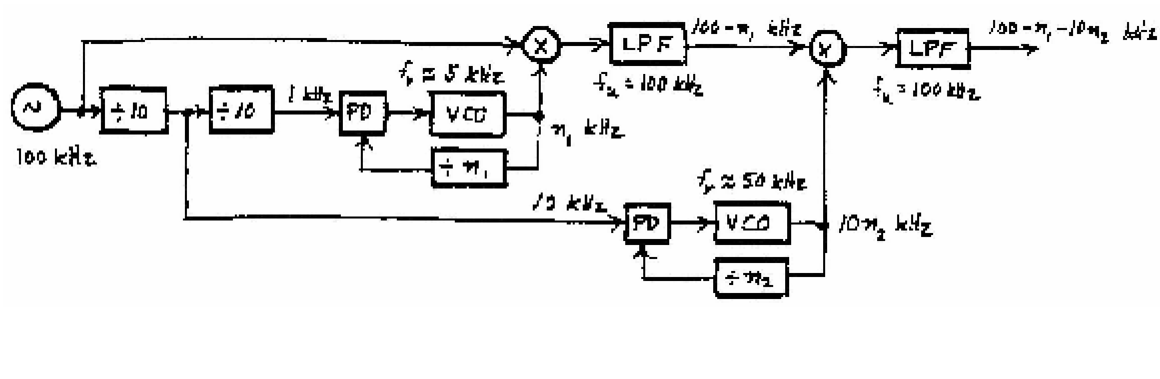
7.3-7

Let subcarrier be $\cos(\omega_{sc}t+\phi_{sc})$ so pilot signal is $\cos\left[(\omega_{sc}t+\phi_{sc})/2\right]$

and output of PLL doubler will be $\cos\phi_v(t)=\cos\left[2(\omega_{sc}t+\phi_{sc})/2+2 \text{ x } 90^0\right]$

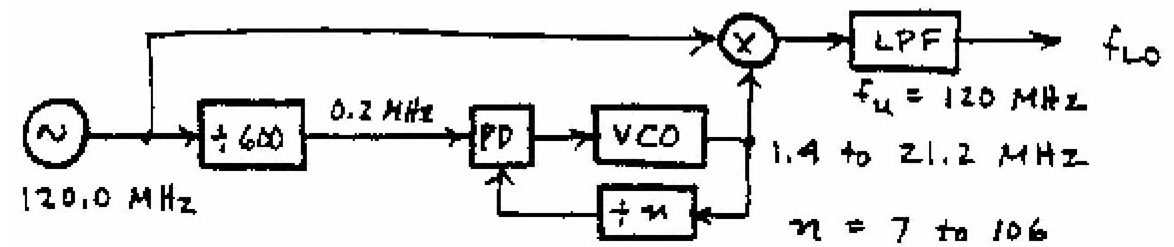


7.3-8



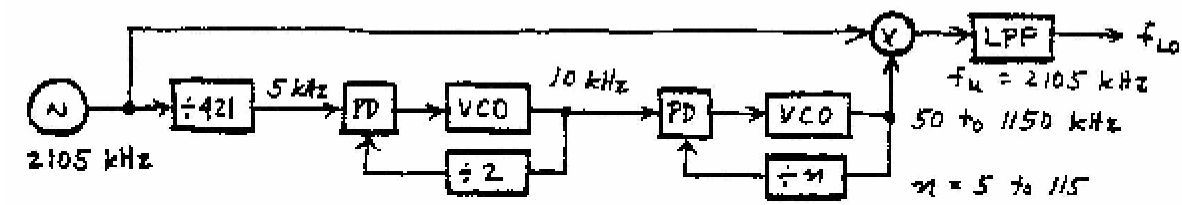
7.3-9

$f_{LO} = f_c + f_{IF} = 98.8$ to 118.6 MHz in steps of 0.2 MHz = 120.0 MHz $\div$ 600
120.0 - 98.8 = 106 x 0.2 MHz,   120.0 - 118.6 = 7 x 0.2 MHz



7.3-10

$f_{LO} = f_c + f_{IF} = 955$ to 2055 kHz in steps of 10 kHz = 2 x 2105 kHz $\div$ 421
2105 - 955 = 115 x 10 kHz,   2105 - 2055 = 5 x 10 kHz



7.3-11

$$Z(f) = \frac{1}{j2\pi f}Y(f) \quad \text{and} \quad \Phi(f) = \phi_\Delta X(f) \quad \text{for PM, so}$$

$$\frac{Z(f)}{X(f)} = \frac{1}{j2\pi f}\frac{1}{K_v}\frac{jfKH(f)}{jf + KH(f)}\phi_\Delta = \frac{\phi_\Delta}{2\pi K_v}\frac{KH(f)}{jf + KH(f)} = \frac{\phi_\Delta}{2\pi K_v}H_L(f)$$

7.4-1

(a) The frame should have an odd number of lines so that each field has a half-line to fill the small wedge at the top and bottom of the raster.



7.4-1 continued

(b) A linear sweep (sawtooth or triangular) is needed to give the same exposure time to each horizontal element. A triangular sweep would result in excessive retrace time, equal to the line time.

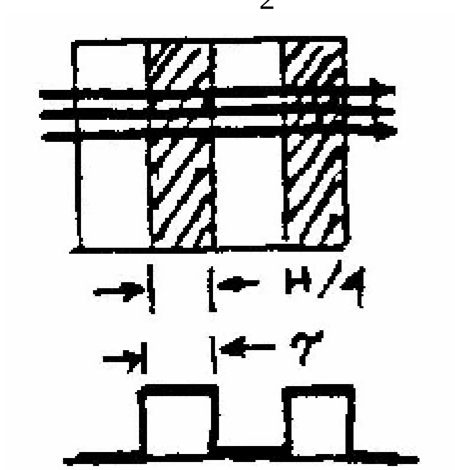
7.4-2

(a) No vertical dependence. Video signal is rectangular pulse train with

$\tau = (H/4)/s_h = 1/4f_h$ and $T_0 = 2\tau$.

Thus, $f_0 = 2f_h$

$c(nf_0) = K\text{sinc}\,\frac{n}{2}$





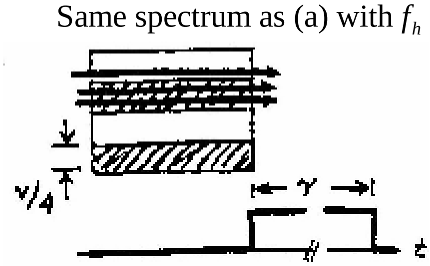
(b) No horizontal dependence. Video signal is rectangular pulse train with

$\tau = (V/4)/s_v = 1/4f_v$ and $T_0 = 2\tau$.

Thus, $f_0 = 2f_v$

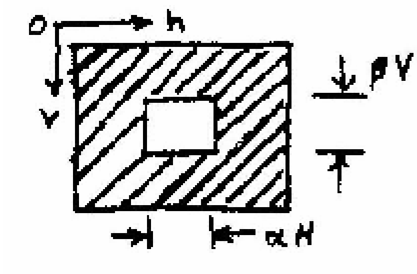
$c(nf_0) = K\text{sinc}\,\frac{n}{2}$

Same spectrum as (a) with f_h replaced by $f_v \ll f_h$, so much smaller bandwidth.



7.4-3

(a)



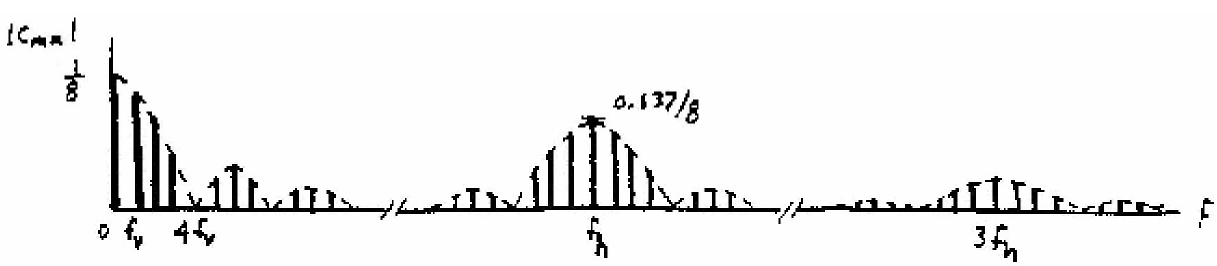
$$I(h,v)=\begin{cases}1 & \frac{H}{2}-\frac{\alpha H}{2}<h<\frac{H}{2}+\frac{\alpha H}{2},\quad \frac{V}{2}-\frac{\beta V}{2}<v<\frac{V}{2}+\frac{\beta V}{2}\\ 0 & \text{otherwise}\end{cases}$$

$$c_{mn}=\frac{1}{HV}\int_{(1-\alpha)H/2}^{(1+\alpha)H/2} e^{-j2\pi mh/H}\,dh\int_{(1-\beta)V/2}^{(1+\beta)V/2} e^{-j2\pi mv/V}\,dv$$

$$=\frac{1}{HV}\left(\frac{e^{-j\pi m\alpha}-e^{j\pi m\alpha}}{-j2\pi m/H}e^{-j\pi m}\right)\left(\frac{e^{-j\pi m\beta}-e^{j\pi m\beta}}{-j2\pi n/V}e^{-j\pi n}\right)$$

Thus, $|c_{mn}|=\left|\frac{\sin\pi m\alpha}{\pi m}\right|\left|\frac{\sin\pi n\beta}{\pi n}\right|=\alpha\beta\left|\text{sinc } m\alpha\ \text{sinc } n\beta\right|$

(b) $|c_{mn}|=\frac{1}{8}\left|\text{sinc }\frac{m}{2}\ \text{sinc }\frac{n}{4}\right|,\quad f_{mn}=mf_h+nf_v=\left(m+\frac{n}{100}\right)f_h$



7.4-4

$n_v=$ 0.7 x 230, $n_p=$ 1 x $n_v^2=25{,}921$

$B=0.35$ x 1 x 230/100 μs = 805 kHz

7.4-5

$n_v = 0.7\,(1125 - N_{vr}) \approx 787, \quad n_p = 5/3 \text{ x } 787^2 = 1.03 \text{ x } 10^6$

$$T_{line} = \frac{(2/60)\text{ sec}}{1125} = 29.6\ \mu\text{s}, \quad B = 0.35 \text{ x } \frac{5}{3} \text{ x } \frac{1125}{(1 - 0.2)29.6\ \mu\text{s}} = 27.7 \text{ MHz}$$

7.4-6

$n_v = 0.7\,(625 - 48) = 404, \quad n_p = 4/3 \text{ x } 404^2 = 2.18 \text{ x } 10^5$

$$T_{line} = \frac{1}{15.625\text{ kHz}} = 64\ \mu\text{s}, \quad B = 0.35 \text{ x } \frac{4}{3} \text{ x } \frac{625 - 48}{(64 - 10)\ \mu\text{s}} = 4.99 \text{ MHz}$$

7.4-7

(a) Since $\tilde{x}(t)$ is proportional to $x(t)$ averaged over the previous τ seconds, the picture will be smeared int he horizontal direction and five vertical lines will be lost.

(b) $\tilde{x}(t) = \int_{-\infty}^{t} x(\lambda)d\lambda - \int_{-\infty}^{t-\tau} x(\lambda)d\lambda = \int_{-\infty}^{t} x(\lambda)d\lambda - \int_{-\infty}^{t} x(\lambda - \tau)d\lambda$

so $\tilde{X}(f) = \dfrac{1}{j2\pi f} X(f) - \dfrac{1}{j2\pi f} X(f)e^{-j2\pi f\tau} = \dfrac{1 - e^{-j2\pi f\tau}}{j2\pi f} X(f)$

$Y(f) = H_{eq}(f)\tilde{X}(f) = KX(f)e^{-j\omega t_d}$

Thus, $H_{eq}(f) = \dfrac{j2\pi f K e^{-j\omega t_d}}{1 - e^{-j2\pi f\tau}} = \dfrac{K}{\tau \text{ sinc } f\tau} e^{-j\omega(t_d - \tau/2)}$

which can only hold for $|f| < 1/\tau$ since $H_{eq}(f) \to \infty$ at $f = 1/\tau,\ 2/\tau, ...$

7.4-8

(a) If gain of the chrominance amp is too high, then $|x_c|$ will be too large and all colors will be saturated and pastel colors will be too bright. If the gain of the chrominance amp is too low, then $|x_c|$ will be too small and all colors will be unsaturated and appear as "washed-out" pastels.

(b) If $+90^0$ error, then red $\to$ blue, blue $\to$ green, green $\to$ red.

If -90^0 error, then red $\to$ green, blue $\to$ red, green $\to$ blue.

If 180^0 error, then red $\to$ blue-green, blue $\to$ yellow (red-green),
green $\to$ purple (red-blue).

7.4-9

Let $x_b^{'}(t)$ be the BPF output in Fig. 7.4-11 so, from Eq. (15),

$$x_b^{'}(t) = x_{YH}(t) + x_Q(t)\sin\omega_{cc}t + x_I(t)\cos\omega_{cc}t + \hat{x}_{IH}(t)\sin\omega_{cc}t$$

where $x_{YH}(t)$ is the high-frequency portion of $x_Y(t)$.

7.4-9 continued

Thus,

$$v_I(t) = x_b^{'}(t) \text{ x } 2\cos\omega_{cc}t = 2x_{YH}\cos\omega_{cc}t + x_Q(t)\sin 2\omega_{cc}t + x_I(t)(1+\cos 2\omega_{cc}t) + \hat{x}_{IH}(t)\sin 2\omega_{cc}t$$

$$= x_I(t) + 2x_{YH}\cos\omega_{cc}t + x_I(t)\cos 2\omega_{cc}t + [x_Q(t) + \hat{x}_{IH}(t)]\sin 2\omega_{cc}t$$

$$v_Q(t) = x_b^{'}(t) \text{ x } 2\sin\omega_{cc}t = 2x_{YH}\sin\omega_{cc}t + x_Q(t)(1-\cos 2\omega_{cc}t) + x_I\sin 2\omega_{cc}t + \hat{x}_{IH}(1-\cos 2\omega_{cc}t)$$

$$= x_Q(t) + \hat{x}_{IH}(t) + 2x_{YH}(t)\sin\omega_{cc}t + x_I(t)\sin 2\omega_{cc}t - [x_Q(t) + \hat{x}_{IH}(t)]\cos 2\omega_{cc}t$$

7.4-10

To modify Eq. (15) to account for asymmetric sidebands in Q channel, let $x_{QH}(t)$ be the high-frequency portion of $x_Q(t)$. Then

$$x_b(t) = x_Y(t) + [x_I(t)\cos\omega_{cc}t + \hat{x}_{IH}(t)\sin\omega_{cc}t] + [x_Q(t)\underbrace{\cos(\omega_{cc}t - 90^0)}_{\sin\omega_{cc}t}$$

$$+\hat{x}_{QH}(t)\underbrace{\sin(\omega_{cc}t - 90^0)}_{-\cos\omega_{cc}t}$$

Let $x_b^{'}(t)$ be the BPF output at the receiver, so

$$x_b^{'}(t) = x_{YH} + [x_I(t)\cos\omega_{cc}t + \hat{x}_{IH}(t)\sin\omega_{cc}t] + [x_Q\sin\omega_{cc}t - \hat{x}_{QH}\cos\omega_{cc}t]$$

Thus

$$v_I(t) = x_b^{'} \text{ x } 2\cos\omega_{cc}t = x_I(t) - \hat{x}_{QH}(t) + 2x_{YH}(t)\cos\omega_{cc}t + [x_I(t) - \hat{x}_{QH}(t)]\cos 2\omega_{cc}t$$
$$+[x_Q(t) + \hat{x}_{IH}(t)]\sin 2\omega_{cc}t$$

$$v_Q(t) = x_b^{'} \text{ x } 2\sin\omega_{cc}t = x_Q(t) + \hat{x}_{IH}(t) + 2x_{YH}(t)\sin\omega_{cc}t + [x_I(t) - \hat{x}_{QH}(t)]\sin 2\omega_{cc}t$$
$$-[x_Q(t) + \hat{x}_{IH}(t)]\cos 2\omega_{cc}t$$

and lowpass filtering with B = 1.5 MHz yields

$$v_I^{'}(t) = x_I(t) - \hat{x}_{QH}(t) + 2x_{YH}(t)\cos\omega_{cc}t$$
$$v_Q^{'}(t) = x_Q(t) + \hat{x}_{IH}(t) + 2x_{YH}(t)\sin\omega_{cc}t$$

Now we have cross talk between I and Q channels since both $\hat{x}_{QH}(t)$ and $\hat{x}_{IH}(t)$ have components in 0.5 MHz $< f <$ 1.5 MHz. This quadrature color cross talk is eliminated by reducing the bandwidth of $x_Q(t)$ to 0.5 MHz so $\hat{x}_{QH}(t) = 0$ and the Q-channel LPF removes $\hat{x}_{IH}(t)$.

Chapter 8

8.1-1

$M = 12$ equally likely outcomes

$P(A) = 6/12$, $P(B) = 4/12$, $P(C) = 3/12$

$P(AB) = 2/12$, $P(AC) = 0$, $P(BC) = 1/12$

$P(A^cB) = 2/12$

A	1	2	3	4	*C*
	5	6	7	8	
B	9	10	11	12	

8.1-2

$M = 16$ equally likely outcomes

$P(A) = 4/16$, $P(B) = 6/16$, $P(C) = 6/16$

$P(AB) = 0$, $P(AC) = 2/16$, $P(BC) = 2/16$

$P(A^cB) = 6/16$

		A	1,1			
		1,2		2,1	*B*	
	1,3		2,2		3,1	
1,4		2,3		3,2		4,1
	2,4		3,3		4,2	
		3,4		4,3		
		C	4,4			

8.1-3

$$P(AB^c) = N_{AB^c}/N = \left(N_A - N_{AB}\right)/N = P(A) - P(AB)$$

8.1-4

$$N_A = N_{AB} + N_{AB^c} \qquad N_B = N_{AB} + N_{A^cB}$$

$$P(A+B) = \frac{N_{AB} + N_{AB^c} + N_{A^cB}}{N} = \frac{N_A + N_B - N_{AB}}{N} = P(A) + P(B) - P(AB)$$

8.1-5

$$N_A = N_{AB} + N_{AB^c} \qquad N_B = N_{AB} + N_{A^cB}$$

$$P(C) = \frac{N_{AB^c} + N_{A^cB}}{N} = \frac{N_A + N_B - 2N_{AB}}{N} = P(A) + P(B) - 2P(AB)$$

8.1-6

$$P(\text{match}) = P(HH+TT) = P(HH) + P(TT) = P(H)P(H) + P(T)P(T)$$

$$P(T) = P(H^c) = 1 - \frac{1+\in}{2} = \frac{1-\in}{2} \qquad P(\text{match}) = \left(\frac{1+\in}{2}\right)^2 + \left(\frac{1-\in}{2}\right)^2 = \frac{1+\in^2}{2} > \frac{1}{2}$$

8.1-7

Let A = "A fails," B = "B fails," C = "computer inoperable"

$P(A) = 0.01$, $P(B) = 0.005$, $P(B|A) = 4 \times 0.005 = 0.02$

$P(C) = P(AB) = P(B|A)P(A) = 0.0002$, $P(A|B) = P(AB)/P(B) = 0.04$

8.1-8

Let M = "match," H_1 = "heads on first toss," etc.

(a) $P(H_1) = ½$, $P(MH_1) = P(H_1H_2) = (½)^2$, $P(M|H_1) = P(MH_1)/P(H_1) = ½$

(b) Let A = "H_1 or H_2," $P(A) = P(H_1T_2 + T_1H_2 + H_1H_2) = ¾$

$P(MA) = ¼$, $P(M|A) = P(MA)/P(A) = 1/3$

(c) $P(M) = P(H_1H_2 + T_1T_2) = ½$, $P(A|M) = P(A)P(M|A)/P(M) = ½$

8.1-9

Let M = "match," H_1 = "heads on first toss," etc.

(a) $P(H_1) = ¼$, $P(MH_1) = P(H_1H_2) = (¼)^2$, $P(M|H_1) = P(MH_1)/P(H_1) = ¼$

(b) Let A = "H_1 or H_2," $P(A) = P(H_1T_2 + T_1H_2 + H_1H_2) = 2 \times ¼ \times ¾ + (¼)^2 = 7/16$

$P(MA) = P(H_1H_2) = (¼)^2$, $P(M|A) = P(MA)/P(A) = 1/7$

(c) $P(M) = P(H_1H_2) + P(T_1T_2) = (¼)^2 + (¾)^2 = 10/16$, $P(A|M) = P(A)P(M|A)/P(M) = 1/10$

8.1-10

Since $P(AB) = P(A|B)P(B) = P(B|A)P(A)$, $P(XYZ) = P(X)P(YZ|X)$ where

$P(YZ|X) = \dfrac{P(XYZ)}{P(X)} = \dfrac{P(XY)}{P(X)}\dfrac{P(XYZ)}{P(XY)} = P(Y|X)P(Z|XY)$ so $P(XYZ) = P(X)P(Y|X)P(Z|XY)$

8.1-11

Let F = "fair coin," L = "loaded coin," A = "all tails," $P(F) = 1/3$, $P(L) = 2/3$, $P(A|F) = (½)^2$, $P(A|L) = (¾)^2$

(a) $P(A) = P(A|F)P(F) + P(A|L)P(L) = 11/24$

(b) $P(L|A) = P(L)P(A|L)/P(A) = 9/11$

8.1-12

Let F = "fair coin," L = "loaded coin," A = "all tails;" $P(F) = 1/3$, $P(L) = 2/3$, $P(A|F) = (½)^2$, $P(A|L) = (¾)^2$

(a) $P(A) = P(A|F)P(F) + P(A|L)P(L) = 31/96$

(b) $P(L|A) = P(L)P(A|L)/P(A) = 27/31$

8.1-13

Let R_1 = "first marble is red," etc., M = "match;" $P(R_1) = 5/10$, $P(W_1) = 3/10$, $P(G_1) =2/10$, $P(M|R_1) = P(R_2|R_1) = (5 - 1)/(10 - 1) = 4/9$, $P(M|W_1) = 2/9$, $P(M|G_1) = 1/9$

(a) $P(M) = P(M|R_1) \times P(R_1) + P(M|W_1) \times P(W_1) + P(M|G_1) \times P(G_1)$

$$= \frac{4}{9}\times\frac{5}{10}+\frac{2}{9}\times\frac{3}{10}+\frac{1}{9}\times\frac{2}{10}=\frac{14}{45}$$

(b) $P(W_1|M) = P(W_1)P(M|W_1)/P(M) = 3/14$

8.1-14

Let R_1 = "first marble is red," etc., M = "match;" $P(R_1) = 5/10$, $P(W_1) = 3/10$, $P(G_1) =2/10$, $P(M|R_1) = P(R_3R_2|R_1) = P(R_3|R_2R_1)P(R_2|R_1) = \frac{5-2}{10-2}\times\frac{5-1}{10-1}=\frac{3}{8}\times\frac{4}{9}$, $P(M|W_1) = \frac{1}{8}\times\frac{2}{9}$,

$P(M|G_1) = \frac{0}{8}\times\frac{1}{9}$

(a) $P(M) = P(M|R_1) \times P(R_1) + P(M|W_1) \times P(W_1) + P(M|G_1) \times P(G_1)$

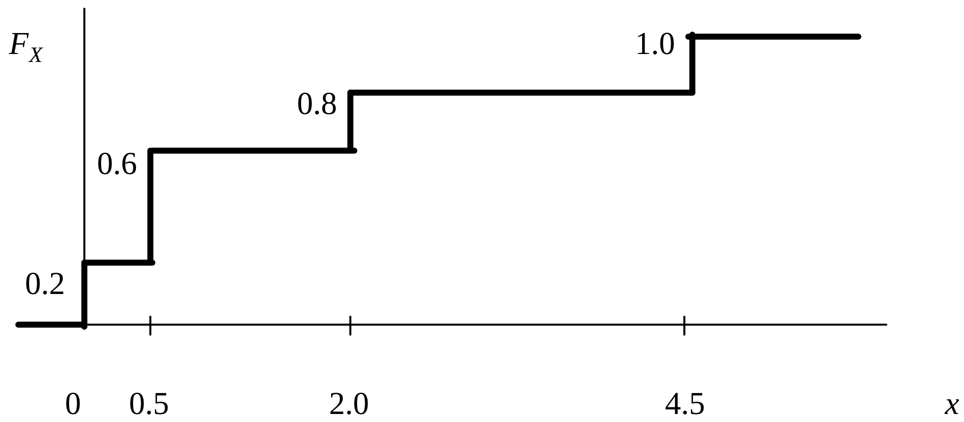
$$= \frac{12}{72}\times\frac{5}{10}+\frac{2}{72}\times\frac{3}{10}+0\times\frac{2}{10}=\frac{11}{120}$$

(b) $P(W_1|M) = P(W_1)P(M|W_1)/P(M) = 1/11$

8.2-1

$P(N_i) = 1/5$

N	x_i	$P_X(x_i)$	$F_X(x_i)$
0	0	0.2	0.2
-1,1	0.5	0.4	0.6
2	2	0.2	0.8
3	4.5	0.2	1.0



(cont.)

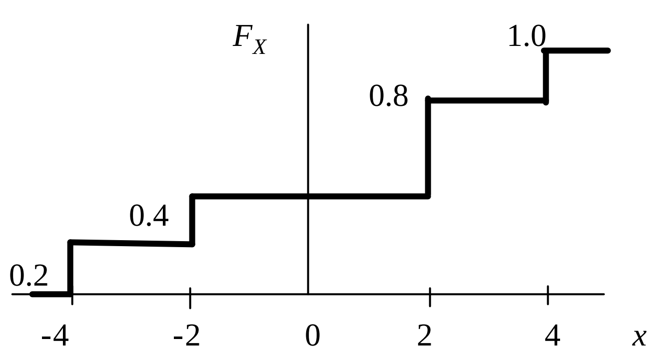
$P(X \leq 0) = F_X(0) = 0.2$, $P(2 < X \leq 3) = F_X(3) - F_X(2) = 0$, $P(X < 2) = F_X(2 - \in) = 0.6$,

$P(X \geq 2) = 1 - 0.6 = 0.4$

8.2-2

$P(N_i) = 1/5$

N	x_i	$P_X(x_i)$	$F_X(x_i)$
3	-4	0.2	0.2
2	-2	0.2	0.4
-1,1	2	0.4	0.8
0	4	0.2	1.0



$P(X \le 0) = F_X(0) = 0.4$, $P(2 < X \le 3) = F_X(3) - F_X(2) = 0$, $P(X < 2) = F_X(2 - \in) = 0.4$,

$P(X \ge 2) = 1 - 0.4 = 0.6$

8.2-3

$$F_X(x) = \int_{-\infty}^{x} p_X(\lambda)\, d\lambda = \begin{cases} 0 & x \le 0 \\ \int_0^x \lambda e^{-\lambda}\, d\lambda = 1 - (x+1)e^{-x} & x > 0 \end{cases}$$

$P(X \le 1) = F_X(1) = 0.264$, $P(X > 2) = 1 - F_X(2) = 0.406$, $P(1 < X \le 2) = F_X(2) - F_X(1) = 0.330$

8.2-4

$$F_X(x) = \int_{-\infty}^{x} p_X(\lambda)\, d\lambda = \begin{cases} \int_{-\infty}^{x} \frac{1}{2} e^{\lambda}\, d\lambda = \frac{1}{2} e^{x} & x \le 0 \\ \frac{1}{2} + \int_0^x \frac{1}{2} e^{-\lambda}\, d\lambda = 1 - \frac{1}{2} e^{-x} & x > 0 \end{cases}$$

$P(X \le 0) = F_X(0) = 1/2$, $P(X > 1) = 1 - F_X(1) = 0.184$, $P(0 < X \le 1) = F_X(1) - F_X(0) = 0.316$

8.2-5

$F_X(\infty) = 100K = 1 \;\Rightarrow\; K = 0.01$ so $p_X(x) = dF_X(x)/dx = 0.2x[u(x) - u(x - 10)]$

$P(X \le 5) = F_X(5) = K \times 5^2 = 0.25$, $P(5 < X \le 7) = F_X(7) - F_X(5) = 0.49 - 0.25 = 0.24$

8.2-6

$F_X(\infty) = K/\sqrt{2} = 1 \;\Rightarrow\; K = \sqrt{2}$ so $p_X(x) = dF_X(x)/dx = \frac{\sqrt{2}\pi}{40} \cos\frac{\pi x}{40} [u(x) - u(x - 10)]$

$P(X \le 5) = F_X(5) = K \sin\frac{\pi}{8} = 0.541$, $P(5 < X \le 7) = F_X(7) - F_X(5) = K \sin\frac{7\pi}{40} - 0.541 = 0.198$

8.2-7

$P(Z < 0) = 0$, $P(Z \le 0) = P(X \le 0) = ½$, $P(Z \le z) = P(X \le z)$ for $z > 0$

$$F_Z(z) = \begin{cases} 0 & z < 0 \\ \frac{1}{2} + \frac{1}{\pi} \arctan z & z \ge 0 \end{cases} \qquad p_Z(z) = \frac{d}{dz} F_Z(z) = \begin{cases} 0 & z < 0 \\ \frac{1}{2}\delta(z) + \frac{1}{\pi(1+z^2)} & z \ge 0 \end{cases}$$

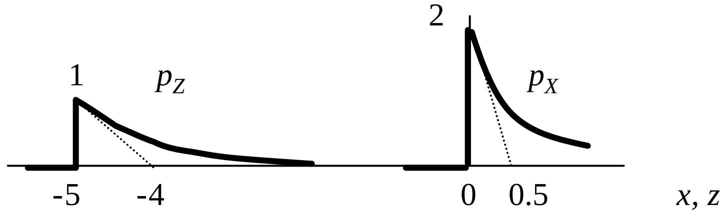
8.2-8

$P(Z < -1) = 0$, $P(Z \le -1) = P(X \le 0) = ½$, $P(Z \le z) = P(X \le z)$ for $z > 0$

$$F_Z(z) = \begin{cases} 0 & z < -1 \\ 1/2 & -1 \le z \le 0 \\ \dfrac{1}{2} + \dfrac{1}{\pi}\arctan z & z \ge 0 \end{cases} \qquad p_Z(z) = \frac{d}{dz} F_Z(z) = \begin{cases} \dfrac{1}{2}\delta(z+1) & z \le 0 \\ \dfrac{1}{\pi(1+z^2)} & z > 0 \end{cases}$$

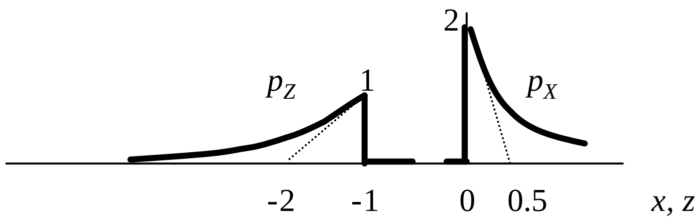
8.2-9

$$p_Z(z) = \frac{1}{|2|} 2e^{-2(z+5)/2} u\left(\frac{z+5}{2}\right) = e^{-(z+5)} u(z+5)$$



8.2-10

$$p_Z(z) = \frac{1}{|-2|} 2e^{-2(z-1)/(-2)} u\left(\frac{z-1}{-2}\right) = e^{(z-1)} u[-(z-1)]$$



8.2-11

Monotonic transformation with $g^{-1}(z) = z^2 - 1$, $dg^{-1}/dz = 2z$, $p_X(x) = ¼$ for $-1 \le x \le 3$, so

$$p_Z(z) = \frac{1}{4}|2z|[u(z) - u(z-2)] = \frac{z}{2}[u(z) - u(z-2)]$$

8.2-12

$g_1(x) = -x[u(x+1) - u(x)]$, $g_1^{-1}(z) = -z[u(z) - u(z-1)]$, $dg_1^{-1}/dz = -1$

$g_2(x) = x[u(x) - u(x-3)]$, $g_2^{-1}(z) = z[u(z) - u(z-3)]$, $dg_2^{-1}/dz = 1$, $p_X(x) = ¼$ for –1 ⦸ x ⦸ 3, so

$$p_Z(z) = \begin{cases} \frac{1}{4}|-1| + \frac{1}{4}|1| = \frac{1}{2} & 0 \le z \le 1 \\ \frac{1}{4}|1| = \frac{1}{4} & 1 < z \le 3 \end{cases}$$

8.2-13

$g_1(x) = \sqrt{-x}\,[u(x+1) - u(x)]$, $g_1^{-1}(z) = -z^2[u(z) - u(z-1)]$, $dg_1^{-1}/dz = -2z$

$g_2(x) = \sqrt{x}\,[u(x) - u(x-3)]$, $g_2^{-1}(z) = z^2[u(z) - u(z - \sqrt{3})]$, $dg_2^{-1}/dz = 2z$, $p_X(x) = ¼$ for –1 ⦸ x ⦸ 3, so

$$p_Z(z) = \begin{cases} \frac{1}{4}|-2z| + \frac{1}{4}|2z| = z & 0 \le z \le 1 \\ \frac{1}{4}|2z| = \frac{z}{2} & 1 < z \le \sqrt{3} \end{cases}$$

8.2-14

$g_1(x) = x^2u(-x)$, $g_1^{-1}(z) = -\sqrt{z}\,u(z)$, $dg_1^{-1}/dz = -1/2\sqrt{z}$, $g_2(x) = x^2u(x)$,

$g_2^{-1}(z) = +\sqrt{z}\,u(z)$, $dg_2^{-1}/dz = +1/2\sqrt{z}$ (cont.)

$p_Z(z) = 0$ for $z < 0$, $p_Z(z) = p_X(\sqrt{z})\left|\frac{1}{2\sqrt{z}}\right| + p_X(-\sqrt{z})\left|-\frac{1}{2\sqrt{z}}\right|$ for $z > 0$, so

$$p_Z(z) = \frac{1}{2\sqrt{z}}\left[p_X(\sqrt{z}) + p_X(-\sqrt{z})\right]u(z)$$

8.2-15

$$p_Y(y) = \int_{-\infty}^{\infty} p_{XY}(x,y)\,dx = ye^{-y}u(y)\int_0^{\infty} e^{-yx}\,dx = e^{-y}u(y),$$

$$p_{XY}(x,y) = ye^{-yx}u(x)e^{-y}u(y) \ne p_X(x)p_Y(y),\ p_X(x|y) = p_{XY}(x,y)/p_Y(y) = ye^{-yx}u(x)$$

8.2-16

$$p_Y(y)=\int_{-\infty}^{\infty} p_{XY}(x,y)\,dx=\frac{1}{40}\Pi\left(\frac{y}{6}\right)\int_{-1}^{1}\left(x^2+2xy+y^2\right)dx=\frac{1}{60}\left(1+3y^2\right)\Pi\left(\frac{y}{6}\right),$$

$$p_{XY}(x,y)=\frac{3}{2}\frac{(x+y)^2}{(1+3y^2)}\Pi\left(\frac{x}{2}\right)\frac{1}{60}(1+3y^2)\Pi\left(\frac{y}{6}\right)\neq p_X(x)p_Y(y)$$

$$p_X(x|y)=\ p_{XY}(x,y)/p_Y(y)=\frac{3(x+y)^2}{2(1+3y^2)}\Pi\left(\frac{x}{2}\right)$$

8.2-17

$$\int_{-\infty}^{\infty} p_X(x|y)\,dx=\int_{-\infty}^{\infty}\frac{p_{XY}(x,y)}{p_Y(y)}dx=\frac{1}{p_Y(y)}\int_{-\infty}^{\infty} p_{XY}(x,y)\,dx=1$$

For any given $Y=y$, X must be somewhere in the range $-\infty < x < \infty$.

8.2-18

$$p_{XY}(x,y) = p_X(x|y)p_Y(y),\ \ p_X(x)=\int_{-\infty}^{\infty} p_{XY}(x,y)\,dy=\int_{-\infty}^{\infty} p_X(x|y)p_Y(y)\,dy$$

Thus, $p_Y(y|x) = p_{XY}(x,y)/p_X(x) = p_X(x|y)p_Y(y)/\int_{-\infty}^{\infty} p_X(x|y)p_Y(y)\,dy$

8.3-1

$$m_X=a\int_0^{\infty} xe^{-ax}\,dx=1/a,\ \ \overline{X^2}=a\int_0^{\infty} x^2e^{-ax}\,dx=2/a^2,\ \text{so}\ \ \sigma_X=\sqrt{\frac{2}{a^2}-\left(\frac{1}{a}\right)^2}=\frac{1}{a}$$

8.3-2

$$m_X=a^2\int_0^{\infty} x^2e^{-ax}\,dx=2/a,\ \ \overline{X^2}=a^2\int_0^{\infty} x^3e^{-ax}\,dx=6/a^2,\ \text{so}\ \ \sigma_X=\sqrt{\frac{6}{a^2}-\left(\frac{2}{a}\right)^2}=\frac{\sqrt{2}}{a}$$

8.3-3

$$m_X=\frac{\sqrt{2}}{\pi}\int_{-\infty}^{\infty}\frac{x}{1+(x-a)^4}dx=\frac{\sqrt{2}}{\pi}\left[\int_{-\infty}^{\infty}\frac{\lambda}{1+\lambda^4}d\lambda+\int_{-\infty}^{\infty}\frac{a}{1+\lambda^4}d\lambda\right]=a,$$

$$\overline{X^2}=\frac{\sqrt{2}}{\pi}\int_{-\infty}^{\infty}\frac{x^2}{1+(x-a)^4}dx=\frac{\sqrt{2}}{\pi}\left[\int_{-\infty}^{\infty}\frac{\lambda^2}{1+\lambda^4}d\lambda+\int_{-\infty}^{\infty}\frac{2\lambda}{1+\lambda^4}d\lambda+\int_{-\infty}^{\infty}\frac{a^2}{1+\lambda^4}d\lambda\right]=1+a^2,\ \text{so}$$

$$\sigma_X=\sqrt{\left(1+a^2\right)-a^2}=1$$

8.3-4

$P(X = b) = 1 - p$, $m_X = ap + b(1 - p)$, $\overline{X^2} = a^2p + b^2(1 - p)$,

$\sigma_X^2 = a^2p + b^2(1 - p) - [ap + b(1 - p)]^2 = (a - b)^2p(1 - p)$, $\sigma_X = |a - b|\sqrt{p(1-p)}$

8.3-5

$$m_X = \sum_{i=0}^{K-1} ai\,\frac{1}{K} = \frac{a}{K}\frac{(K-1)K}{2} = \frac{K-1}{2}a\,,$$

$$\overline{X^2} = \sum_{i=0}^{K-1}(ai)^2\,\frac{1}{K} = \frac{a^2}{K}\frac{(K-1)K[2(K-1)+1]}{6} = \frac{(K-1)(2K-1)}{6}a^2$$

$$\sigma_X{}^2 = \overline{X^2} - m_X{}^2 = \frac{(K-1)a^2}{2}\frac{2(2K-1)-3(K-1)}{6} = \frac{K^2-1}{12}a^2, \quad \sigma_X = \sqrt{\frac{K^2-1}{3}}\,\frac{a}{2}$$

8.3-6

$$m_Y = \int_{-\infty}^{\infty} a\cos x\, p_X(x)\, dx = \frac{a}{2\pi}\int_{\theta}^{\theta+2\pi}\cos x dx = 0\,,$$

$$\overline{Y^2} = \int_{-\infty}^{\infty} a^2\cos^2 x\, p_X(x)\, dx = \frac{a^2}{2\pi}\int_{\theta}^{\theta+2\pi}\cos^2 x dx = \frac{a^2}{2} \quad \sigma_Y = \sqrt{a^2/2 - 0} = a/\sqrt{2}$$

8.3-7

$$m_Y = \int_{-\infty}^{\infty} a\cos x\, p_X(x)\, dx = \frac{a}{\pi}\int_{\theta}^{\theta+\pi}\cos x dx = -\frac{2a}{\pi}\sin\theta\,,$$

$$\overline{Y^2} = \int_{-\infty}^{\infty} a^2\cos^2 x\, p_X(x)\, dx = \frac{a^2}{\pi}\int_{\theta}^{\theta+\pi}\cos^2 x dx = \frac{a^2}{2}$$

$$\sigma_Y = \sqrt{\frac{a^2}{2} - \left(-\frac{2a}{\pi}\sin\theta\right)^2} = a\sqrt{\frac{1}{2} - \frac{4}{\pi^2}\sin^2\theta}$$

8.3-8

$$m_Y = \alpha m_X + \beta, \quad \overline{Y^2} = E\left[(\alpha X + \beta)^2\right] = E\left[\alpha^2 X^2 + 2\alpha\beta X + \beta^2\right] = \alpha^2\,\overline{X^2} + 2\alpha\beta m_X + \beta^2$$

$$\sigma_Y{}^2 = \overline{Y^2} - m_Y{}^2 = \alpha^2\left(\overline{X^2} - m_X{}^2\right) = \alpha^2\sigma_X{}^2, \quad \sigma_Y = |\alpha|\,\sigma_X$$

8.3-9

$$\overline{Y^2} = E\left[(X+\beta)^2\right] = E\left[X^2 + 2\beta X + \beta^2\right] = \overline{X^2} + 2\beta m_X + \beta^2$$

$$\frac{d}{d\beta}\overline{Y^2} = 2m_X + 2\beta = 0 \quad \Rightarrow \quad \beta = -m_X$$

8.3-10

$$P(X \ge a) = \int_a^\infty p_X(x)\,dx \quad \text{and} \quad p_X(x) = 0 \text{ for } x < 0$$

$$E[X] = \int_0^\infty p_X(x)\,dx \ge \int_a^\infty x p_X(x)\,dx \ge a\int_a^\infty p_X(x)\,dx = aP(X \ge a)\text{, so } P(X \ge a) \le m_X / a$$

8.3-11

$$E\left[(X \pm Y)^2\right] = E\left[X^2 \pm 2XY + Y^2\right] = \overline{X^2} \pm 2\overline{XY} + \overline{Y^2} \ge 0$$

so $2\overline{XY} \ge -\left(\overline{X^2} + \overline{Y^2}\right)$ and $2\overline{XY} \le \left(\overline{X^2} + \overline{Y^2}\right)$, $\Rightarrow\ -\dfrac{\overline{X^2}+\overline{Y^2}}{2} \le \overline{XY} \le \dfrac{\overline{X^2}+\overline{Y^2}}{2}$

8.3-12

$$C_{XY} = E\left[XY - m_X Y - m_Y X + m_X m_Y\right] = \overline{XY} - m_X m_Y$$

(a) $\overline{XY} = \overline{X}\,\overline{Y} = m_X m_Y \quad \Rightarrow \quad C_{XY} = 0$

(b) $\overline{XY} = E\left[X(\alpha X + \beta)\right] = \alpha\overline{X^2} + \beta m_X$ and $m_Y = \alpha m_X + \beta$ so $C_{XY} = \alpha\left(\overline{X^2} - m_X\right) = \alpha\sigma_X{}^2$

8.3-13

$$\epsilon^2 = E\left[Y^2 - 2(\alpha X + \beta)Y + (\alpha X + \beta)^2\right] = \overline{Y^2} - 2\alpha\overline{XY} - 2\beta\overline{Y} + \alpha^2\overline{X^2} + 2\alpha\beta\overline{X} + \beta^2$$

$\partial\epsilon^2/\partial\alpha = -2\overline{XY} + 2\alpha\overline{X^2} + 2\beta\overline{Y} = 0$ and $\partial\epsilon^2/\partial\beta = -2\overline{Y} - 2\alpha\overline{X} + 2\beta = 0$ so

$\alpha = \left(\overline{XY} - \overline{X}\,\overline{Y}\right)/\sigma_X{}^2$ and $\beta = \overline{Y} - \alpha\overline{X}$

8.3-14

$$\frac{d^n}{d\nu^n}\Phi_X(\nu) = \frac{d^n}{d\nu^n}E\left[e^{j\nu X}\right] = E\left[(jX)^n e^{j\nu X}\right] = j^n E\left[X^n e^{j\nu X}\right]\text{, so}$$

$$\frac{d^n}{d\nu^n}\Phi_X(0) = j^n E\left[X^n\right] \quad \Rightarrow \quad E\left[X^n\right] = j^{-n}\frac{d^n}{d\nu^n}\Phi_X(\nu)\Big|_{\nu=0}$$

8.3-15

$$\mathcal{F}\left[ae^{-at}u(t)\right] = \frac{a}{a+j2\pi f} \quad \Rightarrow \quad \mathcal{F}^{-1}\left[ae^{-af}u(f)\right] = \frac{a}{a+j2\pi(-t)} \text{ so}$$

$$\Phi_X(2\pi t) = \frac{a}{a-j2\pi t} \quad \text{and} \quad \Phi_X(\nu) = \frac{a}{a-j\nu} = \left(1-j\frac{\nu}{a}\right)^{-1}$$

$$\frac{d\Phi_X}{d\nu} = -\left(1-j\frac{\nu}{a}\right)^{-2}\left(-\frac{j}{a}\right) \quad \Rightarrow \quad \overline{X} = j^{-1}\left(\frac{j}{a}\right) = \frac{1}{a}$$

$$\frac{d^2\Phi_X}{d\nu^2} = 2\left(1-j\frac{\nu}{a}\right)^{-3}\left(-\frac{j}{a}\right)^2 \quad \Rightarrow \quad \overline{X^2} = j^{-2}2\left(\frac{j}{a}\right)^2 = \frac{2}{a^2}$$

$$\frac{d^3\Phi_X}{d\nu^3} = -6\left(1-j\frac{\nu}{a}\right)^{-4}\left(-\frac{j}{a}\right)^3 \quad \Rightarrow \quad \overline{X^3} = j^{-3}(-6)\left(\frac{j}{a}\right)^3 = \frac{6}{a^3}$$

8.3-16

$$\Phi_Y(\nu) = E\left[e^{j\nu x^2}\right] = \int_0^\infty e^{j\nu x^2}2axe^{-ax^2}\,dx = \int_0^\infty e^{j\nu\lambda}ae^{-a\lambda}\,d\lambda \quad \Rightarrow \quad p_Y(y) = ae^{-ay}u(y)$$

8.3-17

$$\Phi_Y(\nu) = E\left[e^{j\nu\sin x}\right] = \int_{-\pi/2}^{\pi/2} e^{j\nu\sin x}\frac{1}{\pi}\,dx$$

Let $\lambda = \sin x$, $d\lambda = \cos x\ dx$ where $\cos x = \sqrt{1-\sin^2 x} = \sqrt{1-\lambda^2}$, so

$$\Phi_Y(\nu) = \int_{\sin(-\pi/2)}^{\sin\pi/2} e^{j\nu\lambda}\frac{1}{\pi}\frac{d\lambda}{\sqrt{1-\lambda^2}} = \int_{-1}^{1} e^{j\nu\lambda}\frac{1}{\pi\sqrt{1-\lambda^2}}\,d\lambda \quad \Rightarrow \quad p_Y(y) = \frac{1}{\pi\sqrt{1-y^2}}\Pi\left(\frac{y}{2}\right)$$

8.4-1

Binomial distribution with $\alpha = (1-\alpha) = ½$, so $m = 10 \times ½ = 5$, $\sigma^2 = 5 \times ½ = 2.5$, $m \pm 2\sigma \approx 2$ to 8

$$P(i<3) = F_I(2) = \left[\binom{10}{0}+\binom{10}{1}+\binom{10}{2}\right]\left(\frac{1}{2}\right)^{10} = \frac{1+10+45}{1024} = 0.0547$$

8.4-2

Binomial distribution with $\alpha = 3/5$ and $(1-\alpha) = 2/5$, so $m = 10 \times 3/5 = 6$, $\sigma^2 = 5 \times 2/5 = 2.4$,

$m \pm 2\sigma \approx 3$ to 9

$$P(i<3) = F_I(2) = \left[\binom{10}{0}3^0 2^2 + \binom{10}{1}3^1 2^1 + \binom{10}{2}3^2 2^0\right]\frac{2^8}{5^{10}} = \frac{(1\times4+10\times6+45\times9)256}{9.87\times10^6} = 0.0122$$

8.4-3

Let I = number of forward steps, binomial distribution with $m_I = 100 \times ¾ = 75$, $\sigma_I^2 = 75 \times ¼$,

$\overline{I^2} = 75/4 + 75^2$, $X = Il - (100 - I)l = (2I - 100)l$ so $m_X = (2m_I - 100)l = 50l$ and

$$\overline{X^2} = \overline{(2I-100)^2}l^2 = (4\overline{I^2} - 400m_I + 10^4)l^2 = 2575l^2, \quad \sigma_X = \sqrt{2575l^2 - (50l)^2} = \sqrt{75}l$$

8.4-4

Binomial distribution with $1 - \alpha = 0.99$ so

$$P(I>1) = 1 - P_I(0) - P_I(1) = 1 - \binom{10}{0} 0.01^0 0.99^{10} - \binom{10}{1} 0.01^1 0.99^9 = 0.0042$$

Poisson approximation with $m = 10 \times 0.01 = 0.1$

$$P(I>1) \approx 1 - e^{-0.1}\frac{(0.1)^0}{0!} - e^{-0.1}\frac{(0.1)^1}{1!} = 0.0047$$

8.4-5

$\mu = 0.5$ particles/sec, $T = 2$ sec, $\mu T = 1$, so

(a) $P_I(1) = e^{-1}\frac{1^1}{1!} = 0.368$ (b) $P(I>1) = 1 - P_I(0) - P_I(1) = 1 - e^{-1}\frac{1^0}{0!} - e^{-1}\frac{1^1}{1!} = 0.264$

8.4-6

$$E[I] = \sum_{i=0}^{\infty} i e^{-m}\frac{m^i}{i!} = e^{-m}\sum_{i=0}^{\infty} i\frac{m^i}{i!}, \quad E[I^2] = e^{-m}\sum_{i=0}^{\infty} i^2\frac{m^i}{i!}$$, where

$$e^m = 1 + m + \frac{1}{2!}m^2 + \cdots = \sum_{i=0}^{\infty}\frac{m^i}{i!} \text{ so } \frac{d}{dm}e^m = \frac{d}{dm}\sum_{i=0}^{\infty}\frac{m^i}{i!} = \sum i\frac{m^{i-1}}{i!} = \frac{1}{m}\sum_{i=0}^{\infty} i\frac{m^i}{i!} \text{ and}$$

$$\frac{d^2}{dm^2}e^m = \frac{d}{dm}\sum_{i=0}^{\infty} i\frac{m^{i-1}}{i!} = \sum i(i-1)\frac{m^{i-2}}{i!} = \frac{1}{m^2}\sum_{i=0}^{\infty}(i^2 - i)\frac{m^i}{i!}$$

(cont.)

But $\frac{d}{dm}e^m = \frac{d^2}{dm^2}e^m = e^m$ so $\sum_{i=0}^{\infty} i\frac{m^i}{i!} = me^m$ and $\sum_{i=0}^{\infty}(i^2 - i)\frac{m^i}{i!} = \sum_{i=0}^{\infty} i^2\frac{m^i}{i!} - me^m = m^2 e^m$

Thus, $E[I] = e^{-m}(me^m) = m$ and $E[I^2] = e^{-m}(m^2e^m + me^m) = m^2 + m$

8.4-7

$\overline{X} = m = 100, \quad \sigma^2 = \overline{X^2} - \overline{X}^2 \quad \Rightarrow \quad \overline{X^2} = \sigma^2 + m^2 = 10{,}004$

$P(X < m-\sigma \text{ or } X > m+\sigma) = P(X \le m-\sigma) + P(X > m+\sigma) = 2\,Q(1) \approx 0.32$

8.4-8

$m = \overline{X} = 2, \quad \sigma = \sqrt{\overline{X^2} - \overline{X}^2} = 3$

$P(X > 5) = P(X > m+\sigma) = Q(1) \approx 0.16\,,$

$P(2 < X \le 5) = P(X > m) - P(X > m+\sigma) = \dfrac{1}{2} - Q(1) \approx 0.34$

8.4-9

$m = 10, \quad \sigma = \sqrt{500-100} = 20, \quad P(X > 20) = P(X > m+\sigma/2) = Q(0.5) \approx 0.31$

$P(10 < X \le 20) = P(X > m) - P(X > m+\sigma/2) = 1/2 - Q(0.5) \approx 0.19$

$P(0 < X \le 20) = P(|X-m| < \sigma/2) = 1 - 2Q(0.5) \approx 0.38$

$P(X > 0) = 1 - P(X \le m-\sigma/2) = 1 - Q(0.5) \approx 0.69$

8.4-10

$m = 100 \times ½ = 50$, $\sigma^2 = 50 \times ½ = 25$, $\sigma = 5$

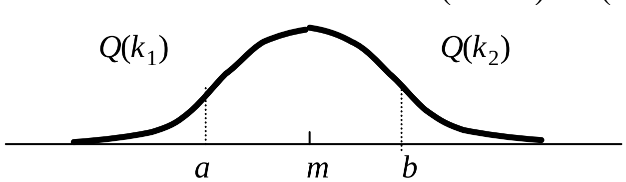
(a) $P(X > 70) = P(X > m+4\sigma) = Q(4) \approx 3.5\times 10^{-5}$

(b) $P(40 < X \le 60) = P(|X-m| \le 2\sigma) = 1 - 2Q(2) = 0.95$

8.4-11

Let $a = m - k_1\sigma$ and $b = m + k_2\sigma$ so

$P(a < X \le b) = 1 - Q(k_1) - Q(k_2) = 1 - Q\left(\dfrac{m-a}{\sigma}\right) - Q\left(\dfrac{m-b}{\sigma}\right)$



8.4-12

$m = 0,\ \sigma = 3,\ P(|X| \le c) = P\left(|X-m| \le \frac{c}{\sigma}\sigma\right) = 1 - 2Q\left(\frac{c}{\sigma}\right)$

(a) $1 - 2Q(c/3) = 0.9 \Rightarrow Q(c/3) = 0.05,\ c \approx 3 \times 1.65 = 4.95$

(b) $1 - 2Q(c/3) = 0.99 \Rightarrow Q(c/3) = 0.005,\ c \approx 3 \times 2.57 = 7.71$

8.4-13

$$Q(k) = \frac{1}{\sqrt{2\pi}}\int_k^\infty e^{-\lambda^2/2}\,d\lambda = \frac{1}{\sqrt{2\pi}}\int_k^\infty \left(-\frac{1}{\lambda}\right) d\left(e^{-\lambda^2/2}\right) = \frac{1}{\sqrt{2\pi}}\left[-\frac{1}{\lambda}e^{-\lambda^2/2}\Big|_k^\infty - \int_k^\infty e^{-\lambda^2/2}\,d\left(-\frac{1}{\lambda}\right)\right]$$

$$= \frac{1}{\sqrt{2\pi k^2}}e^{-k^2/2} - \frac{1}{\sqrt{2\pi}}\int_k^\infty \frac{1}{\lambda^2}e^{-\lambda^2/2}\,d\lambda$$

so $Q(k) < \frac{1}{\sqrt{2\pi k^2}}e^{-k^2/2}$ and

$$\frac{1}{\sqrt{2\pi}}\int_k^\infty \frac{1}{\lambda^2}e^{-\lambda^2/2}\,d\lambda < \frac{1}{\sqrt{2\pi}}\frac{1}{k^2}\int_k^\infty e^{-\lambda^2/2}\,d\lambda = \frac{1}{k^2}Q(k) \ll Q(k) \text{ if } k \gg 1$$

Thus, $Q(k) \approx \frac{1}{\sqrt{2\pi k^2}}e^{-k^2/2}$ for $k \gg 1$

8.4-14

$$E\left[(X-m)^n\right] = \frac{1}{\sqrt{2\pi\sigma^2}}\int_{-\infty}^\infty (x-m)^n e^{-(x-m)^2/2\sigma^2}\,dx = \frac{\left(2\sigma^2\right)^{n/2}}{\sqrt{\pi}}\int_{-\infty}^\infty \lambda^n e^{-\lambda^2}\,d\lambda$$

(a) $E\left[(X-m)^n\right] = 0$ for odd n since $\lambda^n e^{-\lambda^2}$ has odd symmetry

(b) $E\left[(X-m)^n\right] = K_n 2\int_0^\infty \lambda^n e^{-\lambda^2}\,d\lambda$ for even n, where $K_n = \frac{\left(2\sigma^2\right)^{n/2}}{\sqrt{\pi}} = \frac{2^{n/2}\sigma^n}{\sqrt{\pi}}$

But $e^{-\lambda^2}\,d\lambda = -\frac{1}{2\lambda}\,d\left(e^{-\lambda^2}\right)$ so

$$E\left[(X-m)^n\right] = -K_n\int_0^\infty \lambda^{n-1}\,d\left(e^{-\lambda^2}\right) = -K_n\left[\lambda^{n-1}e^{-\lambda^2}\Big|_0^\infty - \int_0^\infty e^{-\lambda^2}\,d\left(\lambda^{n-1}\right)\right] = K_n(n-1)\int_0^\infty \lambda^{n-2}e^{-\lambda^2}\,d\lambda$$

$$= (n-1)\frac{K_n}{2K_{n-2}}K_{n-2}2\int_0^\infty \lambda^{n-2}e^{-\lambda^2}\,d\lambda = (n-1)\sigma^2 E\left[(X-m)^{n-2}\right]$$

(cont.)

Thus, $E\left[(X-m)^4\right]=(4-1)\sigma^2 E\left[(X-m)^2\right]=3\sigma^4$, $E\left[(X-m)^6\right]=(6-1)\sigma^2\left(3\sigma^4\right)=3\cdot 5\sigma^6$, and

$E\left[(X-m)^n\right]=1\cdot 3\cdot 5\cdots(n-1)\sigma^n, \quad n=2,4,6,\ldots$

8.4-15

$$p_X(f)=\frac{1}{b}e^{-\pi[(f-m)/b]^2} \quad \text{where} \quad b=\sqrt{2\pi\sigma^2}$$

If m = 0, $\Phi_X(2\pi t)=e^{-\pi(bt)^2}=e^{-\sigma^2(2\pi t)^2/2}$, so $\Phi_X(\nu)=e^{-\sigma^2\nu^2/2}$

For $m \neq 0$, use frequency-translation theorem with $\omega_c = 2\pi m$, so

$$\Phi_X(2\pi t)=e^{-\pi(bt)^2}e^{j\omega_c t}=e^{-\sigma^2(2\pi t)^2/2}e^{jm(2\pi t)} \quad \text{and} \quad \Phi_X(\nu)=e^{-\sigma^2\nu^2/2}e^{jm\nu}$$

8.4-16

$$\Phi_Z(\nu)=\Phi_X(\nu)\Phi_Y(\nu)=e^{-\sigma_X{}^2\nu^2/2}e^{jm_X\nu}e^{-\sigma_Y{}^2\nu^2/2}e^{jm_Y\nu}=e^{-\sigma_Z{}^2\nu^2/2}e^{jm_Z\nu}$$

where $\sigma_Z{}^2=\sigma_X{}^2+\sigma_Y{}^2$, $m_Z=m_X+m_Y$. Hence, $p_Z(z)=\dfrac{1}{\sqrt{2\pi(\sigma_X{}^2+\sigma_Y{}^2)}}e^{-[z-(m_X+m_Y)]^2/2(\sigma_X{}^2+\sigma_Y{}^2)}$

If $Z=\dfrac{1}{n}\sum_{i=1}^n X_i=Y_1+Y_2+\cdots+Y_n$ where $Y_i=\dfrac{X_i}{n}$ is gaussian with

$$\overline{Y_i}=\frac{\overline{X_i}}{n},\ \overline{Y_i^2}=\frac{\overline{X_i^2}}{n^2},\ \sigma_{Y_i}{}^2=\frac{\overline{X_i^2}-\overline{X_i}^2}{n^2}=\frac{\sigma_{X_i}{}^2}{n^2}$$

Then $\Phi_Z(\nu)=\Phi_{Y_1}(\nu)\Phi_{Y_2}(\nu)\cdots\Phi_{Y_n}(\nu)=e^{-\sigma_Z{}^2\nu^2/2}e^{jm_Z\nu}$ where

$$\sigma_Z{}^2=\sum\sigma_{Y_i}{}^2=\frac{1}{n^2}\sum_{i=1}^n\sigma_{X_i}{}^2, \quad m_Z=\sum m_{Y_i}=\frac{1}{n}\sum_{i=1}^n m_{X_i}$$

8.4-17

$X = \ln Y \Rightarrow Y = e^X$

$$E[Y]=E\left[e^X\right]=E\left[e^{j\nu X}\right]\Big|_{j\nu=1}=\Phi_X(-j)=e^{\sigma_X{}^2/2}e^{m_X}$$

$$E\left[Y^2\right]=E\left[e^{2X}\right]=E\left[e^{j\nu X}\right]\Big|_{j\nu=2}=\Phi_X(-j2)=e^{2\sigma_X{}^2}e^{2m_X}$$

8.4-18

(a) $\Phi_Z(\nu) = \int_{-\infty}^{\infty} e^{j\nu x^2} p_X(x)\,dx = \dfrac{2}{\sqrt{2\pi\sigma^2}} \int_0^{\infty} e^{-\left(\frac{1}{2\sigma^2} - j\nu\right)x^2}\,dx = \left(1 - j2\sigma^2\nu\right)^{-1/2}$

(b) $\dfrac{d\Phi_Z}{d\nu} = j\sigma^2\left(1 - j2\sigma^2\nu\right)^{-3/2} \quad \Rightarrow \quad E[Z] = j^{-1}(j\sigma^2) = \sigma^2$

$\dfrac{d^2\Phi_Z}{d\nu^2} = -3\sigma^4\left(1 - j2\sigma^2\nu\right)^{-5/2} \quad \Rightarrow \quad E\left[Z^2\right] = j^{-2}(-3\sigma^4) = 3\sigma^4$

$\dfrac{d^3\Phi_Z}{d\nu^3} = -j15\sigma^6\left(1 - j2\sigma^2\nu\right)^{-7/2} \quad \Rightarrow \quad E\left[Z^3\right] = j^{-3}(-j15\sigma^6) = 15\sigma^6$

Thus, $E\left[X^2\right] = E[Z] = \sigma^2,\ E\left[X^4\right] = E\left[Z^2\right] = 3\sigma^4,\ E\left[X^6\right] = E\left[Z^3\right] = 15\sigma^6$

8.4-19

$\overline{R^2} = 2\sigma^2 = 32 \quad \Rightarrow \quad \sigma^2 = 16$ so $p_R(r) = \dfrac{r}{16} e^{-r^2/32} u(r)$ and $P(R \le r) = F_R(r) = \left(1 - e^{-r^2/32}\right) u(r)$

Thus, $P(R > 6) = 1 - P(R \le 6) = e^{-6^2/32} = 0.325$ and

$P(4.5 < R < 5.5) = P(R \le 5.5) - P(R \le 4.5) = e^{-4.5^2/32} - e^{-5.5^2/32} = 0.143$

8.4-20

$\overline{X^2} = 2\sigma^2 = 18 \quad \Rightarrow \quad \sigma^2 = 9$ so $p_X(x) = \dfrac{x}{9} e^{-x^2/18} u(x)$ and $P(X \le x) = \left(1 - e^{-x^2/18}\right) u(x)$

Thus, $P(X < 3) = P(X \le 3) = 1 - e^{-3^2/18} = 0.393,\ P(X > 4) = 1 - P(X \le 4) = e^{-4^2/18} = 0.411$, and

$P(3 < X \le 4) = P(X > 3) - P(X > 4) = [1 - P(X \le 3)] - [1 - P(X \le 4)] = e^{-3^2/18} - e^{-4^2/18} = 0.195$

8.4-21

Since $Z \geq 0$ and $X \geq 0$, monotonic transformation with $g(x) = x^2$, $g^{-1}(z) = +\sqrt{z}$, $dg^{-1}/dz = 1/2\sqrt{z}$

$p_X(x) = \dfrac{x}{\sigma^2} e^{-x^2/2\sigma^2} u(x),\ m = \overline{Z} = E\left[X^2\right] = 2\sigma^2$. Thus

$p_Z(z) = \dfrac{\sqrt{z}}{\sigma^2} e^{-\left(\sqrt{z}\right)^2/2\sigma^2} u\left(+\sqrt{z}\right) \dfrac{1}{2\sqrt{z}} = \dfrac{1}{m} e^{-z/m} u(z)$

$P(Z \le km) = \int_0^{km} \dfrac{1}{m} e^{-z/m}\,dz = 1 - e^{-k} = \begin{cases} 0.632 & k = 1 \\ 0.095 & k = 0.1 \end{cases}$

8.4-22

(a) $A = R_1^2 = X^2 + Y^2$ where X and Y are gaussian with $\overline{X} = \overline{Y} = 0,\ \sigma_X^2 = \sigma_Y^2 = \sigma^2$

$$\Phi_A(\nu) = \Phi_{X^2}(\nu)\Phi_{Y^2}(\nu) = \left[(1-\ j2\sigma^2\nu)^{-1/2}\right]^2 = (1-j\,2\sigma^2\nu)^{-1}$$

$$\Phi_A(2\pi t) = \frac{1}{1+j2\sigma^2(-2\pi t)} = \frac{b}{b+j2\pi(-t)},\ b = \frac{1}{2\sigma^2} \ \text{ so }\ p_A(a) = be^{-ba}u(a) = \frac{1}{2\sigma^2}e^{-a/2\sigma^2}u(a)$$

(b) $p_W(w) = p_{R_1^2}(w) * p_{R_2^2}(w),\quad p_{R_1^2} = p_{R_2^2} = p_A$

$$= \begin{cases} 0 & w<0 \\ \int_0^w \frac{1}{2\sigma^2}e^{-\lambda/2\sigma^2}\frac{1}{2\sigma^2}e^{-(w-\lambda)/2\sigma^2}\,d\lambda = \left(\frac{1}{2\sigma^2}\right)^2 e^{-w/2\sigma^2}\int_0^w d\lambda & w>0 \end{cases}$$

so $p_W(w) = \dfrac{w}{4\sigma^4}e^{-w/2\sigma^2}u(w)$

8.4-23

Let $a^2 = \dfrac{1}{2\sigma^2(1-\rho^2)}$ so $p_{XY}(x,y) = \dfrac{a}{\sqrt{2\pi^2\sigma^2}}e^{-a^2(x^2-2\rho xy+y^2)}$

$$p_Y(y) = \int_{-\infty}^{\infty} p_{XY}(x,y)\,dx = \frac{a}{\sqrt{2\pi^2\sigma^2}}e^{-a^2y^2}\int_{-\infty}^{\infty} e^{-a^2(x^2-2\rho xy)}\,dx = \frac{e^{-a^2y^2}}{\sqrt{2\pi^2\sigma^2}}e^{a^2\rho^2y^2}\,2\int_0^{\infty} e^{-\lambda^2}\,d\lambda = \frac{e^{-y^2/2\sigma^2}}{\sqrt{2\pi\sigma^2}}$$

$$p_X(x|y) = \frac{p_{XY}(x,y)}{p_Y(y)} = \frac{1}{\sqrt{2\pi\sigma^2(1-\rho^2)}}e^{-(x-\rho y)^2/2\sigma^2(1-\rho^2)}$$

8.4-24

Since Z is a linear combination of gaussian RVs, $p_Z(z)$ is a gaussian PDF with

$$m_Z = E[X+3Y] = m_X + 3m_Y = 0$$

$$\sigma_Z^2 = E\left[X^2 + 6XY + 9Y^2\right] = (\sigma_X^2 + m_X^2) + 6E[XY] + 9(\sigma_Y^2 + m_Y^2) = 100$$

8.4-25

$E\left[X^n\right] = \int_{-\infty}^{\infty} x^n p_X(x)\,dx = 0$ for n odd, $E[Y] = E\left[X^2\right] = \sigma_X^2$

$$E[(X-m_X)(Y-m_Y)] = E\left[X(X^2-\sigma_X^2)\right] = E\left[X^3\right] - \sigma_X^2E[X] = 0 \quad\Rightarrow\quad \rho = 0$$

Chapter 9

9.1-1

$$E\left[e^{Xt}\right]=\frac{1}{2}\int_0^2 e^{xt}\,dx=\frac{1}{2t}\left(e^{2t}-1\right),\quad \overline{v(t)}=E\left[6e^{Xt}\right]=\frac{3}{t}\left(e^{2t}-1\right)$$

$$R_v(t_1,t_2)=E\left[36e^{X(t_1+t_2)}\right]=\frac{18}{t_1+t_2}\left[e^{2(t_1+t_2)}-1\right],\quad \overline{v^2(t)}=\frac{9}{t}\left(e^{4t}-1\right)$$

9.1-2

$$E\left[\cos Xt\right]=\frac{1}{2}\int_0^2\cos xt\,dx=\frac{1}{2t}\sin 2t,\quad \overline{v(t)}=E\left[6\cos Xt\right]=\frac{3}{t}\sin 2t$$

$$R_v(t_1,t_2)=E\left[36\cos Xt_1\cos Xt_2\right]=18E\left[\cos X(t_1-t_2)+\cos X(t_1+t_2)\right]$$

$$=18\left[\frac{\sin 2(t_1-t_2)}{2(t_1-t_2)}+\frac{\sin 2(t_1+t_2)}{2(t_1+t_2)}\right]$$

$$\overline{v^2(t)}=18\left(1+\frac{\sin 4t}{4t}\right)$$

9.1-3

$$\overline{X}=0,\quad \overline{X^2}=1/3,\quad \overline{v(t)}=E\left[Y+3Xt\right]t=\overline{Y}t+3\overline{X}t^2=2t$$

$$R_v(t_1,t_2)=E\left[Y^2+3YX(t_1+t_2)+9X^2t_1t_2\right]t_1t_2=\left[\overline{Y^2}+3\overline{X}\,\overline{Y}(t_1+t_2)+9\overline{X^2}t_1t_2\right]t_1t_2=6t_1t_2+3\left(t_1t_2\right)^2$$

$$\overline{v^2(t)}=6t^2+3t^4$$

9.1-4

$$E\left[e^{Xt}\right]=\frac{1}{2}\int_{-1}^1 e^{xt}\,dx=\frac{1}{2t}\left(e^{t}-e^{-t}\right),\ \overline{v(t)}=E\left[Ye^{Xt}\right]=\overline{Y}E\left[e^{Xt}\right]=\frac{1}{t}\left(e^{t}-e^{-t}\right)$$

$$R_v(t_1,t_2)=E\left[Y^2e^{X(t_1+t_2)}\right]=\overline{Y^2}E\left[e^{X(t_1+t_2)}\right]=\frac{3}{t_1+t_2}\left[e^{(t_1+t_2)}-e^{-(t_1+t_2)}\right],\ \overline{v^2(t)}=\frac{3}{2t}\left(e^{2t}-e^{-2t}\right)$$

9.1-5

$$E\left[\cos Xt\right]=\frac{1}{2}\int_{-1}^1\cos xt\,dx=\frac{\sin t}{t},\quad \overline{v(t)}=E\left[Y\cos Xt\right]=\overline{Y}E\left[\cos Xt\right]=2\frac{\sin t}{t}$$

(cont.)

$$R_v(t_1,t_2)=E\left[Y^2\cos Xt_1\cos Xt_2\right]=\frac{1}{2}\overline{Y^2}E\left[\cos X(t_1-t_2)+\cos X(t_1+t_2)\right]$$

$$=3\left[\frac{\sin(t_1-t_2)}{t_1-t_2}+\frac{\sin(t_1+t_2)}{t_1+t_2}\right]$$

$$\overline{v^2(t)}=3\left(1+\frac{\sin 2t}{2t}\right)$$

9.1-6

$p_{F\Phi}(f,\varphi)=\dfrac{1}{2\pi}p_F(f),\quad 0\le\varphi\le 2\pi,\qquad R_v(t_1,t_2)=A^2\int_{-\infty}^{\infty}g(f)\,p_F(f)\,df$ where

$$g(f)=\frac{1}{2\pi}\int_0^{2\pi}\cos(2\pi f t_1+\varphi)\cos(2\pi f t_2+\varphi)\,d\varphi$$

$$=\frac{1}{4\pi}\int_0^{2\pi}\cos 2\pi f(t_1-t_2)\,d\varphi+\frac{1}{4\pi}\int_0^{2\pi}\cos[2\pi f(t_1+t_2)+2\varphi]\,d\varphi=\tfrac{1}{2}\cos 2\pi f(t_1-t_2)$$

Thus, with $f=\lambda$, $R_v(t_1,t_2)=\dfrac{A^2}{2}\displaystyle\int_{-\infty}^{\infty}\cos 2\pi\lambda(t_1-t_2)\,p_F(\lambda)\,d\lambda$

$$\overline{v(t)}=A\int_{-\infty}^{\infty}\left[\frac{1}{2\pi}\int_0^{2\pi}\cos(2\pi f t+\varphi)\,d\varphi\right]p_F(f)\,df=0,\quad \overline{v^2(t)}=R_v(t,t)=\frac{A^2}{2}\int_{-\infty}^{\infty}p_F(f)\,df=\frac{A^2}{2}$$

9.1-7

$$v(t_1)w(t_2)=XY(\cos\omega_0t_1\cos\omega_0t_2-\sin\omega_0t_1\sin\omega_0t_2)-X^2\cos\omega_0t_1\sin\omega_0t_2$$
$$+\,Y^2\sin\omega_0t_1\cos\omega_0t_2$$

$E[XY]=\overline{X}\,\overline{Y}=0,\ E\left[X^2\right]=E\left[Y^2\right]=\sigma^2$, so

$$R_{vw}(t_1,t_2)=E[v(t_1)w(t_2)]=\sigma^2(\sin\omega_0t_1\cos\omega_0t_2-\cos\omega_0t_1\sin\omega_0t_2)=\sigma^2\sin\omega_0(t_1-t_2)$$

9.1-8

(*a*) $\overline{v(t)}=E\left[X\cos\omega_0t+Y\sin\omega_0t\right]=\overline{X}\cos\omega_0t+\overline{Y}\sin\omega_0t=0$

$$R_v(t_1,t_2)=E\left[X^2\cos\omega_0t_1\cos\omega_0t_2+XY(\cos\omega_0t_1\sin\omega_0t_2+\sin\omega_0t_1\cos\omega_0t_2)+Y^2\sin\omega_0t_1\sin\omega_0t_2\right]$$
$$=\overline{X^2}\cos\omega_0t_1\cos\omega_0t_2+\overline{Y^2}\sin\omega_0t_1\sin\omega_0t_2=\sigma^2\cos\omega_0(t_1-t_2)$$

so $R_V(\tau)=\sigma^2\cos\omega_0\tau$

(*b*) $\overline{v^2(t)}=R_v(0)=\sigma^2$ (cont.)

$<v_i^2(t)> = <(X_i \cos \omega_0 t + Y_i \sin \omega_0 t)^2> = X_i^2 <\cos^2 \omega_0 t> + 2X_i Y_i <\cos \omega_0 t \sin \omega_0 t>$
$+ Y_i^2 <\sin^2 \omega_0 t>$

$$= \frac{1}{2} X_i^2 + \frac{1}{2} Y_i^2 \neq \sigma^2$$

9.1-9

$$(a)\ \overline{v(t)} = \int_{-\infty}^{\infty} a p_A(a)\, da \int_0^{2\pi} \cos(\omega_0 t + \varphi) \frac{d\varphi}{2\pi} = \overline{A} \times 0 = 0$$

$$R_v(t_1, t_2) = \int_{-\infty}^{\infty} a^2 p_A(a)\, da \int_0^{2\pi} \cos(\omega_0 t_1 + \varphi)\cos(\omega_0 t_2 + \varphi) \frac{d\varphi}{2\pi} = \overline{A^2} \times \frac{1}{2} \cos \omega_0 (t_1 - t_2)$$

so $R_v(\tau) = \dfrac{\overline{A^2}}{2} \cos \omega_0 \tau$

$$(b)\ \overline{v^2(t)} = \overline{A^2}/2, \quad <v_i^2(t)> = <A_i^2 \cos^2(\omega_0 t + \Phi_i)> = A_i^2 <\cos^2(\omega_0 t + \Phi_i)> = A_i^2/2 \neq \overline{A^2}/2$$

9.1-10

$$m_Z = \overline{z(t)} = E\left[v(t) - v(t+T)\right] = \overline{v(t)} - \overline{v(t+T)} = 0$$

$$\sigma_Z^2 = \overline{z^2(t)} = \overline{v^2(t)} - 2\overline{v(t)v(t+T)} + \overline{v^2(t+T)} = R_v(0) - 2R_v(T) + R_v(0) = 2\left[R_v(0) - R_v(T)\right]$$

9.1-11

$$m_Z = \overline{z(t)} = E\left[v(t) + v(t-T)\right] = \overline{v(t)} + \overline{v(t-T)} = 2\sqrt{R_v(\pm\infty)}$$

$$\overline{z^2(t)} = \overline{v^2(t)} + 2\overline{v(t)v(t-T)} + \overline{v^2(t-T)} = R_v(0) + 2R_v(T) + R_v(0),$$

$$\sigma_Z^2 = 2\left[R_v(0) + R_v(T) - 2R_v(\pm\infty)\right]$$

9.2-1

$$\mathcal{F}\left[\boldsymbol{e}^{-(\sqrt{\pi} b t)^2}\right] = \frac{1}{b} e^{-(\sqrt{\pi} f/b)^2} \text{ so } G_V(f) = 2\sqrt{\pi} e^{-(\pi f/8)^2} + 9\delta(f)$$

$<v(t)> = \sqrt{R_v(\pm\infty)} = \pm 3, \quad <v^2(t)> = R_v(0) = 25, \quad v_{\text{rms}} = \sqrt{25 - 9} = 4$

9.2-2

$$G_V(f) = \frac{32}{8} \Lambda\left(\frac{f}{8}\right) + \frac{4}{2}\left[\delta(f-8) + \delta(f+8)\right]$$

$<v(t)> = m_V = 0, \quad <v^2(t)> = R_V(0) = 36, \quad v_{\text{rms}} = \sqrt{36 - 0} = 6$

9.2-3

$$R_v(\tau) = \frac{A^2}{2}\int_{-\infty}^{\infty} \cos 2\pi\lambda\tau \, p_F(\lambda)\, d\lambda \, ,$$

$$G_v(f) = \mathcal{F}_\tau\left[R_v(\tau)\right] = \frac{A^2}{2}\int_{-\infty}^{\infty} \mathcal{F}_\tau\left[\cos 2\pi\lambda\tau\right] p_F(\lambda)\, d\lambda = \frac{A^2}{2}\int_{-\infty}^{\infty} \frac{1}{2}\left[\delta(f-\lambda)+\delta(f+\lambda)\right] p_F(\lambda)\, d\lambda$$

$$= \frac{A^2}{4}\left[p_F(f) + p_F(-f)\right]$$

$$p_F(f) = \delta(f - f_0) \Rightarrow G_v(f) = \frac{A^2}{4}\left[\delta(f - f_0) + \delta(f + f_0)\right] \quad \text{since} \;\; \delta(-f - f_0) = \delta(f + f_0)$$

9.2-4

$$(a)\;\; E\left[\tilde{G}_v(f)\right] = \int_{-T}^{T}\left(1 - \frac{|\tau|}{T}\right) R_v(\tau) e^{-j\omega\tau}\, d\tau = \int_{-\infty}^{\infty} \Lambda\left(\frac{\tau}{T}\right) R_v(\tau) e^{-j\omega\tau}\, d\tau$$

$$= \mathcal{F}_\tau\left[\Lambda\left(\frac{\tau}{T}\right) R_v(\tau)\right] = (T\,\mathrm{sinc}^2\, fT) * G_v(f)$$

$$(b)\;\; \lim_{T\to\infty} \Lambda\left(\frac{\tau}{T}\right) = 1 \quad \text{so} \quad \lim_{T\to\infty} T\,\mathrm{sinc}^2 fT = \mathcal{F}_\tau[1] = \delta(f) \;\; \text{and}$$

$$\lim_{T\to\infty} E\left[\tilde{G}_v(f)\right] = \delta(f) * G_v(f) = G_v(f)$$

9.2-5

$$v_T(t) = A\cos(\omega_0 t + \Phi)\,\Pi\left(\frac{t}{T}\right), \; V_T(f,s) = \frac{AT}{2}\left[\mathrm{sinc}(f - f_0)T\, e^{j\Phi} + \mathrm{sinc}(f + f_0)T\, e^{-j\Phi}\right]$$

$$E\left[\left|V_T(f,s)\right|^2\right] = \frac{A^2T^2}{4}\left\{\mathrm{sinc}^2(f - f_0)T + \mathrm{sinc}^2(f - f_0)T + E\left[e^{j2\Phi} + e^{-j2\Phi}\right]\mathrm{sinc}(f - f_0)T\,\mathrm{sinc}(f + f_0)T\right\}$$

But $E\left[e^{j2\Phi} + e^{-j2\Phi}\right] = E\left[2\cos 2\Phi\right] = 0$ and $\lim_{T\to\infty} T\,\mathrm{sinc}^2 fT = \delta(f)$, so

$$G_v(f) = \lim_{T\to\infty} \frac{A^2}{4}\left[T\,\mathrm{sinc}^2(f - f_0)T + T\,\mathrm{sinc}^2(f + f_0)T\right] = \frac{A^2}{4}\left[\delta(f - f_0) + \delta(f + f_0)\right]$$

9.2-6

$$R_{vw}(t_1, t_2) = E\left[v(t_1)w(t_2)\right] = m_V m_W \quad \text{so} \quad R_{vw}(\tau) = R_{wv}(\tau) = m_V m_W$$

$R_Z(\tau) = R_V(\tau) + R_W(\tau) \pm 2m_V m_W$, $G_Z(f) = G_V(f) + G_W(f) \pm 2m_V m_W \delta(f)$

(cont.)

$R_Z(\pm\infty) = R_V(\pm\infty) + R_W(\pm\infty) \pm 2m_V m_W = m_V^2 + m_W^2 \pm 2m_V m_W = (m_V \pm m_W)^2$

$$\overline{z^2} = R_z(0) = R_v(0) + R_w(0) \pm 2m_V m_W = \overline{v^2} + \overline{w^2} \pm 2m_V m_W$$

$$= \sigma_V{}^2 + m_V{}^2 + \sigma_W{}^2 + m_W{}^2 \pm 2m_V m_W = \sigma_V{}^2 + \sigma_W{}^2 + (m_V \pm m_W)^2 > 0$$

9.2-7

$$R_{wv}(t_1,t_2) = E\left[w(t_1)v(t_2)\right] = E\left[v(t_2)w(t_1)\right] = R_{vw}(t_2 - t_1) \quad \text{so} \quad R_{wv}(\tau) = R_{vw}(-\tau)$$

$$G_{wv}(f) = \mathrm{F}_\tau\left[R_{vw}(-\tau)\right] = \frac{1}{|-1|}G_{vw}(-f) = G_{vw}(-f)$$

9.2-8

$$R_z(t_1,t_2) = E\left[v(t_1)v(t_2) + v(t_1+T)v(t_2+T) - v(t_1+T)v(t_2) - v(t_1)v(t_2+T)\right]$$

$$= R_v(t_1 - t_2) + R_v(t_1 + T - t_2 - T) - R_v(t_1 + T - t_2) - R_v(t_1 - t_2 - T) \text{ so}$$

$$R_z(\tau) = 2R_v(\tau) - R_v(\tau+T) - R_v(\tau-T) \quad \text{and}$$

$$G_z(f) = 2G_v(f) - G_v(f)\left(e^{j\omega T} + e^{-j\omega T}\right) = 2G_v(f)\left(1 - \cos 2\pi fT\right)$$

9.2-9

$$R_z(t_1,t_2) = E\left[v(t_1)v(t_2) + v(t_1-T)v(t_2-T) + v(t_1)v(t_2-T) + v(t_1-T)v(t_2)\right]$$

$$= R_v(t_1 - t_2) + R_v(t_1 - T - t_2 + T) + R_v(t_1 - t_2 + T) + R_v(t_1 - T - t_2) \text{ so}$$

$$R_z(\tau) = 2R_v(\tau) + R_v(\tau+T) + R_v(\tau-T) \quad \text{and}$$

$$G_z(f) = 2G_v(f) + G_v(f)\left(e^{j\omega T} + e^{-j\omega T}\right) = 2G_v(f)\left(1 + \cos 2\pi fT\right)$$

9.2-10

$z(t) = v(t)\cos(2\pi f_2 t + \Phi_2)$ with $v(t) = A\cos(2\pi f_1 t + \Phi_1)$ so

$G_V(f) = (A^2/2)[\delta(f - f_1) + \delta(f + f_1)]$

Thus, $G_z(f) = \dfrac{A^2}{16}[\delta(f - f_1 - f_2) + \delta(f + f_1 - f_2) + \delta(f - f_1 + f_2) + \delta(f + f_1 + f_2)]$

For $f_1 = f_2$, $G_z(f) = \dfrac{A^2}{16}[2\delta(f) + \delta(f - 2f_2) + \delta(f + 2f_2)]$

9.2-11

$R_y(t_1,t_2) = E\left[y(t_1)y(t_2)\right], \quad y(t_2) = \int_{-\infty}^{\infty} h(\lambda)x(t_2-\lambda)\,d\lambda$ so

$R_y(t_1,t_2) = \int_{-\infty}^{\infty} h(\lambda)E\left[y(t_1)x(t_2-\lambda)\right]d\lambda$ (cont.)

But $E\left[y(t_1)x(t_2-\lambda)\right] = R_{yx}(t_1,t_2-\lambda) = R_{yx}(t_1-t_2+\lambda) = R_{yx}(\tau+\lambda)$ so

$$R_y(\tau) = \int_{-\infty}^{\infty} h(\lambda)R_{yx}(\tau+\lambda)\,d\lambda = \int_{-\infty}^{\infty} h(-\mu)R_{yx}(\tau-\mu)\,d\mu = h(-\tau)*R_{yx}(\tau)$$

9.2-12

$$R_y(\tau) = \mathcal{F}_\tau^{-1}\left[(2\pi f)^2 G_x(f)\right] = -\mathcal{F}_\tau^{-1}\left[(j2\pi f)^2 G_x(f)\right] = -d^2R_x(\tau)/d\tau^2$$

$G_{yx}(f) = \mathcal{F}_\tau\left[h(\tau)*R_x(\tau)\right] = H(f)G_x(f)$ where $H(f) = j2\pi f$,

so $R_{yx}(\tau) = \mathcal{F}_\tau^{-1}\left[(j2\pi f)G_x(f)\right] = dR_x(\tau)/d\tau$

9.2-13

If $x(t)$ is deterministic, then $Y(f) = X(f) - \alpha X(f)e^{-j\omega T} \quad \Rightarrow \quad H(f) = 1-\alpha e^{-j\omega T}$

$\left|H(f)\right|^2 = 1+\alpha^2 - \alpha(e^{j\omega T}+e^{-j\omega T}) = 1+\alpha^2 - 2\alpha\cos\omega T$ so

$$G_y(f) = (1+\alpha^2 - 2\alpha\cos\omega T)G_x(f), \quad R_y(\tau) = (1+\alpha^2)R_x(\tau) - \alpha\left[R_x(\tau+T) + R_x(\tau-T)\right]$$

9.2-14

$$R_{\hat{x}x}(\tau) = h_Q(\tau)*R_x(\tau) = \int_{-\infty}^{\infty} \frac{1}{\pi(\tau-\lambda)}R_x(\lambda)\,d\lambda$$

$$R_{x\hat{x}}(\tau) = R_{\hat{x}x}(-\tau) = \int_{-\infty}^{\infty} \frac{1}{\pi(-\tau-\lambda)}R_x(\lambda)\,d\lambda = -\int_{-\infty}^{\infty} \frac{1}{\pi\mu}R_x(\mu-\tau)\,d\mu = -\int_{-\infty}^{\infty} \frac{1}{\pi\mu}R_x(\tau-\mu)\,d\mu$$

$$= -\left[h_Q(\tau)*R_x(\tau)\right] = -\hat{R}_x(\tau)$$

9.2-15

Let $x(t) = \delta(t)$ so $y(t) = h(t) = \dfrac{1}{T}\int_{t-T/2}^{t+T/2} \delta(\lambda)\,d\lambda = \begin{cases} 1/T & t - T/2 < 0 < t + T/2 \\ 0 & \text{otherwise} \end{cases}$

Thus, $h(t) = \begin{cases} 1 & |t| < T/2 \\ 0 & |t| > T/2 \end{cases} \Rightarrow H(f) = \text{sinc}\, fT$, and

$$R_y(\tau) = \mathcal{F}_\tau^{-1}\left[\text{sinc}^2 fT G_x(f)\right] = \frac{1}{T}\Lambda\left(\frac{\tau}{T}\right) * R_x(\tau) = \frac{1}{T}\int_{-T}^{T}\left(1 - \frac{|\lambda|}{T}\right) R_x(\tau - \lambda)\,d\lambda$$

9.3-1

Let $x = h|f|/kT$, $\quad e^x - 1 = x + \frac{1}{2}x^2 + \cdots \approx x\left(1 + \frac{1}{2}x\right)$ for $|x| \ll 1$. Then

$$\left(e^x - 1\right)^{-1} \approx \frac{1}{x}\left(1 + \frac{1}{2}x\right)^{-1} = \frac{1}{x}\left(1 - \frac{1}{2}x + \frac{1}{2}x^2 + \cdots\right) \approx \frac{1}{x}\left(1 - \frac{1}{2}x\right), \text{ so}$$

$$G_v(f) \approx \frac{2Rh|f|}{h|f|/kT}\left(1 - \frac{1}{2}\frac{h|f|}{kT}\right) = 2RkT\left(1 - \frac{h|f|}{2kT}\right)$$

9.3-2

$$R_{yx}(\tau) = h(\tau) * \frac{N_0}{2}\delta(\tau) = \frac{N_0}{2}h(\tau)$$

$$R_y(\tau) = h(-\tau) * \frac{N_0}{2}h(\tau) = \frac{N_0}{2}\int_{-\infty}^{\infty} h(-\lambda)h(\tau - \lambda)\,d\lambda = \frac{N_0}{2}\int_{-\infty}^{\infty} h(t)h(t + \tau)\,dt$$

9.3-3

$$G_y(f) = \frac{N_0 T^2}{2}\text{sinc}^2 fT, \quad R_y(\tau) = \frac{N_0 T}{2}\Lambda\left(\frac{\tau}{T}\right), \quad \overline{y^2} = R_y(0) = \frac{N_0 T}{2}$$

9.3-4

$$G_y(f) = \frac{N_0 K^2}{2}e^{-2(af)^2} = \frac{N_0 K^2}{2}e^{-\pi\left(f/\sqrt{\pi/2a^2}\right)^2},$$

$$R_y(\tau) = \frac{N_0 K^2}{a}\sqrt{\frac{\pi}{8}}\,e^{-(\pi\tau/\sqrt{2}a)^2}, \quad \overline{y^2} = R_y(0) = \frac{N_0 K^2}{a}\sqrt{\frac{\pi}{8}}$$

9.3-5

$$G_y(f) = \frac{N_0 K^2}{2}\left[\Pi\left(\frac{f-f_0}{B}\right) + \Pi\left(\frac{f+f_0}{B}\right)\right],$$

$$R_y(\tau) = N_0 K^2 B \,\text{sinc}^2 B\tau \cos 2\pi f_0 \tau, \quad \overline{y^2} = R_y(0) = N_0 K^2 B$$

9.3-6

$$G_y(f) = \frac{N_0 K^2}{2}\left[1 - \Pi\left(\frac{f}{2f_0}\right)\right], \; R_y(\tau) = \frac{N_0 K^2}{2}\left[\delta(\tau) - 2f_0 \,\text{sinc}\, 2f_0\tau\right], \quad \overline{y^2} = R_y(0) = \infty$$

9.3-7

$$H(f) = \frac{R}{R + j\omega L} = \frac{1}{1 + j(f/B)}, \; B = \frac{R}{2\pi L}, \text{ so } G_y(f) = \left|H(f)\right|^2 G_x(f) = \frac{N_{0v}/2}{1 + (f/B)^2}$$

$$R_y(\tau) = \frac{N_{0v}}{2}\pi B e^{-2\pi B|\tau|} = \frac{N_{0v} R}{4L} e^{-R|\tau|/L}, \quad \overline{y^2} = R_y(0) = \frac{N_{0v} R}{4L}$$

9.3-8

$$H(f) = \frac{j\omega L}{R + j\omega L} = \frac{j2\pi f}{b + j2\pi f}, \; b = \frac{R}{L}, \text{ so}$$

$$G_y(f) = \left|H(f)\right|^2 G_x(f) = \frac{N_{0v}}{2}\frac{(2\pi f)^2}{b^2 + (2\pi f)^2} = -\frac{N_{0v}}{4b}(j2\pi f)^2 \frac{2b}{b^2 + (2\pi f)^2}.$$

$$R_y(\tau) = -\frac{N_{0v}}{4b}\frac{d^2}{d\tau^2}\left[e^{-b|\tau|}\right] = -\frac{N_{0v}}{4b}\frac{d}{d\tau}\left[-be^{-b\tau}u(\tau) + be^{b\tau}u(-\tau)\right]$$

$$= -\frac{N_{0v}}{4b}\left[b^2 e^{-b\tau}u(\tau) - b\delta(\tau) + b^2 e^{b\tau}u(-\tau) - b\delta(-\tau)\right]$$

$$= \frac{N_{0v}}{4}\left[2\delta(\tau) - be^{-b|\tau|}\right], \quad \overline{y^2} = R_y(0) = \infty$$

9.3-9

$$\overline{i^2} = \int_{-\infty}^{\infty} G_i(f)\,df = N_{0v}\int_0^{\infty}\frac{df}{R^2 + (2\pi f L)^2} = \frac{N_{0v}}{4LR}. \text{ Thus, } \frac{1}{2}L\frac{N_{0v}}{4LR} = \frac{1}{2}kT \quad \Rightarrow \quad N_{0v} = 4RkT$$

9.3-10

y is gaussian with $\overline{y}=0,\ \overline{y^2}=\sigma^2=4RkT_0B=4\times10^{-10}$

$$\overline{z}=\int_{-\infty}^{\infty}|y|\,p_Y(y)\,dy=2\int_0^{\infty}\frac{y}{\sqrt{2\pi\sigma^2}}e^{-y^2/2\sigma^2}\,dy=\sqrt{\frac{2}{\pi}}\sigma\int_0^{\infty}e^{-\lambda^2}\,d(\lambda^2)=\sqrt{\frac{2}{\pi}}\sigma\approx16\ \mu\text{V}$$

$z^2=y^2$ so $\overline{z^2}=\overline{y^2}=\sigma^2,\quad \sigma_Z=\sqrt{\sigma^2-\left(\sqrt{\frac{2}{\pi}}\sigma\right)^2}\approx12\,\mu\text{V}$

9.3-11

y is gaussian with $\overline{y}=0,\ \overline{y^2}=\sigma^2=4RkT_0B=4\times10^{-10}$ and $z=y\,u(y)$

(cont.)

$$\overline{z}=\int_0^{\infty}yp_Y(y)\,dy=\int_0^{\infty}\frac{y}{\sqrt{2\pi\sigma^2}}e^{-y^2/2\sigma^2}\,dy=\frac{\sigma}{\sqrt{2\pi}}\int_0^{\infty}e^{-\lambda^2}\,d(\lambda^2)=\frac{\sigma}{\sqrt{2\pi}}\approx8\ \mu\text{V}$$

$$\overline{z^2}=\int_0^{\infty}\frac{y^2}{\sqrt{2\pi\sigma^2}}e^{-y^2/2\sigma^2}\,dy=\frac{2\sigma^2}{\sqrt{\pi}}\int_0^{\infty}\lambda^2e^{-\lambda^2}\,d\lambda=\frac{\sigma^2}{2},\ \sigma_Z=\sqrt{\frac{\sigma^2}{2}-\left(\frac{\sigma}{\sqrt{2\pi}}\right)^2}\approx12\,\mu\text{V}$$

9.3-12

$\overline{y}=\overline{z}=0,\ \overline{y^2}=\overline{z^2}=\sigma^2=4RkT_0B=4\times10^{-10}$

$E\left[(Y-m_Y)(Z-m_Z)\right]=E\left[y(t)y(t-T)\right]=R_y(T)=\sigma^2\text{sinc}2BT=0$ since $2BT=5$

Thus, $\rho=0$ and $p_{YZ}(y,z)=\frac{1}{2\pi\sigma^2}e^{(y^2+z^2)/2\sigma^2}$

9.3-13

$\overline{y}=\overline{z}=0,\ \overline{y^2}=\overline{z^2}=\sigma^2=4RkT_0B=4\times10^{-10}$

$E\left[(Y-m_Y)(Z-m_Z)\right]=E\left[y(t)y(t-T)\right]=R_y(T)=\sigma^2\text{sinc}2BT$ so $\rho=\frac{\sigma^2}{\sigma\sigma}$ sinc 0.5 = 0.637

Thus, $p_{YZ}(y,z)=\frac{1}{2\pi0.77\sigma^2}e^{-(y^2+z^2-1.274\,yz)/1.19\sigma^2}$

9.3-14

$$g = |H(0)|^2 = K^2, \quad B_N = \int_0^\infty e^{-2a^2f^2}\,df = \frac{\sqrt{\pi}}{2\sqrt{2}a}$$

At $f = B$, $|H(B)|^2 = K^2 e^{-2a^2B^2} = \dfrac{K^2}{2} \;\Rightarrow\; B = \sqrt{\dfrac{\ln 2}{2}}\dfrac{1}{a}$ so $\dfrac{B_N}{B} = \dfrac{\sqrt{\pi}}{2\sqrt{\ln 2}} = 1.06$

9.3-15

$$X_T(f,s) = \int_{-T/2}^{T/2}\left[\sum_{k=-K_1}^{K_2} A_k\delta(t-T_k)\right]e^{-j\omega t}\,dt = \sum_{k=-K_1}^{K_2} A_k e^{-j\omega T_k} \quad \text{where} \quad T_{-K_1} > -\frac{T}{2} \text{ and } T_{K_2} < \frac{T}{2}$$

$$|X_T(f,s)|^2 = \sum_k\sum_m A_kA_m e^{-j\omega(T_k-T_m)},\ E\left[|X_T(f,s)|^2\right] = \sum_k\sum_m E[A_kA_m]E\left[e^{-j\omega(T_k-T_m)}\right]$$

where $E[A_kA_m] = \begin{cases}\sigma^2 & m=k\\ 0 & m\neq k\end{cases}$. So $E\left[|X_T(f,s)|^2\right] = \sum_k \sigma^2 E\left[e^{-j\omega(T_k-T_k)}\right] = \sigma^2(K_1+K_2)$

with $K_1 + K_2$ = expected number of impulses in T seconds = μT (cont.)

Thus, $G_x(f) = \lim_{T\to\infty}\dfrac{1}{T}\sigma^2\mu T = \mu\sigma^2$

9.4-1

$$10\log_{10}\left(\frac{T_N}{T_0}W\right) = 10\log_{10}\left(4\times10^6\right) \approx 66\text{ dB},\ S_R = 2\times10^{-5}\text{ mW} \approx -47\text{ dBm}$$

$$(S/N)_{D_{\text{dB}}} \approx -47 + 174 - 66 = 61\text{ dB}$$

9.4-2

$$10\log_{10}\left(\frac{T_N}{T_0}W\right) = 10\log_{10}\left(5\times2\times10^6\right) = 70\text{ dB},\ S_R = 4\times10^{-6}\text{ mW} \approx -54\text{ dBm}$$

$$(S/N)_{D_{\text{dB}}} \approx -54 + 174 - 70 = 50\text{ dB}$$

9.4-3

S_T/LN_0W = 46 dB = $4 \times 10^4 \Rightarrow LN_0 = 5 \times 10^{-10}$

(a) W = 20 kHz, $(S/N)_D$ = 55-65 dB, $S_{T_{\text{dBm}}} = (S/N)_D - 10\log_{10}\left(5\times10^{-10}\times20\times10^3\right)+30=$

35 to 45 dBm, S_T = 3.2-32 W

(b) W = 3.2 kHz, $(S/N)_D$ = 25-35 dB, $S_{T_{\text{dBm}}} = (S/N)_D - 10\log_{10}\left(5\times10^{-10}\times3.2\times10^3\right)+30=$

-3 to +7 dBm, S_T = 0.5-5 mW

9.4-4

$(S/N)_D = S_R/N_0B_N = (W/B_N)(S_R/N_0W)$

(a) $B_N = \dfrac{\pi}{2}B = 23.6$ kHz = $2.36W \Rightarrow (S/N)_D = 0.424(S_R/N_0W)$

(b) $B_N = \dfrac{\pi B}{4\sin\pi/4} = 13.4$ kHz = $1.34W \Rightarrow (S/N)_D = 0.746(S_R/N_0W)$

9.4-5

$$S_D = \int_{-\infty}^{\infty}|H_C(f)|^2|H_R(f)|^2 G_x(f)\,df = K^2\int_{-\infty}^{\infty}G_x(f)\,df = K^2S_T$$

$$N_D = \frac{N_0}{2}\int_{-\infty}^{\infty}|H_R(f)|^2\,df = N_0K^2L\int_0^W\left[1+\left(\frac{f}{W}\right)^2\right]df = N_0K^2L\frac{3W}{2} \text{ so } \left(\frac{S}{N}\right)_D = \frac{2}{3}\frac{S_T}{LN_0W}$$

9.4-6

$$S_D = \int_{-\infty}^{\infty}|H_C(f)|^2|H_R(f)|^2 G_x(f)\,df = K^2\int_{-\infty}^{\infty}G_x(f)\,df = K^2S_T$$

$$N_D = \frac{N_0}{2}\int_{-\infty}^{\infty}|H_R(f)|^2\,df = N_0K^2L\int_0^W\left[1+\left(\frac{2f}{W}\right)^4\right]df = N_0K^2L\frac{21W}{5} \text{ so } \left(\frac{S}{N}\right)_D = \frac{5}{21}\frac{S_T}{LN_0W}$$

9.4-7

(a) $S_{T_{\text{dBm}}} - L + 174 - 10\log_{10}(10\times5\times10^3) = 60$ dB,

$L = 3 \times 40$ = 120 dB, S_T = 53 dBm = 200 W

(b) L_1 = 60 dB = 10^6, L = 120 dB = 10^{12}, $S_T = \dfrac{2L_1}{L}\times 200$ W = 0.4 mW

9.4-8

$$L_1 = \frac{240}{6} = 40 \text{ dB} = 10^4, \left(\frac{S}{N}\right)_D = \frac{1}{6\times10^4}\frac{S_T}{N_0W} = 10^3 \quad\Rightarrow\quad \frac{S_T}{N_0W} = 6\times10^7$$

(a) $L_1 = 20 \text{ dB} = 100, \left(\frac{S}{N}\right)_D = \frac{1}{12\times100}\times6\times10^7 = 5\times10^4 = 47 \text{ dB}$

(b) $L_1 = 60 \text{ dB} = 10^6, \left(\frac{S}{N}\right)_D = \frac{1}{4\times10^6}\times6\times10^7 = 15 \approx 12 \text{ dB}$

9.4-9

$L = 0.5\times400 = 200$ dB, $L_1 = 200/m$ dB,

$$\left(\frac{S}{N}\right)_D = 10\log_{10}\left(\frac{S_T}{mL_1N_0W}\right) = 80 - 10\log_{10} m - \frac{200}{m} \geq 30 \text{ dB so } \log_{10} m + \frac{20}{m} \leq 5$$

m	$\log m + 20/m$
10	1.0 + 2 = 3
5	0.7 + 4 = 4.7 $\Rightarrow$ $m_{\min} = 5$
4	0.6 + 5 = 5.6

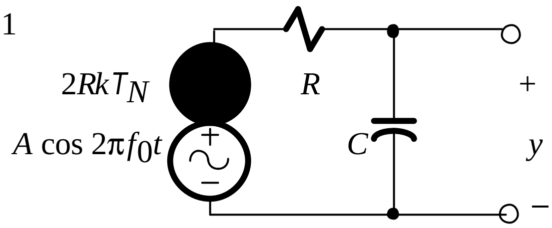
9.4-10

$$L_{1_{\text{dB}}} = \frac{L_{\text{dB}}}{m} \quad\Rightarrow\quad L_1 = L^{1/m}, \text{ so } \left(\frac{S}{N}\right)_D = Km^{-1}L^{-m^{-1}} \text{ where } K = \frac{S_T}{N_0W} \qquad \text{(cont.)}$$

$$\frac{d}{dm}\left(\frac{S}{N}\right)_D = K\left[(-m)^{-2}L^{-m^{-1}} + m^{-1}L^{-m^{-1}}(\ln L)(m^{-2})\right] = 0 \quad \text{so}$$

$$m = \ln L = \frac{\ln 10}{10}\left(10\log_{10} L\right) = 0.23L_{\text{dB}}$$

9.4-11



$N = kT_N/C$ (from Example 9.3-1)

$S = (A^2/2)[1 + (2\pi f_0 RC)^2]^{-1}$

$$\frac{S}{N} = \frac{A^2}{2kT_N}\frac{C}{1+(2\pi f_0 RC)^2} \quad \text{so} \quad \frac{d}{dC}\left(\frac{S}{N}\right) = \frac{A^2}{2kT_N}\frac{1+(2\pi f_0 RC)^2 - (2\pi f_0 RC)^2\, 2C\times C}{\left[1+(2\pi f_0 RC)^2\right]^2} = 0$$

and $C = \dfrac{1}{2\pi f_0 R}$

9.5-1

$$\left(\frac{\sigma_A}{A}\right)^2 = \frac{N_0 B_N}{A^2} = \frac{kT_N B_N \tau}{E_p} = \frac{4\times 10^{-21}\times 1}{10^{-20}} = 0.4$$

9.5-2

$$\sigma_t^{\,2} = \frac{t_r^{\,2} N_0 B_N}{A^2} = \frac{t_r^{\,2} N_0 B_N \tau}{A^2 \tau} \quad \Rightarrow \quad \left|\frac{\sigma_t}{t_r}\right| = \sqrt{\frac{N_0 B_N \tau}{E_p}}$$

$$\sigma_A^{\,2} = N_0 B_N \quad \Rightarrow \quad \left|\frac{\sigma_A}{A}\right| = \sqrt{\frac{N_0 B_N}{A^2}} = \sqrt{\frac{N_0 B_N \tau}{E_p}}$$

9.5-3

Take $B_N \approx 1/2\tau = 100$ kHz « B_T, so $\sigma_A^{\,2} \approx \dfrac{N_0}{2E_p} A^2 \le \left(\dfrac{A}{100}\right)^2 \quad \Rightarrow \quad E_p \ge \dfrac{10^4 N_0}{2} = 5\times 10^{-9}$

Then $\sigma_t / \tau \approx \sqrt{N_0 / 4B_N E_p} = 0.01$

9.5-4

Take $B_N \approx B_T = 1$ MHz, so $\sigma_t^{\,2} \approx \dfrac{N_0 \tau}{4B_T E_p} \le \left(\dfrac{\tau}{100}\right)^2 \quad \Rightarrow \quad E_p \ge \dfrac{10^4 N_0}{4B_T \tau} = 5\times 10^{-11}$

Then $\sigma_A / A = \sqrt{N_0 B_N \tau / E_p} = 1$

9.5-5

$$\sigma_A{}^2 = N_0 B_N \leq \left(\frac{A}{100}\right)^2 = \frac{E_p/\tau}{10^4} \quad \Rightarrow \quad B_N \leq \frac{E_p}{10^4 N_0 \tau} = 10^{11} E_p$$

$$\sigma_t{}^2 = \frac{N_0 \tau}{4 B_N E_p} \leq \left(\frac{\tau}{1000}\right)^2 \quad \Rightarrow \quad B_N \geq \frac{10^6 N_0}{4\tau E_p} = \frac{1}{4\times 10^3 E_p}. \text{ Thus,}$$

$$10^{11} E_p \geq \frac{1}{4\times 10^3 E_p} \quad \Rightarrow \quad E_p \geq 5\times 10^{-8}$$

and $B_N = 10^{11} E_{p_{\min}} = 5\,\text{kHz}$ so $\dfrac{1}{2\tau} < B_N < B_T$

9.5-6

$B \gg 1/\tau \;\Rightarrow\; y(t) \approx x_R(t)$, $E_p = A_p{}^2\tau$, so $A^2 \approx A_p{}^2 = E_p/\tau$

$$B_N = \pi B/2 \;\Rightarrow\; \sigma^2 = N_0 B_N = N_0 \pi B/2, \text{ so } \left(\frac{A}{\sigma}\right)^2 = \frac{A_p{}^2}{N_0 \pi B/2} = \frac{2E_p}{\pi N_0 B\tau} \;\square\; \frac{2E_p}{N_0}$$

9.5-7

Assuming pulse arrives at $t = 0$, $y(t) = A_p\left(1 - e^{-2\pi B t}\right)$, $0 < t < \tau$, so $A = y(\tau) = A_p\left(1 - e^{-2\pi B\tau}\right)$

$$\sigma^2 = N_0 B_N = N_0 \pi B/2, \text{ so } \left(\frac{A}{\sigma}\right)^2 = \frac{A_p{}^2\left(1 - e^{-2\pi B\tau}\right)^2}{N_0 \pi B/2} = \frac{\left(1 - e^{-2\pi B\tau}\right)^2}{\pi B\tau}\frac{2E_p}{N_0}$$

9.5-8

$$P(f) = \tau \operatorname{sinc}^2 f\tau \;\Rightarrow\; H_{\text{opt}}(f) = \frac{2K\tau}{N_0}\operatorname{sinc}^2 f\tau\, e^{-j\omega t_d} \text{ and } h_{\text{opt}}(t) = \frac{2K\tau}{N_0}\Lambda\left(\frac{t - t_d}{\tau}\right)$$

Want $h_{\text{opt}}(t) = 0$ for $t < 0$ for realizability, so t_d ☚ τ.

9.5-9

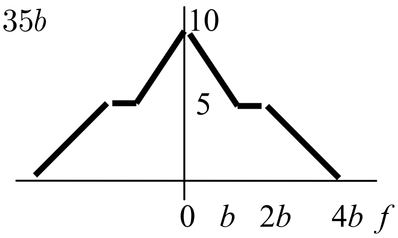
$$P(f) = \frac{1}{b + j2\pi f} \quad \Rightarrow \quad H_{\text{opt}}(f) = \frac{2K}{N_0}\frac{1}{b - j2\pi f}e^{-j\omega t_d} \text{ and } h_{\text{opt}}(t) = \frac{2K}{N_0}e^{-b(t_d - t)}u(t_d - t)$$

Want $h_{\text{opt}}(t) \approx 0$ for $t < 0$ for approximate realizability, so take t_d ☚ $5/b$ which yields $h_{\text{opt}}(t)$ « $2K/N_0$ for $t < 0$.

Chapter 10

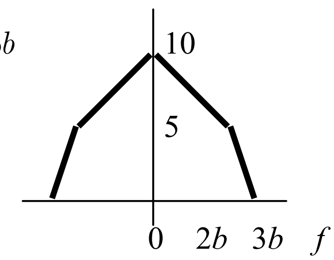
10.1-1

$$\overline{n^2} = 2\times 5\left[\frac{b}{2}+2b+\frac{2b}{2}\right]=35b,\ \overline{n_i^2} = 2\times 5\left[2b+\frac{b}{2}+\frac{2b}{2}\right]=35b$$



10.1-2

$$\overline{n^2} = 2\times 5\left[\frac{b}{2}+2b+\frac{2b}{2}\right]=35b,\ \overline{n_i^2} = 2\times 5\left[2b+\frac{b}{2}+\frac{2b}{2}\right]=35b$$



10.1-3

$$G_n(f)=\frac{N_0/2}{1+\left[\frac{4}{3}\left(\frac{f}{f_c}-\frac{f_c}{f}\right)\right]^2}$$

f/f_c	0	±0.5	±1	±1.5	±2
G_n/N_0	0	0.1	0.5	0.22	0.1

10.1-4

(*a*) $G_{lp}(f)=\frac{N_0}{2}\left|H_R(f+f_c)u(f+f_c)\right|^2=0$ for $f < -f_c$

For $f > 0$, $G_n(f)=\frac{N_0}{2}\left|H_R(f)u(f)\right|^2=G_{lp}(f-f_c)$

For $f < 0$, $G_n(f)=G_n(-f)=\frac{N_0}{2}\left|H_{lp}(-f-f_c)\right|^2=\frac{N_0}{2}\left|H_{lp}(f+f_c)\right|^2=G_{lp}(f+f_c)$

But $G_{lp}(f-f_c)=0$ for $f < 0$ and $G_{lp}(f+f_c)=0$ for $f > 0$, so

(cont.)

$$G_n(f) = G_{lp}(f - f_c) + G_{lp}(f + f_c) = \begin{cases} G_{lp}(f - f_c) & f > 0 \\ G_{lp}(f + f_c) & f < 0 \end{cases}$$

(b) $G_n(f)u(f) = G_{lp}(f - f_c) \quad \Rightarrow \quad G_n(f + f_c)u(f + f_c) = G_{lp}(f - f_c + f_c) = G_{lp}(f)$

$G_n(f)[1 - u(f)] = G_{lp}(f + f_c) \quad \Rightarrow \quad G_n(f - f_c)[1 - u(f - f_c)] = G_{lp}(f - f_c + f_c) = G_{lp}(f)$

Thus, $G_{ni}(f) = G_{lp}(f) + G_{lp}(f) = 2G_{lp}(f)$

10.1-5

(a) $H_{lp}(f) = H_R(f + f_c)u(f + f_c) \approx 1/(1 + j2f/B_T)$ for $f > -f_c$. Thus,

$$G_{lp}(f) = \frac{N_0}{2}\left|\frac{1}{1 + j2f/B_T}\right|^2 = \frac{N_0/2}{1 + (2f/B_T)^2}$$ for $f > -f_c$, which looks like the output of a 1st-order LPF with $B = B_T/2$.

(b) $G_{ni}(f) = 2G_{lp}(f)$

$$\overline{n_i^2} = \int_{-\infty}^{\infty} 2G_{lp}(f)\,df \approx N_0\int_{-f_c}^{\infty}\frac{df}{1 + (2f/B_T)^2} = \frac{N_0 B_T}{2}\left[\frac{\pi}{2} + \arctan\frac{2f_c}{B_T}\right]$$

$\approx N_0 \pi B_T/2$ since $2f_c/B_T = 2Q \gg 1$

10.1-6

$$y(t) = 2\left[n_i(t)\cos\omega_c t - n_q(t)\sin\omega_c t\right]\cos(\omega_c t + \theta)$$

$$= \underbrace{n_i(t)\cos\theta + n_q(t)\sin\theta}_{y_{lp}(t)} + \underbrace{n_i(t)\cos(2\omega_c t + \theta) - n_q(t)\sin(2\omega_c t + \theta)}_{y_{bp}(t)}$$

Since n_i and n_q are independent and $\overline{n_i} = \overline{n_q} = 0$

$$\overline{y_{lp}} = 0,\ \overline{y_{lp}^2} = \overline{n_i^2}\cos^2\theta + \overline{n_q^2}\sin^2\theta = \overline{n^2}$$

$$\overline{y_{bp}} = 0,\ \overline{y_{bp}^2} = \overline{n_i^2}\cos^2(2\omega_c t + \theta) + \overline{n_q^2}\sin^2(2\omega_c t + \theta) = \overline{n^2}[\cos^2(2\omega_c t + \theta) + \sin^2(2\omega_c t + \theta)]$$

$$= \overline{n^2}$$

10.1-7

$y(t) = A_n(t) - \overline{A_n}$ with $\overline{A_n^2} = 2\sigma_n^2 = 8$ and $\overline{A_n} = \sqrt{\pi\sigma_n^2/2} = \sqrt{2\pi}$

$p_Y(y) = p_{A_n}\left(y + \overline{A_n}\right) = \frac{1}{4}(y + \sqrt{2\pi})e^{-(y+\sqrt{2\pi})^2/8}u(y + \sqrt{2\pi})$ (cont.)

$$\overline{y} = 0,\ \sigma_y^2 = \overline{y^2} = \overline{\left(A_n - \overline{A_n}\right)^2} = \overline{A_n^2} - \overline{A_n}^2 = 8 - 2\pi \quad \Rightarrow \quad \sigma_y = \sqrt{8-2\pi} = 1.3$$

10.1-8

$$\overline{y} = \overline{A_n^2} = 2\sigma_n^2$$

$$\overline{y^2} = \overline{A_n^4} = \int_0^\infty A_n^4 \frac{A_n}{\sigma_n^2} e^{-A_n^2/2\sigma_n^2}\, dA_n = 4\sigma_n^4 \int_0^\infty \lambda^2 e^{-\lambda}\, d\lambda = 8\sigma_n^4$$

Since $A_n \geq 0$ and $y \geq 0$, transformation with $g^{-1}(A_n) = y^{1/2}$ yields

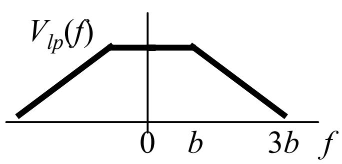
$$p_Y(y) = p_{A_n}(y^{1/2})\left|\frac{dy^{1/2}}{dy}\right| = \frac{y^{1/2}}{\sigma_n^2} e^{-y/2\sigma_n^2} u(y^{1/2}) \tfrac{1}{2} y^{-1/2} = \frac{1}{2\sigma_n^2} e^{-y/2\sigma_n^2} u(y)$$

10.1-9

$$\mathcal{F}[j\hat{v}(t)] = j(-j\,\mathrm{sgn}\, f)V(f) = (\mathrm{sgn}\, f)V(f) \text{ so}$$

$$V_{lp}(f) = \tfrac{1}{2}V(f+f_c) + \tfrac{1}{2}\mathrm{sgn}(f+f_c)V(f+f_c) = \tfrac{1}{2}[1+\mathrm{sgn}(f+f_c)]V(f+f_c)$$

$$= u(f+f_c)V(f+f_c) \text{ since } 1+\mathrm{sgn}(f+f_c) = \begin{cases} 0 & f < -f_c \\ 2 & f > -f_c \end{cases}$$



10.1-10

$$G_n(f) = \frac{N_0}{2}\left[\Pi\left(\frac{f-f_c}{B_T}\right) + \Pi\left(\frac{f+f_c}{B_T}\right)\right],$$

$$R_n(\tau) = \frac{N_0}{2} B_T \mathrm{sinc}\ B_T\tau\left(e^{j\omega_c\tau} + e^{-j\omega_c\tau}\right) = N_0 B_T \mathrm{sinc}\ B_T\tau \cos\omega_c\tau$$

$$(-j\,\mathrm{sgn}\, f)G_n(f) = \frac{-jN_0}{2}\left[\Pi\left(\frac{f-f_c}{B_T}\right) - \Pi\left(\frac{f+f_c}{B_T}\right)\right],$$

$$\hat{R}_n(\tau) = \frac{-jN_0}{2} B_T \mathrm{sinc}\ B_T\tau\left(e^{j\omega_c\tau} - e^{-j\omega_c\tau}\right) = N_0 B_T \mathrm{sinc}\ B_T\tau \sin\omega_c\tau$$

(cont.)

$$R_{n_i}(\tau) = \left(N_0 B_T \text{sinc } B_T\tau \cos\omega_c\tau\right)\cos\omega_c\tau + \left(N_0 B_T \text{sinc } B_T\tau \sin\omega_c\tau\right)\sin\omega_c\tau$$

$$= N_0 B_T \text{sinc } B_T\tau\left(\cos^2\omega_c\tau + \sin^2\omega_c\tau\right) = N_0 B_T \text{sinc } B_T\tau$$

$$G_{n_i}(f) = \frac{N_0}{2}\Pi\left(\frac{f}{B_T}\right) + \frac{N_0}{2}\Pi\left(\frac{f}{B_T}\right) = N_0\Pi\left(\frac{f}{B_T}\right) \text{ so } R_{n_i}(\tau) = \mathcal{F}_\tau^{-1}\left[G_{n_i}(f)\right]$$

10.1-11

Let $f_0 = f_c + B_T/2$ so $\omega_0 = \omega_c + \pi B_T$. Then $G_n(f) = \frac{N_0}{2}\left[\Pi\left(\frac{f-f_0}{B_T}\right) + \Pi\left(\frac{f+f_0}{B_T}\right)\right]$ and

$$R_n(\tau) = \frac{N_0}{2}B_T \text{sinc } B_T\tau\left(e^{j\omega_0\tau} + e^{-j\omega_0\tau}\right) = N_0 B_T \text{sinc } B_T\tau\cos\omega_0\tau$$

$$(-j\,\text{sgn}\, f)G_n(f) = \frac{-jN_0}{2}\left[\Pi\left(\frac{f-f_0}{B_T}\right) - \Pi\left(\frac{f+f_0}{B_T}\right)\right]$$

$$\hat{R}_n(\tau) = \frac{-jN_0}{2}B_T \text{sinc } B_T\tau\left(e^{j\omega_0\tau} - e^{-j\omega_0\tau}\right) = N_0 B_T \text{sinc } B_T\tau\sin\omega_0\tau$$

$$R_{n_i}(\tau) = \left[N_0 B_T \text{sinc } B_T\tau\cos\left(\omega_c + \pi B_T\right)\tau\right]\cos\omega_c\tau + \left[N_0 B_T \text{sinc } B_T\tau\sin\left(\omega_c + \pi B_T\right)\tau\right]\sin\omega_c\tau$$

$$= N_0 B_T \text{sinc } B_T\tau\cos\pi B_T\tau = \frac{N_0}{\pi\tau}\sin\pi B_T\tau\cos\pi B_T\tau = \frac{N_0}{2\pi\tau}\sin 2\pi B_T\tau = N_0 B_T \text{sinc } 2B_T\tau$$

$$G_{n_i}(f) = \frac{N_0}{2}\left[\Pi\left(\frac{f - B_T/2}{B_T}\right) + \Pi\left(\frac{f + B_T/2}{B_T}\right)\right] = \frac{N_0}{2}\Pi\left(\frac{f}{2B_T}\right) \text{ so } R_{n_i}(\tau) = \mathcal{F}_\tau^{-1}\left[G_{n_i}(f)\right]$$

10.1-12

$$R_{n_i}(\tau) = \mathcal{F}_\tau^{-1}\left[2G_{lp}(f)\right] = 2\int_{-\infty}^{\infty} G_{lp}(f)e^{j\omega\tau}\,df$$

$$R_n(\tau) = \mathcal{F}_\tau^{-1}\left[G_{lp}(f-f_c) + G_{lp}(f+f_c)\right] = \int_{-\infty}^{\infty} G_{lp}(f-f_c)e^{j\omega\tau}\,df + \int_{-\infty}^{\infty} G_{lp}(f+f_c)e^{j\omega\tau}\,df$$

$$= \int_{-\infty}^{\infty} G_{lp}(\lambda_1)e^{j2\pi(\lambda_1+f_c)}\,d\lambda_1 + \int_{-\infty}^{\infty} G_{lp}(\lambda_2)e^{j2\pi(\lambda_2-f_c)}\,d\lambda_2$$

$$= 2\int_{-\infty}^{\infty} G_{lp}(\lambda)e^{j2\pi\lambda\tau}\,d\lambda\,\tfrac{1}{2}\left(e^{j\omega_c\tau} + e^{-j\omega_c\tau}\right) = R_{n_i}(\tau)\cos\omega_c\tau$$

$$\hat{R}_n(\tau)\sin\omega_c\tau = R_{n_i}(\tau) - \left[R_{n_i}(\tau)\cos\omega_c\tau\right]\cos\omega_c\tau = R_{n_i}(\tau)\sin^2\omega_c\tau \text{ so } \hat{R}_n(\tau) = R_{n_i}(\tau)\sin\omega_c\tau$$

10.1-13

$E[n_q(t)n_q(t-\tau)] = E_1 - E_2 - E_3 + E_4$ where

$E_1 = E[\hat{n}(t)\cos\omega_c t \times \hat{n}(t-\tau)\cos\omega_c(t-\tau)] = \frac{1}{2}R_{\hat{n}}(\tau)[\cos\omega_c\tau + \cos\omega_c(2t-\tau)]$

$E_2 = E[\hat{n}(t)\cos\omega_c t \times n(t-\tau)\sin\omega_c(t-\tau)] = \frac{1}{2}R_{\hat{n}n}(\tau)[-\sin\omega_c\tau + \sin\omega_c(2t-\tau)]$

$E_3 = E[n(t)\sin\omega_c t \times \hat{n}(t-\tau)\cos\omega_c(t-\tau)] = \frac{1}{2}R_{n\hat{n}}(\tau)[\sin\omega_c\tau + \sin\omega_c(2t-\tau)]$

$E_4 = E[n(t)\sin\omega_c t \times n(t-\tau)\sin\omega_c(t-\tau)] = \frac{1}{2}R_n(\tau)[\cos\omega_c\tau - \cos\omega_c(2t-\tau)]$

Thus, $R_{n_q}(t,t-\tau) = \frac{1}{2}[R_{\hat{n}}(\tau) + R_n(\tau)]\cos\omega_c\tau + \frac{1}{2}[R_{\hat{n}n}(\tau) - R_{n\hat{n}}(\tau)]\sin\omega_c\tau$

$$+\frac{1}{2}[R_{\hat{n}}(\tau) - R_n(\tau)]\cos\omega_c(2t-\tau) - \frac{1}{2}[R_{\hat{n}n}(\tau) + R_{n\hat{n}}(\tau)]\sin\omega_c(2t-\tau)$$

But $R_{\hat{n}} = R_n$ so $R_{\hat{n}} + R_n = 2R_n$ and $R_{\hat{n}} - R_n = 0$ and

$-R_{n\hat{n}} = R_{\hat{n}n} = \hat{R}_n$ so $R_{\hat{n}n} - R_{n\hat{n}} = 2\hat{R}_n$ and $R_{\hat{n}n} + R_{n\hat{n}} = 0$. Hence,

$R_{n_q}(t,t-\tau) = R_{n_q}(\tau) = R_n(\tau)\cos\omega_c\tau + \hat{R}_n(\tau)\sin\omega_c\tau$

10.1-14

$E[n_i(t)n_q(t-\tau)] = E_1 - E_2 + E_3 - E_4$ where

$E_1 = E[n(t)\cos\omega_c t \times \hat{n}(t-\tau)\cos\omega_c(t-\tau)] = \frac{1}{2}R_{n\hat{n}}(\tau)[\cos\omega_c\tau + \cos\omega_c(2t-\tau)]$

$E_2 = E[n(t)\cos\omega_c t \times n(t-\tau)\sin\omega_c(t-\tau)] = \frac{1}{2}R_n(\tau)[-\sin\omega_c\tau + \sin\omega_c(2t-\tau)]$

$E_3 = E[\hat{n}(t)\sin\omega_c t \times \hat{n}(t-\tau)\cos\omega_c(t-\tau)] = \frac{1}{2}R_{\hat{n}}(\tau)[\sin\omega_c\tau + \sin\omega_c(2t-\tau)]$

$E_4 = E[\hat{n}(t)\sin\omega_c t \times n(t-\tau)\sin\omega_c(t-\tau)] = \frac{1}{2}R_{\hat{n}n}(\tau)[\cos\omega_c\tau - \cos\omega_c(2t-\tau)]$

Thus, $R_{n_in_q}(t,t-\tau) = \frac{1}{2}[R_{n\hat{n}}(\tau) + R_{\hat{n}n}(\tau)]\cos\omega_c\tau + \frac{1}{2}[R_n(\tau) - R_{\hat{n}}(\tau)]\sin\omega_c\tau$

$$+\frac{1}{2}[R_{n\hat{n}}(\tau) - R_{\hat{n}n}(\tau)]\cos\omega_c(2t-\tau) - \frac{1}{2}[R_n(\tau) + R_{\hat{n}}(\tau)]\sin\omega_c(2t-\tau)$$

But $R_{\hat{n}} = R_n$ so $R_{\hat{n}} + R_n = 2R_n$ and $R_{\hat{n}} - R_n = 0$ and

$R_{n\hat{n}} = -R_{\hat{n}n} = -\hat{R}_n$ so $R_{n\hat{n}} - R_{\hat{n}n} = -2\hat{R}_n$ and $R_{n\hat{n}} + R_{\hat{n}n} = 0$. Hence,

$R_{n_in_q}(t,t-\tau) = R_{n_in_q}(\tau) = R_n(\tau)\sin\omega_c\tau + \hat{R}_n(\tau)\cos\omega_c\tau$

10.1-15

(*a*) $R_{n_in_q}(\tau) = [R_{n_i}(\tau)\cos\omega_c\tau]\sin\omega_c\tau - [R_{n_i}(\tau)\sin\omega_c\tau]\cos\omega_c\tau = 0$ (cont.)

(*b*) $G_n(f)u(f) = G_{lp}(f - f_c) \quad \Rightarrow \quad G_n(f + f_c)u(f + f_c) = G_{lp}(f + f_c - f_c) = G_{lp}(f)$

$G_n(f)[1 - u(f)] = G_{lp}(f + f_c) \quad \Rightarrow \quad G_n(f - f_c)[1 - u(f - f_c)] = G_{lp}(f - f_c + f_c) = G_{lp}(f)$

Thus, $R_{n_i n_q}(\tau) = \mathcal{F}_\tau^{-1}\{j[G_{lp}(f) - G_{lp}(f)]\} = 0$

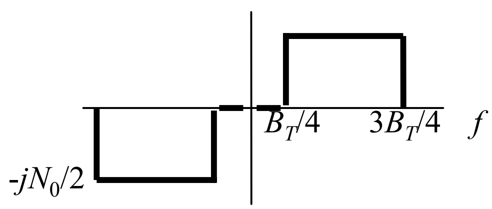
10.1-16

$$G_n(f + f_c)u(f + f_c) = \frac{N_0}{2}\Pi\left(\frac{f - B_T/2}{B_T}\right),\ G_n(f - f_c)[1 - u(f - f_c)] = \frac{N_0}{2}\Pi\left(\frac{f + B_T/2}{B_T}\right)$$

$$R_{n_i n_q}(\tau) = \mathcal{F}_\tau^{-1}\left\{j\frac{N_0}{2}\left[\Pi\left(\frac{f - B_T/2}{B_T}\right) - \Pi\left(\frac{f + B_T/2}{B_T}\right)\right]\right\} = j\frac{N_0}{2}B_T \operatorname{sinc} B_T\tau\left(e^{j\pi B_T\tau} - e^{-j\pi B_T\tau}\right)$$

$$= -N_0 B_T \operatorname{sinc} B_T\tau \sin \pi B_T\tau = -\pi N_0 B_T^{\,2}\tau \operatorname{sinc}^2 B_T\tau$$

10.1-17



$$G_n(f + f_c)u(f + f_c) - G_n(f - f_c)[1 - u(f - f_c)] = \frac{N_0}{2}\left[\Pi\left(\frac{f - B_T/2}{B_T/2}\right) - \Pi\left(\frac{f + B_T/2}{B_T/2}\right)\right]$$

$$R_{n_i n_q}(\tau) = \mathcal{F}_\tau^{-1}\left\{j\frac{N_0}{2}\left[\Pi\left(\frac{f - B_T/2}{B_T/2}\right) - \Pi\left(\frac{f + B_T/2}{B_T/2}\right)\right]\right\} = j\frac{N_0}{2}\frac{B_T}{2}\operatorname{sinc}\frac{B_T\tau}{2}\left(e^{j\pi B_T\tau} - e^{-j\pi B_T\tau}\right)$$

$$= -\frac{N_0 B_T}{2}\operatorname{sinc}\frac{B_T\tau}{2}\sin \pi B_T\tau = -\frac{\pi N_0 B_T^{\,2}\tau}{2}\operatorname{sinc}\frac{B_T\tau}{2}\operatorname{sinc} B_T\tau$$

10.2-1

$$N_0 = k\mathcal{T}_N = k\mathcal{T}_0(\mathcal{T}_N/\mathcal{T}_0) = 4\times 10^{-21}\times 10 = 4\times 10^{-20}$$

$$(S/N)_D = S_R/N_0 W = 20\times 10^{-9}/(4\times 10^{-20}\times 5\times 10^{6}) = 10^5 = 50 \text{ dB}$$

10.2-2

$$N_0 = k\mathcal{T}_N = k\mathcal{T}_0(\mathcal{T}_N/\mathcal{T}_0) = 4\times 10^{-21}\times 10 = 4\times 10^{-20}$$

$$\left(\frac{S}{N}\right)_D = \frac{0.4}{1 + 0.4}\frac{20\times 10^{-9}}{4\times 10^{-20}\times 5\times 10^{6}} = 2.86\times 10^4 = 44.6 \text{ dB}$$

10.2-3

$$y(t) = \left\{ \left[A_c x(t) + n_i(t) \right] \cos \omega_c t - n_q(t) \sin \omega_c t \right\} 2 \cos(\omega_c t + \phi')$$

$$= A_c x(t) \cos\phi' + n_i(t) \cos\phi' + n_q(t) \sin\phi' + \text{ high-frequency terms}$$

$y_D(t) = A_c x(t) \cos\phi' + n_i(t)\cos\phi' + n_q(t)\sin\phi'$ so $S_D = A_c^2 \overline{x^2} \cos^2\phi'$

$$N_D = E\left[n_i^2 \cos^2\phi' + 2 n_i n_q \cos\phi' \sin\phi' + n_q^2 \sin^2\phi' \right]$$

$$= \overline{n_i^2} \cos^2\phi' + \overline{n_q^2} \sin^2\phi' = \overline{n^2}(\cos^2\phi' + \sin^2\phi') = \overline{n^2} = 2N_0 W$$

Thus, $(S/N)_D = A_c^2 \overline{x^2} \cos^2\phi' / 2N_0 W = S_R / N_0 W \times \cos^2\phi' = \gamma \cos^2\phi'$

10.2-4

DSB: $S_p = x_c^2\big|_{\max} = A_c^2 \quad \Rightarrow \quad S_D = A_c^2 \overline{x^2} = S_p S_x$ so $(S/N)_D = S_x S_p / 2N_0 W = \frac{1}{2} S_x \gamma_p$

AM: $S_p = A_c^2(1 + x^2)\big|_{\max} = 4A_c^2 \quad \Rightarrow \quad S_D = A_c^2 \overline{x^2} = \frac{1}{4} S_p S_x$ so

$$(S/N)_D = \tfrac{1}{4} S_x S_p / 2N_0 W = \tfrac{1}{8} S_x \gamma_p$$

10.2-5

$v(t) = \left[A_c x_1(t) + n_i(t) \right] \cos\omega_c t \pm \left[A_c x_2(t) \mp n_q(t) \right] \sin\omega_c t$ so $y_{D_1}(t) = A_c x_1(t) + n_i(t)$ and

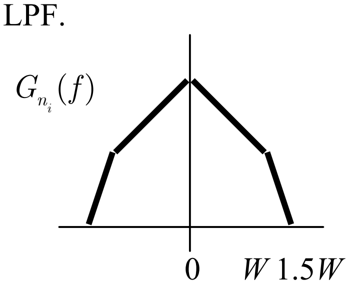
$y_{D_2}(t) = A_c x_2(t) \mp n_q(t)$ where $\overline{x_1^2} = \overline{x_2^2} = S_x$, $S_R = \frac{1}{2} A_c^2 \overline{x_1^2} + \frac{1}{2} A_c^2 \overline{x_2^2} = A_c^2 S_x$, and

$\overline{n_i^2} = \overline{(\mp n_q)^2} = \overline{n^2} = 2N_0 W$. Thus, both outputs have $\left(\dfrac{S}{N}\right)_D = \dfrac{A_c^2 S_x}{2N_0 W} = \dfrac{S_R}{2N_0 W} = \frac{1}{2}\gamma$

10.2-6

For USSB, any noise component in $f_c - W < |f| < f_c$ will be translated to $|f| < W$ and cannot be removed by LPF; similarly, for LSSB, noise components in $f_c < |f| < f_c + W$ cannot be removed by LPF. For DSB, noise components outside $f_c - W < |f| < f_c + W$ are translated to $|f| > W$ and can be removed by LPF.

10.2-7



(cont.)

With ideal LPF at output, $N_D = \int_{-W}^{W} G_{n_i}(f)\,df = \frac{3}{2}N_0 W$ so $\left(\frac{S}{N}\right)_D = \frac{S_R}{(\frac{3}{2}N_0 W)} = \frac{2}{3}\gamma$

10.2-8

USSB: $y_D(t) = \frac{1}{2}A_c x(t) + n_i(t),\ S_D = S_R$

$$N_D = 2\int_0^W \frac{N_0 f_c}{2(f+f_c)}\,df = N_0 f_c\left[\ln(W+f_c) - \ln f_c\right] = N_0 f_c \ln(1 + W/f_c)$$

$$\left(\frac{S}{N}\right)_D = \frac{S_R}{N_0 f_c \ln(1+W/f_c)} = \frac{W/f_c}{\ln(1+W/f_c)}\gamma = \begin{cases} 1.10\gamma & W/f_c = 1/5 \\ 1.01\gamma & W/f_c = 1/50 \end{cases}$$

DSB: $y_D(t) = A_c x(t) + n_i(t),\ S_D = 2S_R,\ G_{n_i}(f) = \frac{N_0 f_c}{2}\left(\frac{1}{f+f_c} - \frac{1}{f-f_c}\right) = \frac{N_0 f_c^2}{f_c^2 - f^2}$

$$N_D = 2\int_0^W \frac{N_0 f_c^2}{f_c^2 - f^2}\,df = 2N_0 f_c \frac{1}{2f_c}\ln\left(\frac{f_c + W}{f_c - W}\right) = N_0 f_c \ln\left(\frac{1+W/f_c}{1-W/f_c}\right)$$

$$\left(\frac{S}{N}\right)_D = \frac{2S_R}{N_0 f_c \ln\left(\frac{1+W/f_c}{1-W/f_c}\right)} = \frac{2W/f_c}{\ln\left(\frac{1+W/f_c}{1-W/f_c}\right)}\gamma = \begin{cases} 0.99\gamma & W/f_c = 1/5 \\ 1.00\gamma & W/f_c = 1/50 \end{cases}$$

Note: no significant difference between LSSB and DSB when $W/f_c \ll 1$.

10.2-9

$v(t) = \left[\frac{1}{2}A_c x(t) + n_i(t)\right]\cos\omega_c t - \left[\frac{1}{2}A_c \hat{x}(t) + n_q(t)\right]\sin\omega_c t$

$y(t) = v(t)2\cos\left[\omega_c t + \phi(t)\right] = \left[\frac{1}{2}A_c x(t) + n_i(t)\right]\cos\phi(t) + \left[\frac{1}{2}A_c \hat{x}(t) + n_q(t)\right]\sin\phi(t) +$

high-frequency terms. Since $\phi(t)$ has slow variations compared to $x(t)$,

$y_D(t) \approx \left[\frac{1}{2}A_c x(t) + n_i(t)\right]\cos\phi(t) + \left[\frac{1}{2}A_c \hat{x}(t) + n_q(t)\right]\sin\phi(t) = \frac{1}{2}A_c x(t)$ when $\phi = n_i = n_q = 0$

which implies that $K = 2/A_c$. Then

$$E\left\{\left[x(t) - Ky_D(t)\right]^2\right\} = E\left\{x^2 + \left(x + \frac{2}{A_c}n_i\right)^2\cos^2\phi + \left(\hat{x} + \frac{2}{A_c}n_q\right)^2\sin^2\phi - 2x\left(x + \frac{2}{A_c}n_i\right)\cos\phi\right.$$

$$\left. -2x\left(\hat{x} + \frac{2}{A_c}n_q\right)\sin\phi + 2\left(x + \frac{2}{A_c}n_i\right)\left(\hat{x} + \frac{2}{A_c}n_q\right)\cos\phi\sin\phi\right\}$$

(cont.)

where $\overline{x^2} = \overline{\hat{x}^2} = S_x$, $\overline{x\hat{x}} = 0$, $\dfrac{4}{A_c^2} = \dfrac{S_x}{S_R}$, $\overline{n_i^2} = \overline{n_q^2} = N_0 W$, $\overline{n_i} = \overline{n_q} = 0$, $\overline{n_i n_q} = 0$. Thus,

$$\in^2 = \left[S_x + \left(S_x + S_x N_0 W / S_R\right)\overline{\cos^2\phi} + \left(S_x + S_x N_0 W / S_R\right)\overline{\sin^2\phi} - 2S_x\overline{\cos\phi}\right] / S_x$$

$$= 1 + \overline{\cos^2\phi} + \overline{\sin^2\phi} - 2\overline{\cos\phi} + \frac{1}{S_R / N_0 W}\left(\overline{\cos^2\phi} + \overline{\sin^2\phi}\right)$$

But $\overline{\cos^2\phi} + \overline{\sin^2\phi} = \overline{\cos^2\phi + \sin^2\phi} = 1$ so $\in^2 = 2\left(1 - \overline{\cos\phi}\right) + 1/\gamma$

10.2-10

Envelope detection without mutilation requires $S_R \gg N_R = N_0 B_N$, where B_N is the noise equivalent bandwidth of $H_R(f)$, so B_N should be as small as possible, namely $B_N = B_T = 2W$ for an ideal BPF. With synchronous detection, there is no mutilation and noise components outside $f_c - W < |f| < f_c + W$ are translated to $|f| > W$ and can be removed by the LPF.

10.2-11

With $S_x = \dfrac{1}{2}$, $\left(\dfrac{S}{N}\right)_D = \dfrac{½}{1+½}\gamma = 10^4 \quad \Rightarrow \quad \gamma = 3\times 10^4$, whereas $\gamma_{th} \approx 20$.

Thus, $g_T = \gamma / \gamma_{th} \approx 1500 = 32\ \text{dB}$

10.2-12

$$\left(\frac{S}{N}\right)_D = \frac{½}{1+½}\gamma = 10^3 \quad \Rightarrow \quad \gamma = \frac{S_R}{N_0 W} = 3\times 10^3 \quad \Rightarrow \quad \frac{S_R}{N_0} = \gamma W = 24\times 10^6$$

$$\gamma_{th} \approx 20 = \left(\frac{S_R}{N_0 W}\right)_{\min} \quad \Rightarrow \quad W_{\max} = \frac{1}{20}\frac{S_R}{N_0} = 1.2\ \text{MHz}$$

10.2-13

$$y_D(t) = y(t) = A_n(t) + A_c x(t)\cos\phi_n(t) - \overline{A_n}$$

$$= A_c x(t) \text{ if } n(t) = 0, \text{ so } K = 1/A_c$$

$$\in^2 = E\left\{\left[x - \frac{1}{A_c}A_n - x\cos\phi_n + \frac{1}{A_c}\overline{A_n}\right]^2\right\} / S_x,\ \ S_x = 1$$

(cont.)

$$\epsilon^2 = E\left\{\overline{x^2} + \frac{A_n^{\,2}}{A_c^{\,2}} + x^2\cos^2\phi_n + \frac{\overline{A_n^{\,2}}}{A_c^{\,2}} - \frac{2}{A_c}A_n x - 2x^2\cos\phi_n + \frac{2}{A_c}\overline{A_n}x\right.$$

$$\left. - \frac{2}{A_c}xA_n\cos\phi_n - \frac{2}{A_c^{\,2}}\overline{A_n}A_n - \frac{2}{A_c}\overline{A_n}x\cos\phi_n\right\}$$

where $\overline{x^2} = 1,\ \overline{x} = 0,\ \overline{A_n^{\,2}} = 2\overline{n^2} = 4N_0W,\ \overline{A_n^{\,2}} = \sqrt{\pi N_0 W},\ \overline{A_n\cos\phi_n} = \overline{n_i} = 0,\ \overline{\cos\phi_n} = 0,.$

$$\overline{\cos^2\phi_n} = \frac{1}{2\pi}\int_{-\pi}^{\pi}\cos^2\phi_n\,d\phi_n = \frac{1}{2},\ A_c^{\,2} = 2S_R/(1+S_x) = S_R.\ \text{Thus,}$$

$$\epsilon^2 = 1 + \frac{4N_0W}{S_R} + \frac{1}{2} + \frac{\pi N_0 W}{S_R} - \frac{2}{S_R}\pi N_0 W = \frac{3}{2} + \frac{4-\pi}{\gamma}\quad \text{with}\quad \gamma < \gamma_{th} \approx 20$$

10.3-1

$$N_0 = k\mathcal{T}_N = k\mathcal{T}_0(\mathcal{T}_N/\mathcal{T}_0) = 4\times10^{-21}\times10 = 4\times10^{-20}$$

PM: $N_D = N_0W/S_R = 4\times10^{-20}\times500\times10^3/10\times10^{-9} = 2\times10^{-6}$

FM: $N_D = N_0W^3/3S_R = 4\times10^{-20}(500\times10^3)^3/3\times10\times10^{-9} = 1.67\times10^5$

Deemphasized FM: $N_D \approx N_0B_{de}^{\,2}W/S_R = 4\times10^{-20}(5\times10^3)^2 500\times10^3/10\times10^{-9} = 50$

10.3-2

$$N_D = \int_{-\infty}^{\infty}\left|H_D(f)\right|^2 G_\xi(f)\,df \le \int_{-\infty}^{\infty}\frac{1}{1+(f/2W)^{2n}}\frac{N_0f^2}{2S_R}df = \frac{N_0W^3}{S_R}\int_0^{\infty}\frac{\lambda^2}{1+\lambda^{2n}}d\lambda$$

$$\le \frac{N_0W^3}{S_R}\frac{\pi/2n}{\sin(3\pi/2n)} \approx \frac{N_0W^3}{3S_R}\ \text{ if } n \gg 1$$

10.3-3

$$G_{n_q}(f) = 2\frac{N_0}{2}\left|H_R(f+f_c)\right|^2 = \frac{N_0}{1+(2f/B_T)^2},\ G_\xi(f) = \frac{N_0}{2S_R}\frac{f^2}{1+(2f/B_T)^2}$$

$$N_D = 2\frac{N_0}{2S_R}\int_0^W\frac{f^2}{1+(2f/B_T)^2}df = \frac{N_0B_T^{\,3}}{8S_R}\left[\frac{2W}{B_T} - \arctan\left(\frac{2W}{B_T}\right)\right]$$

$$= \frac{N_0B_T^{\,3}}{8S_R}\left\{\frac{2W}{B_T} - \left[\frac{2W}{B_T} - \frac{1}{3}\left(\frac{2W}{B_T}\right)^3 + \cdots\right]\right\} \approx \frac{N_0B_T^{\,3}}{8S_R}\frac{1}{3}\left(\frac{2W}{B_T}\right)^3 = \frac{N_0W^3}{3S_R}\ \text{ if }\ B_T \gg W$$

10.3-4

$$N_0 = k\mathcal{T}_N = k\mathcal{T}_0(\mathcal{T}_N / \mathcal{T}_0) = 4\times 10^{-21}\times 10 = 4\times 10^{-20},\;\; D = f_\Delta / W = 2\times 10^6 / 500\times 10^3 = 4$$

FM: $\left(\frac{S}{N}\right)_D = 3D^2 S_x \frac{S_R}{N_0 W} = 3\times 4^2\times 0.1\frac{10^{-9}}{4\times 10^{-20}\times 500\times 10^3} = 240\times 10^3 = 53.8\text{ dB}$

Deemphasized FM:

$$\left(\frac{S}{N}\right)_D \approx \left(\frac{f_\Delta}{B_{de}}\right)^2 S_x \frac{S_R}{N_0 W} = \left(\frac{2\times 10^6}{5\times 10^3}\right)^2 0.1\frac{10^{-9}}{4\times 10^{-20}\times 500\times 10^3} = 800\times 10^6 = 89.0\text{ dB}$$

10.3-5

$$N_D = \int_{-W}^{W}\frac{1}{1+(f/B_{de})^2}\frac{N_0}{2S_R}\,df = \frac{N_0 B_{de}}{S_R}\arctan\frac{W}{B_{de}}$$

$$\left(\frac{S}{N}\right)_D = \frac{\phi_\Delta{}^2 S_x S_R}{N_0 B_{de}\arctan(W/B_{de})} = \frac{W/B_{de}}{\arctan(W/B_{de})}\phi_\Delta{}^2 S_x\gamma \approx \left(\frac{2}{\pi}\frac{W}{B_{de}}\right)\phi_\Delta{}^2 S_x\gamma$$

10.3-6

$S_D = f_\Delta{}^2\overline{x^2} = f_\Delta{}^2/2$ and $N_D = 2\int_{100}^{300}\frac{N_0 f^2}{2S_R}\,df = \frac{N_0}{S_R}\times\frac{26}{3}\times 10^6$ so

$$\left(\frac{S}{N}\right)_D = \frac{1}{2}f_\Delta{}^2\frac{3\times 10^{-6}}{26}\frac{S_R}{N_0} = 290 = 24.6\text{ dB}$$

10.3-7

$$N_D = 2\int_0^W e^{-(f/B_{de})^2}\frac{N_0 f^2}{2S_R}\,df = \frac{N_0 B_{de}{}^3}{S_R}\int_0^{W/B_{de}}\lambda^2 e^{-\lambda^2}\,d\lambda < \frac{N_0 B_{de}{}^3}{S_R}\int_0^\infty \lambda^2 e^{-\lambda^2}\,d\lambda = \frac{N_0 B_{de}{}^3}{S_R}\frac{\sqrt{\pi}}{4}$$

$\left(\frac{S}{N}\right)_D > \frac{4f_\Delta{}^2 S_x S_R}{\sqrt{\pi}N_0 B_{de}{}^3} = \frac{4}{\sqrt{\pi}}\left(\frac{W}{B_{de}}\right)^3\left(\frac{f_\Delta}{W}\right)^2 S_x\gamma$ so

Improvement factor $> (4/\sqrt{\pi})(W/B_{de})^3 \approx 770$ when $B_{de} = W/7$

10.3-8

PM: $(S/N)_D = \phi_\Delta{}^2 S_x\gamma = 10^3$

FM: $D = \phi_\Delta$ for same B_T, so $(S/N)_D = (W/B_{de})^2\phi_\Delta{}^2 S_x\gamma = 10^2\times 10^3 = 50\text{ dB}$

10.3-9

$(S/N)_{D_{th}} = (W/B_{de})^2 D^2 S_x \gamma_{th} \approx 20(W/B_{de})^2 D^2 (D+2) S_x, \;\; D > 2$

$20 \times 5^2 D^2 (D+2)/2 = 10^5 \quad \Rightarrow \quad D^3 + 2D^2 = 400 \quad \Rightarrow \quad D \approx 6.7$ (by trial and error)

$B_T \approx 2(6.7+2) \times 10 = 174$ kHz, $S_R \geq 20(6.7+2) \times 10^{-8} \times 10^4 = 17.4$ mW

10.3-10

$\gamma_{th} = 20M(\phi_\Delta), \;\; (S/N)_{D_{th}} = \phi_\Delta{}^2 S_x \gamma_{th} = 20M(\phi_\Delta)\phi_\Delta{}^2 S_x$

$\phi_\Delta \leq \pi, \;\; M(\pi) \approx 2(\pi+2), \;\; S_x \leq 1,$ so $(S/N)_{D_{th}} \leq 40(\pi+2)\pi^2 \approx 2030 \approx 33$ dB

10.3-11

IF input $= A_c \cos[\omega_c t + \phi(t)] \times 2\cos\left[(\omega_c - \omega_{IF})t + K\phi_D(t)\right]$ so

$v_{IF}(t) = A_c \cos\left[\omega_{IF} t + \phi_{IF}(t)\right] \quad$ where $\quad \phi_{IF}(t) = \phi(t) - K\phi_D(t)$

Thus, $y_D(t) = \dfrac{1}{2\pi}\dot{\phi}_{IF}(t) = f_\Delta x(t) - K y_D(t)$ so $y_D(t) = \dfrac{f_\Delta}{1+K} x(t) = \dfrac{1}{2\pi}\dot{\phi}_{IF}(t)$

and $D_{IF} = \dfrac{f_\Delta}{(1+K)W} = \dfrac{D}{1+K}$

10.4-1

$S_x = 1/2, \;\; (S/N)_D = 10^4, \;\; \gamma = S_T / LN_0 W = 10 S_T$

(*a*) $(S/N)_D = 10 S_T \quad \Rightarrow \quad S_T = 1$ kW

(*b*) $\mu = 1, \;\; \left(\dfrac{S}{N}\right)_D = \dfrac{1/2}{1+1/2} 10 S_T \quad \Rightarrow \quad S_T = 3$ kW

$\mu = \tfrac{1}{2}, \;\; \left(\dfrac{S}{N}\right)_D = \dfrac{1/8}{1+1/8} 10 S_T \quad \Rightarrow \quad S_T = 9$ kW

(*c*) $(S/N)_D = \pi^2 \times \tfrac{1}{2} \times 10 S_T \quad \Rightarrow \quad S_T \approx 200$ W

(*d*) $(S/N)_D = 3D^2 \times \tfrac{1}{2} \times 10 S_T$ provided that $\gamma \geq \gamma_{th} = 20M(D)$ so $S_T \geq 2M(D)$

(cont.)

D	$10^4/15D^2$	$2M(D)$	S_T	
1	667	5	667 W	
5	26.7	14	26.7 W	
10	6.7	24	24 W	Threshold limited

10.4-2

$S_x = 1,\ (S/N)_D = 10^4,\ \gamma = S_T / LN_0W = 5S_T$

$(a)\ (S/N)_D = 5S_T \quad \Rightarrow \quad S_T = 2\ \text{kW}$

$(b)\ \mu = 1,\ \left(\frac{S}{N}\right)_D = \frac{1}{1+1}5S_T \quad \Rightarrow \quad S_T = 4\ \text{kW}$

$\mu = \frac{1}{2},\ \left(\frac{S}{N}\right)_D = \frac{1/4}{1+1/4}5S_T \quad \Rightarrow \quad S_T = 10\ \text{kW}$

$(c)\ (S/N)_D = \pi^2 \times 1 \times 5S_T \quad \Rightarrow \quad S_T \approx 200\ \text{W}$

$(d)\ (S/N)_D = 3D^2 \times 1 \times 5S_T$ provided that $\gamma \geq \gamma_{th} = 20M(D)$ so $S_T \geq 4M(D)$

D	$10^4/15D^2$	$4M(D)$	S_T	
1	667	10	667 W	
5	26.7	28	28 W	Threshold limited
10	6.7	48	48 W	Threshold limited

10.4-3

$L = 10^{\alpha\ell/10} = 10^{\ell/10},\ \gamma = S_T / LN_0W = 10^{10} \times 10^{-\ell/10},\ S_x = 1/2$

$(a)\ (S/N)_D = \gamma = 10^{10} \times 10^{-\ell/10} = 10^4 \quad \Rightarrow \quad \ell = 10(10-4) = 60\ \text{km}$

$(b)\ \left(\frac{S}{N}\right)_D = \frac{1/2}{1+1/2}\gamma = \frac{1}{3} \times 10^{10} \times 10^{-\ell/10} = 10^4 \quad \Rightarrow \quad \ell = 10(10 - 4 - \log_{10} 3) = 55.2\ \text{km}$

$(c)\ (S/N)_D = 3 \times 2^2 \times \frac{1}{2}\gamma = 6 \times 10^{10} \times 10^{-\ell/10} = 10^4 \ \Rightarrow\ \ell = 10(10 - 4 + \log_{10} 6) = 67.8\ \text{km}$

$(S/N)_D = 3 \times 8^2 \times \frac{1}{2}\gamma = 96 \times 10^{10} \times 10^{-\ell/10} = 10^4 \ \Rightarrow\ \ell = 10(10 - 4 + \log_{10} 96) = 79.8\ \text{km}$

10.4-4

$$L = \left(\frac{4\pi \times 3 \times 10^8}{3 \times 10^5} \ell \right)^2 = 1.58 \times 10^8 \ell^2 \text{ and } g_T \times g_R = 52\,\text{dB} = 1.58 \times 10^5 \text{ so}$$

$$\gamma = \frac{g_T g_R S_T}{L N_0 W} \approx \frac{10^7}{\ell^2}$$

(*a*) $(S/N)_D = \gamma = 10^7 / \ell^2 = 10^4 \quad \Rightarrow \quad \ell = \sqrt{1000} = 31.6 \text{ km}$

(*b*) $\left(\frac{S}{N} \right)_D = \frac{1/2}{1+1/2} \gamma = 10^7 / 3\ell^2 = 10^4 \quad \Rightarrow \quad \ell = \sqrt{1000/3} = 18.3 \text{ km}$

(*c*) $(S/N)_D = 3 \times 2^2 \times \frac{1}{2} \gamma = 6 \times 10^7 / \ell^2 = 10^4 \quad \Rightarrow \quad \ell = \sqrt{6000} = 77.5 \text{ km}$

$(S/N)_D = 3 \times 8^2 \times \frac{1}{2} \gamma = 96 \times 10^7 / \ell^2 = 10^4 \quad \Rightarrow \quad \ell = \sqrt{96{,}000} = 310 \text{ km}$

10.4-5

AM: $\left(\frac{S}{N} \right)_D = \frac{1/2}{1+1/2} \gamma = 20 \quad \Rightarrow \quad \gamma = 60$

FM: $\gamma \geq \gamma_{th} = 20M(D) \quad \Rightarrow \quad M(D_{\max}) = 60/20 \quad \Rightarrow \quad D_{\max} \approx 1$

$(S/N)_D = 3D^2 \times \frac{1}{2} \gamma = 90 = 19.5 \text{ dB}$

10.4-6

At output of the *k*th BPF,

$$S_k = (f_\Delta \alpha_k)^2 \overline{x_k^2} = f_\Delta^2 \alpha_k^2, \;\; N_k = 2 \int_{(k-1)W_0}^{kW_0} \frac{N_0 f^2}{2S_R} df = \frac{N_0}{3S_R} \left[k^3 W_0^3 - (k-1)^3 W_0^3 \right]. \text{ Thus,}$$

$$\left(\frac{S}{N} \right)_k = \frac{f_\Delta^2 \alpha_k^2}{(3k^2 - 3k + 1)(N_0 W_0^3 / 3S_R)}$$

10.4-7

$$\left(\frac{S}{N} \right)_k = \frac{\alpha_k^2}{3k^2 - 3k + 1} \frac{3S_R f_\Delta^2}{N_0 W_0^3} = \frac{S}{N} \quad \Rightarrow \quad \alpha_k^2 = C(3k^2 - 3k + 1) \text{ with } C = \frac{N_0 W_0^3}{3S_R f_\Delta^2} \frac{S}{N}$$

$$\overline{x_b^2} = E\left[\sum_{k=1}^{K} \sum_{i=1}^{K} \alpha_k x_k \alpha_i x_i \right] = \sum_{k=1}^{K} \sum_{i=1}^{K} \alpha_k \alpha_i \overline{x_k x_i} \text{ where } \overline{x_k x_i} = \begin{cases} \overline{x_k^2} = 1 & i = k \\ \overline{x_k}\,\overline{x_i} = 0 & i \neq k \end{cases} \quad \text{(cont.)}$$

so

$$\overline{x_b^2} = \sum_{k=1}^{K} \alpha_k^2 = 1 \text{ where} \sum_{k=1}^{K} \alpha_k^2 = C \sum_{k=1}^{K} \left[3k^2 - 3k + 1\right]$$

$$= C\left[3\frac{K(K+1)(2K+1)}{6} - 3\frac{K(K+1)}{2} + K\right] = CK^3 = 1 \Rightarrow C = \frac{1}{K^3}$$

Thus, $\left(\frac{S}{N}\right)_k = \frac{S}{N} = \frac{3S_R f_\Delta^2}{N_0 W_0^3} \frac{1}{K^3} = 3\left(\frac{f_\Delta}{W}\right)^2 \frac{S_R}{N_0 W}$, where $W = KW_0$

10.4-8

(*a*) $G_{n_1}(f) = \left|H_{de}(f)\right|^2 G_{\xi_1}(f)$, $\overline{n_1^2} = \frac{N_0 B_{de}^3}{S_R}\left[\frac{W}{B_{de}} - \arctan\frac{W}{B_{de}}\right] \approx 5.3 \times 10^{10} \frac{N_0}{S_R}$

$$G_{n_2}(f) = \left|H_{de}(f)\right|^2 \left[G_{\xi_2}(f - f_0) + G_{\xi_2}(f + f_0)\right]$$

$$= \left|H_{de}(f)\right|^2 \frac{N_0}{2S_R}\left[(f - f_0)^2 + (f + f_0)^2\right] = \left|H_{de}(f)\right|^2 \frac{N_0}{2S_R}(f^2 + f_0^2)\Pi\left(\frac{f}{2W}\right)$$

$$\overline{n_2^2} = \int_{-W}^{W} \frac{N_0}{S_R} \frac{f^2 + f_0^2}{1 + (f/B_{de})^2} df = \frac{2N_0}{S_R}\left[B_{de}^2 \int_0^{W/B_{de}} \frac{\lambda^2}{1+\lambda^2} d\lambda + B_{de} f_0^2 \int_0^{W/B_{de}} \frac{d\lambda}{1+\lambda^2}\right]$$

$$= \frac{2N_0 B_{de}^3}{S_R}\left\{\frac{W}{B_{de}} + \left[\left(\frac{f_0}{B_{de}}\right)^2 - 1\right]\arctan\frac{W}{B_{de}}\right\} \approx 8.8 \times 10^{12} \frac{N_0}{S_R} \gg \overline{n_1^2}$$

(*b*) $y_1 - y_2 = f_\Delta\left(x_L + x_R - x_L + x_R\right) + n_1 - n_2 \approx 2f_\Delta x_R - n_2$, since $\overline{n_2^2} \gg \overline{n_1^2}$

$y_1 + y_2 = f_\Delta\left(x_L + x_R + x_L - x_R\right) + n_1 + n_2 \approx 2f_\Delta x_L + n_2$. Thus,

$$\left(S/N\right)_{D_L} = \left(S/N\right)_{D_R} \approx (2f_\Delta)^2 \overline{x_R^2} / \overline{n_2^2} = 1.5 \times 10^{-13} f_\Delta^2 S_R / N_0$$

For mono signal with $\overline{x^2} = 1$, $(S/N)_D = f_\Delta^2 \overline{x^2} / \overline{n_1^2} = 1.9 \times 10^{-11} f_\Delta^2 S_R / N_0$. Thus,

$$(S/N)_D(\text{stereo})/(S/N)_D(\text{mono}) \approx 8 \times 10^{-3} = -21 \text{ dB}$$

10.6-1

$N_0 = k\mathcal{T}_N = k\mathcal{T}_0(\mathcal{T}_N/\mathcal{T}_0) = 4 \times 10^{-21} \times 10 = 4 \times 10^{-20}$, $\mu_p = t_0 = 0.1/f_s = 1/12 \times 10^6$

(cont.)

$$\left(\frac{S}{N}\right)_D = 4\mu_p{}^2 B_T\left(\frac{W}{f_s\tau_0}\right)\frac{S_x S_R}{N_0 W} = 4\left(\frac{1}{12\times10^6}\right)^2 10^7\left(\frac{500\times10^3}{1.2\times10^6/12\times10^6}\right)\frac{0.4\times10\times10^{-9}}{4\times10^{-20}\times500\times10^3}$$

$$= 2.78\times10^5 = 54.4 \text{ dB}$$

10.6-2

$$N_0 = k\mathcal{T}_N = k\mathcal{T}_0(\mathcal{T}_N/\mathcal{T}_0) = 4\times10^{-21}\times10 = 4\times10^{-20},\ \mu_p = \mu\tau_0 = 0.2/50\times250 = 16\times10^{-6}$$

$$\left(\frac{S}{N}\right)_D = 4\mu_p{}^2 B_T\left(\frac{W}{f_s\tau_0}\right)\frac{S_x S_R}{N_0 W} = 4\left(16\times10^{-6}\right)^2 3\times10^3\left(\frac{100}{250/50\times250}\right)\frac{0.1 S_R}{4\times10^{-20}\times100}$$

$$= 384\times10^{12} S_R \geq 10^4 \quad\Rightarrow\quad S_R \geq 26 \text{ pW}$$

10.6-3

$\left(\frac{S}{N}\right)_D \leq \frac{1}{8}\times20^2 S_x\gamma = 50 S_x\gamma$. But with $\mu_p = t_0 = \frac{0.3}{f_s},\ \tau_0 = \tau = \frac{0.2}{f_s},$ and $B_T = 20W$

$$\frac{4\mu_p{}^2 B_T W}{f_s\tau_0} = 36\left(\frac{W}{f_s}\right)^2 \quad\Rightarrow\quad \left(\frac{S}{N}\right)_D = 36\left(\frac{1}{2.5}\right)^2 S_x\gamma \approx 5.8 S_x\gamma$$

10.6-4

$$S_R = 9 f_s A^2\tau + f_s A^2 3\tau \ \Rightarrow\ A^2 = \frac{S_R}{12 f_s\tau},\ \mu_p = t_0 = 3.6\,\mu\text{s},\ \tau_0 = \tau = 2.5\,\mu\text{s},$$

and $B_T = 400$ kHz, so $\left(\frac{S}{N}\right)_D = \frac{4\mu_p{}^2 B_T A^2}{N_0} S_x = \frac{4 t_0{}^2 B_T W}{12 f_s\tau} S_x \frac{S_R}{N_0 W} = 2.49\left(\frac{\gamma}{9}\right)$

10.6-5

$$S_R = M f_s A^2\tau \geq M f_s A^2/B_T \quad\Rightarrow\quad A^2 \leq B_T S_R/M f_s$$

$$\mu_p = t_0 = \frac{1}{2}\left(\frac{T_s}{M} - \tau - T_g\right) \leq \frac{1}{2}\left(\frac{1}{M f_s} - \frac{2}{B_T}\right) = \frac{1}{20}\frac{B_T - 2M f_s}{M f_s B_T}$$

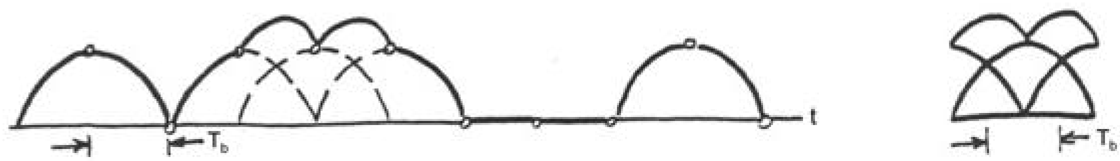
$$\left(\frac{S}{N}\right)_D \leq \left(\frac{B_T - 2M f_s}{M f_s B_T}\right)^2 \frac{B_T{}^2 S_R}{M f_s} S_x = \left(\frac{B_T - 2M f_s}{M f_s B_T}\right)^2 \left(\frac{W}{f_s}\right) S_x \frac{S_R}{M N_0 W} < \frac{1}{8}\left(\frac{B_T}{MW}\right)^2 S_x \frac{\gamma}{M},\ f_s \geq 2W$$

Chapter 11

11.1-1



11.1-2



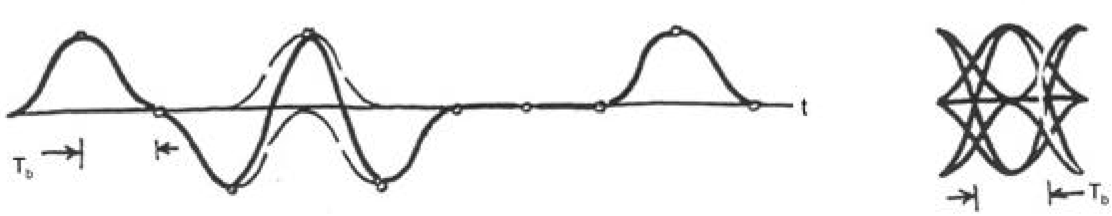
11.1-3



11.1-4



11.1-5



11.1-6

a_k	Nat. code	Gray codes
$7A/2$	111	100 010 001
$5A/2$	110	101 110 011
$3A/2$	101	111 100 111
$A/2$	100	110 101 101
$-A/2$	011	010 111 100
$-3A/2$	010	011 011 110
$-5A/2$	001	001 001 010
$-7A/2$	000	000 000 000

11.1-7

(a) $r_b =$ 16 x 20,000 = 320 kpbs, $B \geq \frac{1}{2} r_b = 160$ kHz

(b) $r = 320\ kpbs/\log_2 M \leq 2B = 120$ kbaud

$\log_2 M \geq 320/120 = 2.67 \Rightarrow M \geq 2^3 = 8$

11.1-8

(a) $128 = 2^7 \Rightarrow$ 7 bits/character

$r_b =$ 7 x 3000 = 21 kbps, $B \geq \frac{1}{2} r_b =$ 10.5 kHz

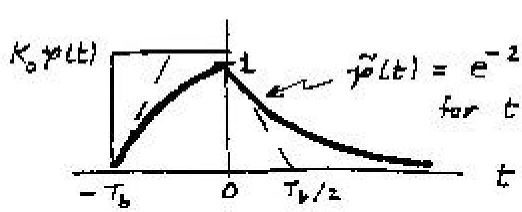
(b) $r = 21$ kbps/$\log_2 M \leq 2B = 6$ kHz

$\log_2 M \geq 21/6 = 3.5 \Rightarrow M \geq 2^4 = 16$

11.1-9

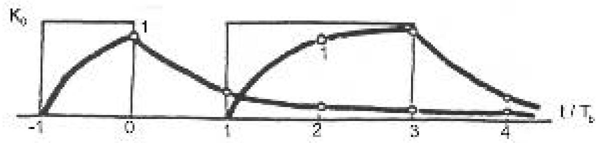
(a) $\tilde{p}(t) = g(t + T_b) - g(t)$

$\tilde{p}(0) = K_0(1 - e^{-2}) = 1 \Rightarrow K_0 = \frac{1}{1 - e^{-2}} = 1.157$



11.9 continued

(b)

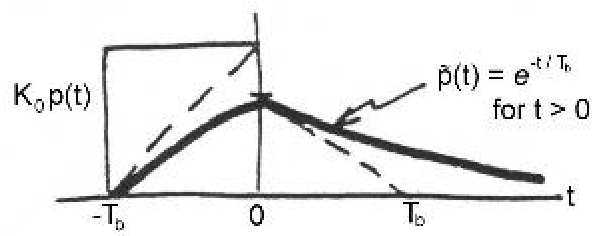


t_k	$y(t_k)$	ISI
0	1	0
1	$1e^{-2}=0.135$	0.135
2	$1+1e^{-4}=1.018$	0.018
3	$K_0(1-e^{-4})+1e^{-6}=1.138$	0.138
4	$K_0(1-e^{-4})e^{-2}+1e^{-8}=0.154$	0.138

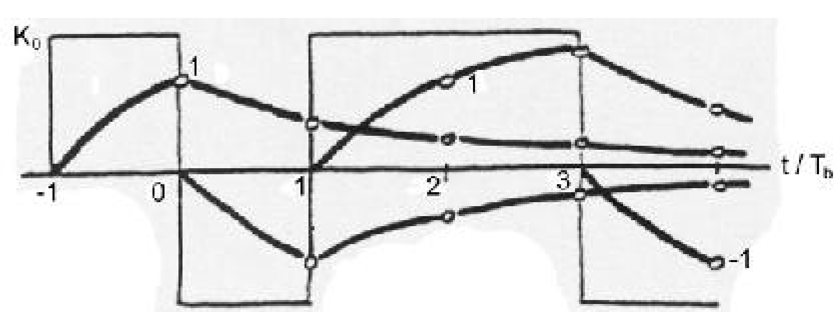
11.1-10

(a) $\tilde{p}(t)=g(t+T_b)-g(t)$

$$\tilde{p}(0)=K_0(1-e^{-1})=1 \quad \Rightarrow \quad K_0=\frac{1}{1-e^{-1}}=1.582,$$



(b)



11.1-10 (b) continued

t_k	$y(t_k)$	ISI
0	1	0
1	$-1 + 1e^{-1} = -0.632$	+0.368
2	$1 - 1e^{-1} + 1e^{-2} = 0.767$	-0.233
3	$K_0(1 - e^{-2}) - 1e^{-2} + 1e^{-3} = 1.282$	+0.282
4	$-1 + K_0(1-e^{-2})e^{-1} - 1e^{3} + 1e^{-4} = -0.528$	+0.472

11.1-11

$$p(D) = e^{-\pi(B/b)^2} \leq 0.01 \quad \Rightarrow \quad \pi(bD)^2 \geq \ln 100 \text{ where } D = 1/r$$

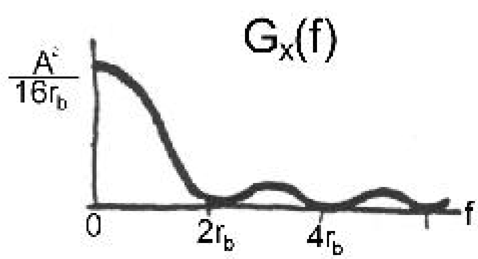
$$\frac{P(B)}{P(0)} = e^{-\pi(B/b)^2} \leq 0.01 \quad \Rightarrow \quad \pi(B/b)^2 \geq \ln 100$$

$$\text{thus } \pi(b/r)^2 \text{ x } \pi(B/b)^2 \geq (\ln 100)^2 \quad \Rightarrow \quad r \leq \frac{\pi}{\ln 100} B \approx 0.7B$$

11.1-12

$$m_a = \overline{a}_k = 0, \ \sigma_a^2 = \overline{a_k^2} = (A/2)^2, \ P(f) = \frac{1}{2r_b} \operatorname{sinc} \frac{f}{2r_b}$$

$$\Rightarrow \quad G_x(f) = \frac{A^2}{16r_b} \operatorname{sinc}^2 \frac{f}{2r_b}$$



11.1-12 continued

$$\overline{x^2} = \int_{-\infty}^{\infty} G_x(f)\,df = 2\frac{A^2}{16r_b}\int_0^{\infty} \text{sinc}^2 \frac{f}{2r_b}\,df = \frac{A^2}{8}$$

A waveform with polar RZ format, amplitude = $\pm A/2$, period = T_b and 50% duty cycle

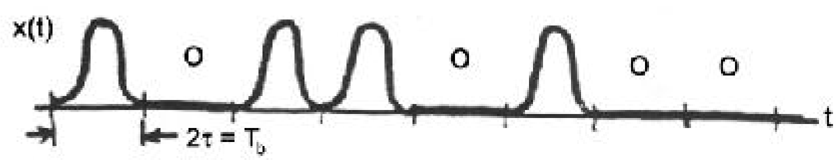
$$\Rightarrow \overline{x^2} = \frac{T_b/2}{T_b}\left(\frac{A}{2}\right)^2 = \frac{A^2}{8}$$

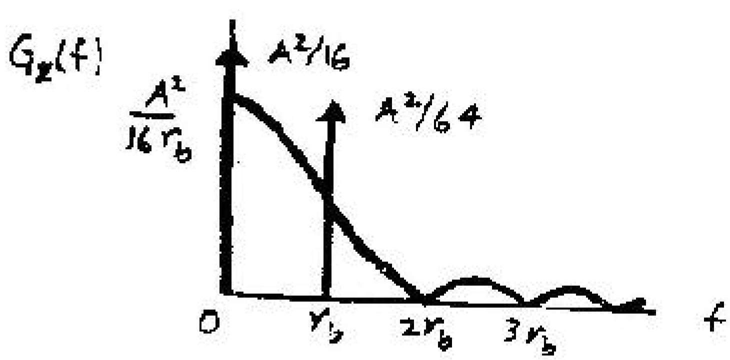
11.1-13

$$m_a = A/2,\ \overline{a_k^2} = \frac{1}{2}A^2,\ \sigma_a^2 = A^2/4,\ P(f) = (\tau\ \text{sinc}\,2f\tau)/[1-(2f\tau)^2]$$

(a) $\tau = 1/2r_b,\ P(0) = 1/2r_b,\ P(\pm r_b) = 1/4r_b,\ P(nr_b) = 0,\ |n| \geq 2$

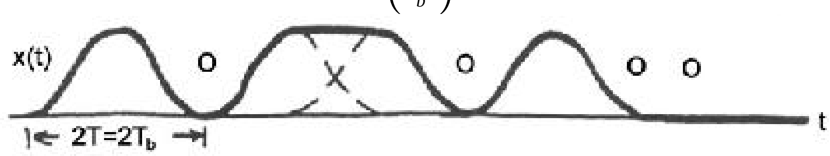
$$G_x(f) = \frac{A^2}{4}r_b^2|P(f)|^2 + \frac{A^2}{4}r_b^2\left(\frac{1}{2r_b^2}\right)^2\delta(f) + \frac{A^2}{4}r_b^2\left(\frac{1}{4r_b^2}\right)^2[\delta(f-r_b)+\delta(f+r_b)]$$



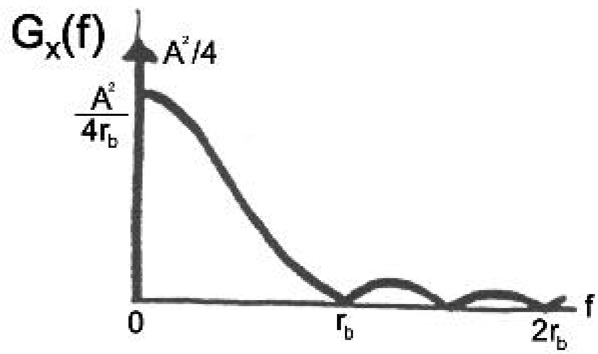


$\tau = 1/r_b,\ P(0) = 1/r_b,\ P(nr_b) = 0,\ n \neq 0$

$$G_x(f) = \frac{A^2}{4}r_b|P(f)|^2 + \frac{A^2}{4}r_b^2\left(\frac{1}{r_b}\right)^2\delta(f)$$



11.1-13 continued

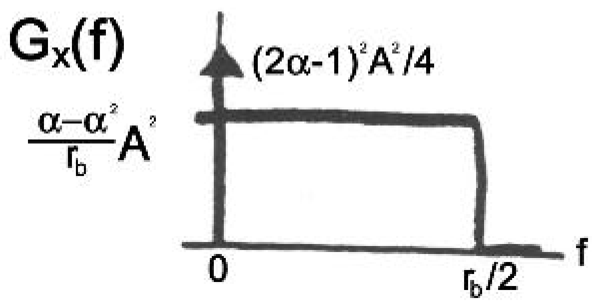


Note larger dc component and smoother waveform.

11.1-14

$$\left.\begin{aligned} m_a &= \overline{a_k} = \alpha\frac{A}{2} + (1-\alpha)(-A/2) = (2\alpha-1)A/2 \\ \overline{a_k^2} &= \alpha(A/2)^2 + (1-\alpha)(-A/2)^2 = A^4/4 \end{aligned}\right\} \sigma_a^2 = [1-(2\alpha-1)^2]A^2/4 = (\alpha-\alpha^2)A^2$$

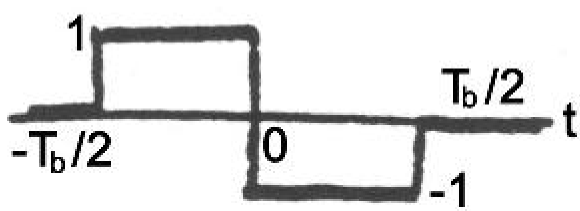
$$P(f) = \frac{1}{r_b}\Pi(f/r_b), \; G_x(f) = \frac{(\alpha-\alpha^2)}{r_b}\Pi(f/r_b) + \left(\frac{2\alpha-1}{4}\right)^2 \delta(f)$$



$$\overline{x^2} = \int_{-\infty}^{\infty} G_x(f)df = \frac{(\alpha-\alpha^2)}{r_b}A^2 \text{ x } 2\, r_b/2 \; + (2\alpha-1)^2 A^2/4 = A^2/4$$

11.1-15

$$p(t) = \Pi\left(\frac{t+T_b/4}{T_b/2}\right) - \Pi\left(\frac{t-T_b/4}{T_b/2}\right)$$

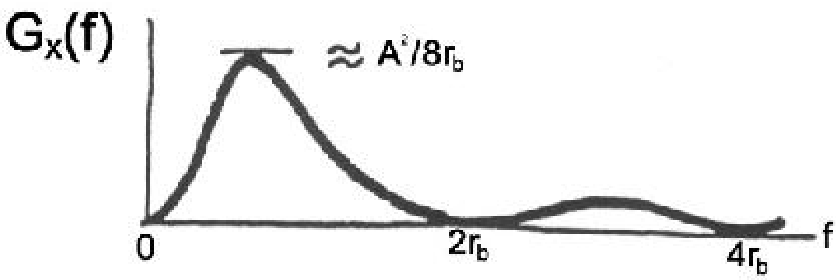


11.1-15 continued

$$P(f)=\frac{T_b}{2}\operatorname{sinc}\left(\frac{fT_b}{2}\right)(e^{j2\pi T_b/4}-e^{-j2\pi T_b/4})=jT_b\operatorname{sinc}\frac{f}{2r_b}\sin\frac{\pi f}{2r_b}$$

$$a_k=\pm A/2\Rightarrow m_a=0,\ \sigma_a^2=\overline{a_k^2}=A^2/4$$

$$G_x(f)=\frac{A^2}{4r_b}\operatorname{sinc}^2\frac{f}{2r_b}\sin^2\frac{\pi f}{2r_b}$$



11.1-16

$$R_a(n)=E[a_{k-n}a_k]$$

n=0

a_k	$P(a_k)$
0	1/2
± A	1/4

$$\Rightarrow R_a(0)=\frac{1}{2}\text{ x }0+2\text{ x }\frac{1}{4}A^2=A^2/2$$

11.1-16 continued

$n = 1$

a_{k-1}	a_k	$P(a_{k-1}a_k)$
0	0	1/2 x 1/2
0	A	1/2 x 1/4
0	-A	1/2 x 1/4
A	0	1/4 x 1/2
-A	0	1/4 x 1/2
A	- A	1/2 x $(1/2)^2$ } $= \frac{1}{2}P(1,1)$
-A	A	1/2 x $(1/2)^2$ }
A	A	0 } not allowed
-A	- A	0 }

$$R_a(\pm 1) = \frac{1}{4} \text{ x } 0 + 4 \text{ x } \frac{1}{8} \text{ x } 0 + 2 \text{ x } \frac{1}{8}(A)(-A) = -A^2/4$$

$n \geq 2$

a_{k-n}	a_k	$P(a_{k-n}a_k)$
0	0	1/2 x 1/2
0	A	1/2 x 1/4
0	-A	1/2 x 1/4
A	0	1/4 x 1/2
-A	0	1/4 x 1/2
A	- A	$(1/4)^2$ }
-A	A	$(1/4)^2$ } $= P(1)P(1)$
A	A	$(1/4)^2$ }
$-A$	-A	$(1/4)^2$ }

$$R_a(n) = \frac{1}{4} \text{ x } 0 + 4 \text{ x } \frac{1}{8} \text{ x } 0 + 2 \text{ x } \frac{1}{16}(A)(-A) + 2 \text{ x } \frac{1}{16}(\pm A)^2 = 0 \qquad |n| \geq 2$$

11.1-17

$$\sum_{k=-K}^{K}\sum_{i=-K}^{K} g(k-i) = \sum_{k=-K}^{K} [g(k-K)+g(k-K-1)+\ldots+g(k+K)]$$

$$\left.\begin{aligned}&[g(-2K)+g(-2K+1)+\ldots+g(0)]\\ &\qquad +[g(-2K+1)+\ldots g(0)+g(1)]\\ &\qquad\qquad +g(0)+g(1)+\ldots g(2K)]\end{aligned}\right\} 2K+1 \text{ sums}$$

$$= g(-2K)+2g(-2K+1)+\ldots+(2K+1)g(0)+2Kg(1)+\ldots 2g(2K-1)+g(2K)$$

$$= \sum_{n=-2K}^{0}(2K+1+n)g(n)+\sum_{n=1}^{2K}(2K+1-n)g(n)$$

$$= \sum_{n=-2K}^{2K}(2K+1-|n|)g(n) = (2K+1)\sum_{n=-2K}^{2K}\left(1-\frac{|n|}{2K+1}\right)g(n)$$

$$E[a_k a_i] = E[a_k a_{k-(k-i)}] = R_a(k-i)$$

$$\text{thus, } \rho_k(f) = \sum_{k=-K}^{K}\sum_{i=-K}^{K} g(k-i) \text{ where } g(k-i) = R_a(k-i)e^{-j\omega(k-i)D}$$

$$= (2K+1)\sum_{n=-2K}^{2K}\left(1-\frac{|n|}{2K+1}\right)R_a(n)e^{-j\omega nD}$$

11.1-18

(a) $\overline{a_k} = \frac{1}{2} \text{ x } 1 + \frac{1}{2} \text{ x } 0 = \frac{1}{2},\ \overline{a_k^2} = \frac{1}{2} \text{ x } 1^2 + \frac{1}{2} \text{ x } 0^2 = \frac{1}{2}$

$\overline{b_k} = 1-\overline{a_k} = \frac{1}{2},\ \overline{b_k^2} = 1-2\,\overline{a_k}+\overline{a_k^2} = \frac{1}{2},\ \overline{a_k b_k} = \overline{a_k}-\overline{a_k^2} = 0$

for $i \neq k$, $\overline{a_k a_i} = \overline{a_k}\ \overline{a_i} = \frac{1}{4},\ \overline{b_k b_i} = \overline{b_k}\ \overline{b_i} = \frac{1}{4},\ \overline{a_k b_i} = \overline{a_k}\ \overline{b_i} = \frac{1}{4}$

(b) $x_T(t) = \sum_{k=-K}^{K}[a_k \rho_1(t-kT_b)+b_k\rho_0(t-kT_b)]$

$$X_T(f) = \sum_{k=-K}^{K}[a_k P_1(f)+b_k P_0(f)]e^{-j\omega kT_b}$$

$$|X_T(f)|^2 = \sum_k\sum_i [a_k P_1 + b_k P_0]e^{-j\omega kT_b}[a_i P_1^* + b_i P_0^*]e^{+j\omega iT_b}$$

$$= \sum_k\sum_i [a_k a_i P_1 P_1^* + a_k b_i P_1 P_0^* + a_i b_k P_1^* P_0 + b_k b_i P_0 P_0^*]e^{+j\omega(k-i)T_b}$$

11.1-18 (b) continued

let $A = P_1P_0^* + P_0P_0^*$ and $B = P_1P_1^* + P_1P_0^* + P_1^*P_0 + P_0P_0^* = |P_1 + P_0|^2$, so

$$E\left[|X_T(f)|^2\right] = \sum_{k=-K}^{K}\left\{\frac{1}{2}A + \sum_{\substack{i=-K\\ i\neq k}}^{K}\frac{1}{4}Be^{-j\omega(k-i)T_b}\right\} = \sum_{k=-K}^{K}\left\{\frac{1}{2}A - \frac{1}{4}B + \sum_{i=-K}^{K}\frac{1}{4}Be^{-j\omega(k-i)T_b}\right\}$$

where $\frac{1}{2}A - \frac{1}{4}B = \frac{1}{4}[P_1P_1^* + P_1P_0^* + P_1^*P_0 + P_0P_0^* = \frac{1}{4}|P_1 - P_0|^2$

thus, $E[|X_T(f)|^2] = \frac{2K+1}{4}|P_1 - P_0|^2 + \frac{1}{4}|P_1 + P_0|^2 \sum_{k=-K}^{K}\sum_{i=-k}^{K} e^{-j\omega(k-i)T_b}$

Hence, $G_x(f) = \frac{r_b}{4}|P_1(f) - P_0(f)|^2 + \frac{r_b^2}{4}\sum_{n=-\infty}^{\infty}|P_1(nr_b) + P_0(nr_b)|^2\,\delta(f - nr_b)$

(c) $\sum_{k=-K}^{K}\sum_{i=-K}^{K} e^{-j\omega(k-i)T_B} = (2K+1)\sum_{n=-2K}^{2K}\left(1 - \frac{|n|}{2K+1}\right)e^{-j\omega nT_b}$

so $E[|X_T(f)|^2] = \frac{2K+1}{4}\left[|P_1 - P_0|^2 + |P_1 + P_0|^2 \sum_{n=-2K}^{2K}\left(1 - \frac{|n|}{2K+1}\right)e^{-j\omega nT_b}\right]$

$$G_x(f) = \lim_{K\to\infty}\frac{1}{(2K+1)T_b}E[|X_T(f)|^2$$

$$= \frac{1}{4T_b}|P_1 - P_0|^2 + \frac{1}{4T_b}|P_1 + P_0|^2\left\{\lim_{K\to\infty}\left[\sum_{n=-2K}^{2K} e^{-j\omega nT_b} - \frac{1}{2K+1}\sum_{n=-2K}^{2K}|n|e^{-j\omega nT_b}\right]\right\}$$

$$= \frac{1}{4T_b}|P_1 - P_0|^2 + \frac{1}{4T_b}|P_1 + P_0|^2\sum_{n=-\infty}^{\infty} e^{-j\omega nT_b}$$

where $\frac{1}{T_b} = r_b$ and $\sum_{n=-\infty}^{\infty} e^{-j\omega nT_b} = \frac{1}{T_b}\sum_{n=-\infty}^{\infty}\delta\left(f - \frac{n}{T_b}\right)$

Hence, $G_x(f) = \frac{r_b}{4}|P_1(f) - P_0(f)|^2 + \frac{r_b^2}{4}\sum_{n=-\infty}^{\infty}|P_1(nr_b) + P_0(nr_b)|^2\,\delta(f - nr_b)$

11.2-1

$P_e = Q\left(\sqrt{\frac{1}{2}(S/N)_R}\right) = 10^{-3} \quad\Rightarrow\quad (S/N)_R \approx 2 \text{ x } 3.1^2 = 19.2$

Polar: $P_e = Q\left(\sqrt{(S/N)_R}\right) = Q(4.38) \approx 6 \text{ x } 10^{-6}$

11.2-2

$P_e = Q(A/2\sigma) \le 10^{-6} \quad \Rightarrow \quad A/2\sigma \ \ge 4.76$

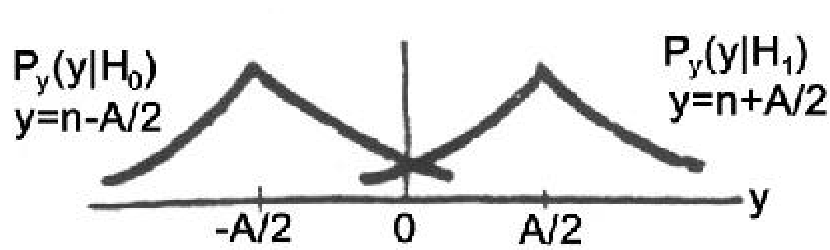
Polar: $4.76^2 \le \ (A/2\sigma)^2 \ \le \ S_R/(N_0 r_b/2) \quad \Rightarrow \quad S_R \ \ge 4.76^2 \text{ x } \frac{1}{2} \text{ x } 10^{-8} = 0.113\ \mu\text{W}$

Unipolar: $(A/2\sigma)^2 \le \ \frac{1}{2} S_R/(N_0 r_b/2) \quad \Rightarrow \quad S_R \ge 0.226\ \mu\text{W}$

11.2-3

(a) From symmetry

$$V_{opt} = 0 \quad \Rightarrow \quad P_e = P_{e0} = \int_0^\infty p_n(y + A/2)\, dy = \frac{1}{\sqrt{2s^2}} \int_0^\infty e^{-\sqrt{2}(y+A/2)} dy = \frac{1}{2} e^{-A\sqrt{2s^2}}$$



(b) $P_e \le 10^{-3} \quad \Rightarrow \quad A/\sqrt{2s^2} \ \ \ge \ \ \ln \frac{1}{2 \text{ x } 10^{-3}} \quad \Rightarrow \quad A \ \ge \ 8.8s$

Gaussian noise: $P_e = Q(A/2s) \ \ \le \ \ 10^{-3} \ \Rightarrow \ A \ \ge \ 6.2s$

Note that impulse noise requires more signal power for same P_e

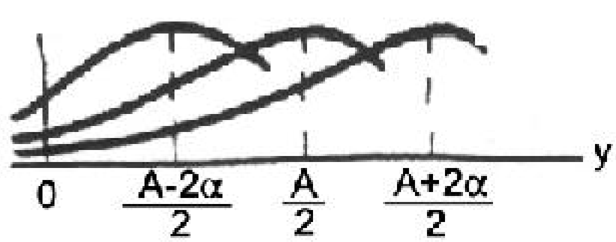
11.2-4

(a) From symmetry, $P_e = P_{e1}$

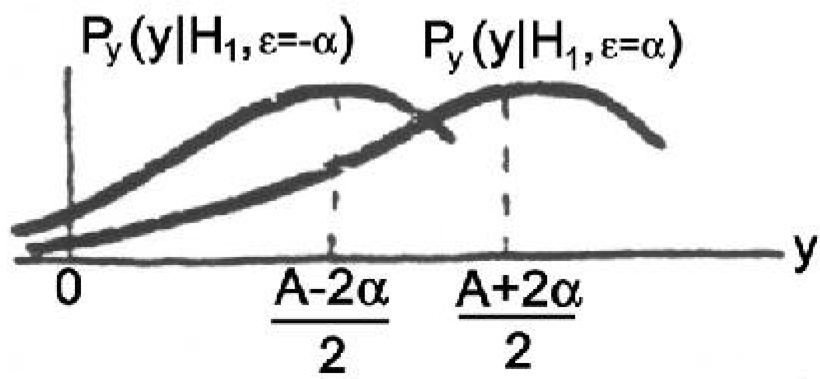
$= P(y < 0 \mid e = -\infty) + P(e = -\infty) + P(y < 0 \mid e = \infty) + P(e = \infty)$

$= \frac{1}{2} Q\left(\frac{A - 2a}{2s}\right) + \frac{1}{2} Q\left(\frac{A + 2a}{2s}\right)$

$p_Y(y \mid H_1, e = -a) \quad p_Y(y \mid H_1, e = 0) \quad p_Y(y \mid H_1, e = a)$



11.2-4 continued



(b) $(A \pm 2a)/2s = (1 \pm 0.2)4$,

$$P_e = \frac{1}{2}Q(3.2) + \frac{1}{2}Q(4.8)$$

$$= \frac{1}{2}(7.4 \text{ x } 10^{-4} + 8.5 \text{ x } 10^{-7}) \approx 3.7 \text{ x } 10^{-4}$$

If $e = 0$, then $P_e = Q(4.0) = 3.4 \text{ x } 10^{-5}$

11.2-5

(a) From symmetry, $P_e = P_{e1}$

$$= P(y < 0 \mid e = -\infty)P(e = -\infty) + P(y < 0 \mid e = 0)P(e = 0) + P(y < 0 \mid e = a)P(e = a)$$

$$= \frac{1}{4}Q\left(\frac{A - 2a}{2s}\right) + \frac{1}{2}Q\left(\frac{A}{2s}\right) + \frac{1}{4}Q\left(\frac{A + 2a}{2s}\right)$$

$p_Y(y \mid H_1, e = -a)$ $p_Y(y \mid H_1, e = 0)$ $p_Y(y \mid H_1, e = a)$

(b) $(A \pm 2a)/2s = (1 \pm 0.2)4$,

$$P_e = \frac{1}{4}Q(3.2) + \frac{1}{2}Q(4) + \frac{1}{2}Q(4.8)$$

$$= \frac{1}{4}(7.4 \text{ x } 10^{-4} + \frac{1}{2} \text{x } 3.4 \text{ x } 10^{-5} + \frac{1}{4} \text{ x } 8.5 \text{ x } 10^{-7}) \approx 2 \text{ x } 10^{-4}$$

If $e = 0$, then $P_e = Q(4.0) = 3.4 \text{ x } 10^{-5}$

11.2-6

$p_y(y|H_0)=p_n(y+A/2)$, $p_y(y|H_1)=p_n(y-A/2)$, $p_n(\text{n})=\frac{1}{\sqrt{2\pi\sigma^2}}\text{e}^{-\text{n}^2/2\sigma^2}$

so $P_0\frac{1}{\sqrt{2\pi\sigma^2}}\text{e}^{-(V+A/2)^2/2\sigma^2}=P_1\frac{1}{\sqrt{2\pi\sigma^2}}\text{e}^{-(V-A/2)^2/2\sigma^2}$

$P_0/P_1=\text{e}^{[(V+A/2)^2-(V-A/2)]^2/2\sigma^2}=e^{VA/\sigma^2}$

Hence, $VA/\sigma^2=\ln(P_0/P_1)\ \Rightarrow\ V_{opt}=\frac{\sigma^2}{A}\ln(P_0/P_1)$

11.2-7

$\frac{dP_e}{dV}=P_0\frac{dP_{e0}}{dV}+P_1\frac{dP_{e1}}{dV}=0$ when $V=V_{opt}$

$\frac{dP_{e0}}{dV}=\frac{d}{dV}\int_{a(V)}^{b(V)}g(v,y)dy$ where $a(V)=V$, $b(V)=\infty$, $g(V,y)=p_y(y|H_0)$

$=0-p_y(V|H_0)+0$ since $\frac{d\,b(V)}{dV}=0$, $\frac{\partial}{\partial\text{V}}[p_y(y|H_0)]=0$

similarly, $\frac{dP_{e1}}{dV}=0+p_y(V|H_1)+0$

hence, $-P_0p_y(V|H_0)+P_1p_y(V|H_1)=0\ \Rightarrow\ P_0p_y(V|H_0)=P_1p_y(V|H_1)$

11.2-8

$(S/N)_1=20\text{ dB}=100$

Regenerative: $P_e\approx 20Q\left[\sqrt{100}\right]\approx 20\frac{1}{\sqrt{2\pi 100}}e^{-100/2}\approx\ 1.5\text{ x }10^{-22}$

Nonregenerative: $P_e\approx Q\left[\sqrt{\frac{1}{20}\text{ x }100}\right]=Q(2.24)\approx\ 1.2\text{ x }10^{-2}$

11.2-9

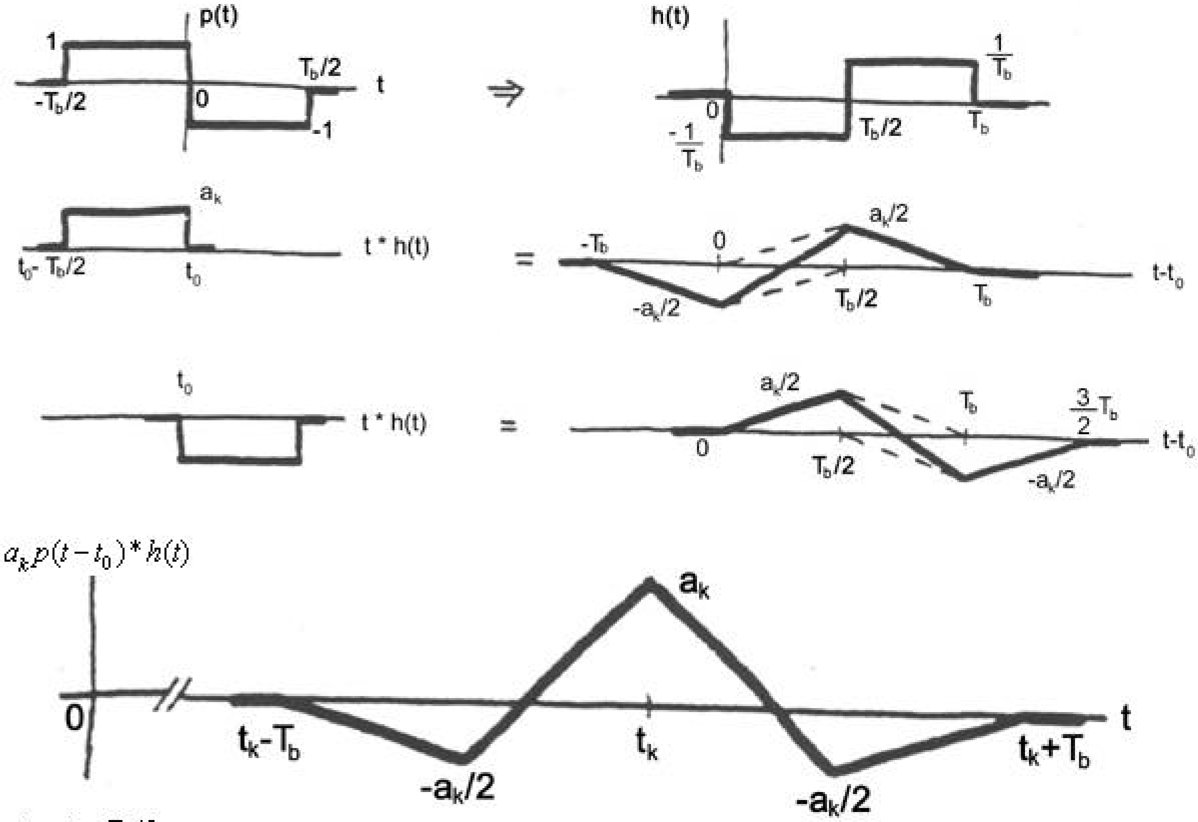
Regenerative: $P_e=50Q\left[\sqrt{(S/N)_1}\right]=10^{-4}\ \Rightarrow\ \sqrt{(S/N)_1}\approx 4.62$

so $(S/N)_1=21.3=13.3\text{ dB}$

Nonregenerative: $P_e=Q\left[\sqrt{\frac{1}{50}(S/N)_1}\right]=10^{-4}\ \Rightarrow\ \sqrt{\frac{1}{50}(S/N)_1}\approx 3.73$

so $(S/N)_1=50\text{ x }3.73^2=696=28.4\text{ dB}$

11.2-10

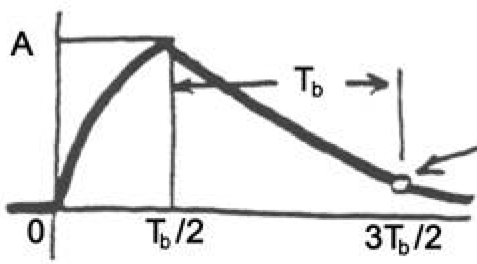


$t_k = t_0 + T_b/2$

11.2-11

(a) $0 \le t \le T_b/2: Ap(t)*h(t) = AK_0 \int_0^t e^{-b\lambda} d\lambda = \frac{AK_0}{b}(1-e^{-bt}) = \frac{A}{(1-e^{-bT_b/2})}(1-e^{-bt})$

$t \ge T_b/2: \quad Ap(t)*h(t) = AK_0 \int_{t-T_b/2}^{t} e^{-b\lambda} d\lambda = \frac{AK_0}{b}[e^{-b(t-T_b/2)} - e^{-bt}] = Ae^{-b(t-T_b/2)}$



$Ae^{-bT_b} \le 0.1A \Rightarrow b \ge \frac{\ln(10)}{T_b} = 2.3 r_b$

11.2-11 continued

(b) $\sigma^2 = \frac{N_0}{2}\int_{-\infty}^{\infty}|h(t)|^2\, dt = \frac{K_0^2 N_0}{2}\int_0^{\infty} e^{-2bt} dt = \frac{K_0^2 N_0}{4b}$

$E_b = \overline{a_k^2}\int_{-\infty}^{\infty} p(t)^2\, dt = \frac{A^2}{2}\;\frac{T_b}{2} = \frac{A^2}{4r_b}$

thus, $\left(\frac{A}{2\sigma}\right)^2 = \frac{4E_b r_b}{4(K_0^2 N_0 / 4b)} = \frac{4br_b}{K_0^2}\gamma_b = \frac{4r_b}{b}(1 - e^{-bT_b/2})^2\gamma_b \le 0.812\gamma_b$ if $b \ge 2.3r_b$

11.2-12

(a) $Q(\sqrt{2\gamma_b}) = 10^{-4} \;\Rightarrow\; \gamma_b = \frac{1}{2}$ x 3.7^2, $S_R \ge N_0 r_b \gamma_b = 34.2$ pW

(b) $2\;\frac{7}{8 \text{ x } 3} Q\left(\sqrt{\frac{6 \text{ x } 3}{63}\gamma_b}\right) = 10^{-4} \;\Rightarrow\; Q(\sqrt{0.286\gamma_b}) = 1.7 \text{ x } 10^{-4}$

$\gamma_b = \frac{1}{0.286}$ x 3.6^2, $S_R \ge N_0 r_b \gamma_b = 227$ pW

11.2-13

$r = \frac{500 \text{ x } 10^3}{\log_2 M} \le 2B = 160 \text{ x } 10^3 \;\Rightarrow\; \log_2 M \ge \frac{50}{16} = 3.125$ so $M_{\min} = 2^4 = 16$

$2\;\frac{15}{16 \text{ x } 4} Q\left(\sqrt{\frac{6 \text{ x } 4}{255}\gamma_b}\right) = 10^{-4} \;\Rightarrow\; Q(\sqrt{0.0941\gamma_b}) = 2.13 \text{ x } 10^{-4}$

so $\gamma_b = \frac{1}{0.0941}$ x 3.55^2, and $S_R \ge N_0 r_b \gamma_b = 670$ pW

11.2-14

With matched filtering; $P_{be} \approx 2\frac{M-1}{M\log_2 M} Q\left(\sqrt{\frac{600\log_2 M}{M^2-1}}\right)$

M	$2\frac{M-1}{M\log_2 M}$	$\sqrt{\frac{600\log_2 M}{M^2-1}}$	Q	P_{be}
4	0.75	8.94	$<10^{-12}$	$<10^{-12}$
8	0.583	5.35	4.5×10^{-8}	$2.6 \times 10^{-8} \Rightarrow$ take $M = 8$
16	0.469	3.07	$>10^{-3}$	$>10^{-4}$

11.2-15

$$\overline{a_k^2} = \frac{1}{M}\left\{\left[-\frac{(M-1)A}{2}\right]^2 + \ldots + \left(\frac{-3A}{2}\right)^2 + \left(-\frac{A}{2}\right)^2 + \left(\frac{3A}{2}\right)^2 + \ldots + \left[\frac{(M-1)A}{2}\right]^2\right\}$$

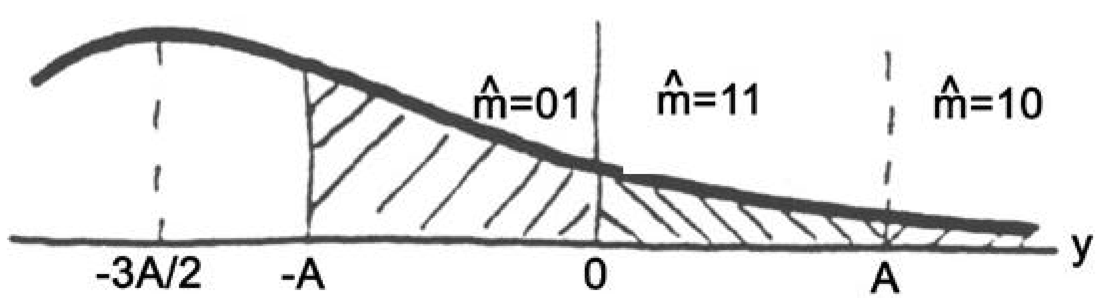
$$= 2 \text{ x } \frac{1}{M}\left[1^2 + 3^2 + \ldots + (M-1)^2\right]\left(\frac{A}{2}\right)^2$$

$$= \frac{\mathrm{A}^2}{2M}\sum_{i=1}^{M/2}(2i-1)^2 = \frac{\mathrm{A}^2}{2M}\sum_{i=1}^{M/2}(4i^2 - 4i + 1)$$

$$= \frac{\mathrm{A}^2}{2M}\left[4\frac{(M/2)(1+M/2)(M+1)}{6} - 4\frac{(M/2)(1+M/2)}{2} + \frac{M}{2}\right] = (M^2-1)\frac{A^2}{12}$$

11.2-16

Let $\tilde{m}$ = regenerated m and consider m = 00



$P(\tilde{m} = 01) = Q(A/2\sigma) - Q(3A/2\sigma)$ 1 bit error

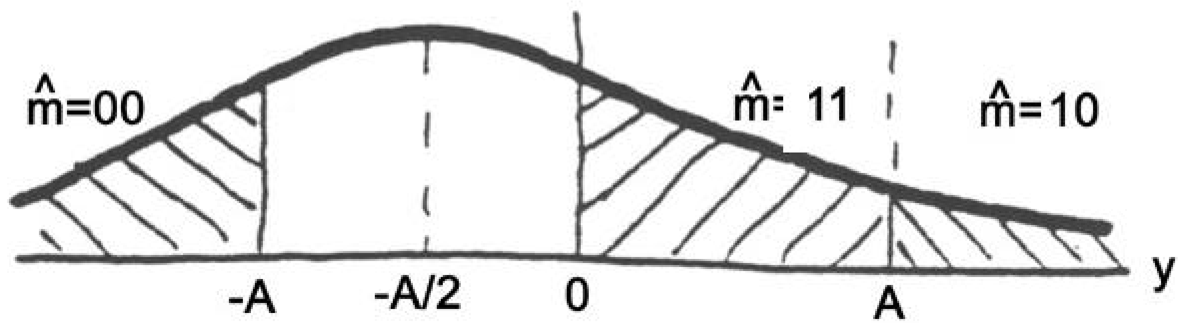
$P(\tilde{m} = 11) = Q(3A/2\sigma) - Q(5A/2\sigma)$ 2 bit errors

$P(\tilde{m} = 10) = Q(5A/2\sigma)$ 1 bit error

Similarly when m = 10.

11.2-16 continued

Now consider m = 01, and similarly m = 11.



$P(\tilde{m}=00)=Q(A/2\sigma)$ 1 bit error

$P(\tilde{m}=11)=Q(A/2\sigma)-Q(3A/2\sigma)$ 1 bit error

$P(\tilde{m}=10)=Q(3A/2\sigma)$ 2 bit errors

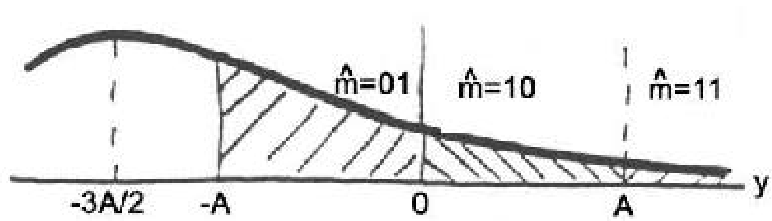
Thus, $P_{be} = 2 \text{ x } \frac{1}{4}\{[Q(k)-Q(3k)]+2[Q(3k)-Q(5k)]+Q(5k)\}$

$+ 2 \text{ x } \frac{1}{4}\{Q(k)+[Q(k)-Q(3k)]+2Q(3k)\}$

$= \frac{3}{2}Q(k)+Q(3k)-\frac{1}{2}Q(5k) \approx \frac{3}{2}Q(k)$ when $k>1$ since $Q(5k) << Q(3k) << Q(k)$

11.2-17

Let $\tilde{m}$ = regenerated m and consider m = 00 (similarly for m = 11)



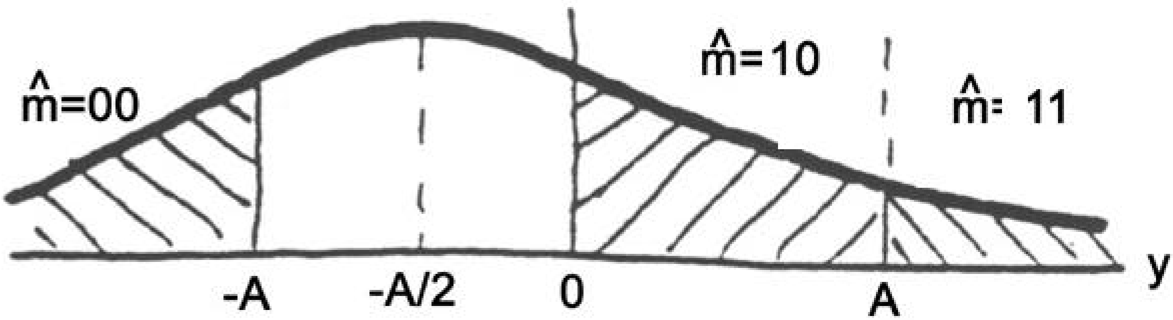
$P(\tilde{m}=01)=Q(A/2\sigma)-Q(3A/2\sigma)$ 1 bit error

$P(\tilde{m}=10)=Q(3A/2\sigma)-Q(5A/2\sigma)$ 1 bit error

$P(\tilde{m}=11)=Q(5A/2\sigma)$ 2 bit errors

11.2-17 continued

Now consider m = 01 (similarly m = 10)



$P(\tilde{m}=00)=Q(A/2\sigma)$ 1 bit error
$P(\tilde{m}=10)=Q(A/2\sigma)-Q(3A/2\sigma)$ 2 bit errors
$P(\tilde{m}=11)=Q(3A/2\sigma)$ 1 bit error

Thus, $P_{be}=\ 2 \text{ x } \frac{1}{4}\{[Q(k)-Q(3k)]+2[Q(3k)-Q(5k)]+2Q(5k)\}$

$$+\ 2 \text{ x } \frac{1}{4}\{Q(k)+2[Q(k)-Q(3k)]+2Q(3k)\}$$

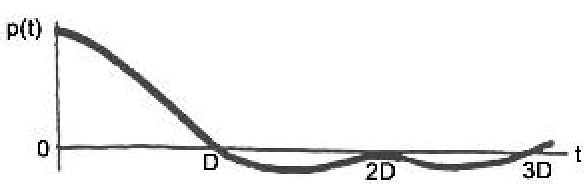
$$=\ 2Q(k)-\frac{1}{2}Q(3k)+\frac{1}{2}Q(5k)\ \approx\ 2Q(k) \quad \text{when } k>1 \text{ since } Q(5k)<<Q(3k)<<Q(k)$$

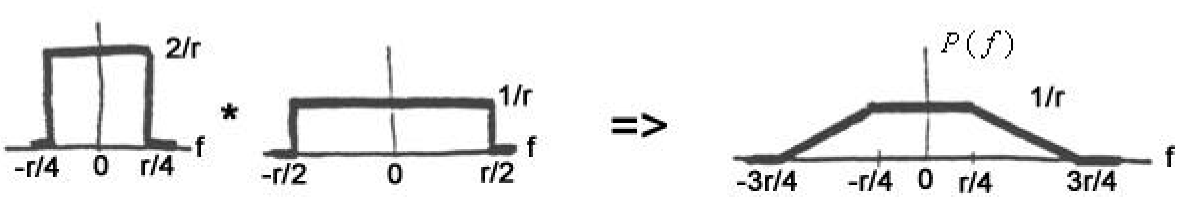
11.3-1

$p_\beta(t)=\ \text{sinc}\, rt\ /2$

$p(t)=\ \text{sinc}\ \frac{rt}{2}\ \text{sinc}\ rt\,,\ \frac{1}{r}\ =\ D$

No additional zero crossings.



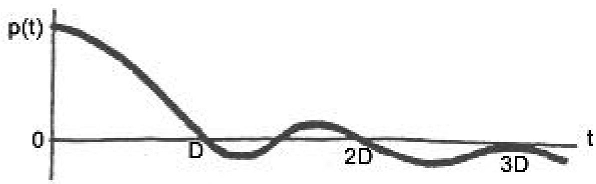


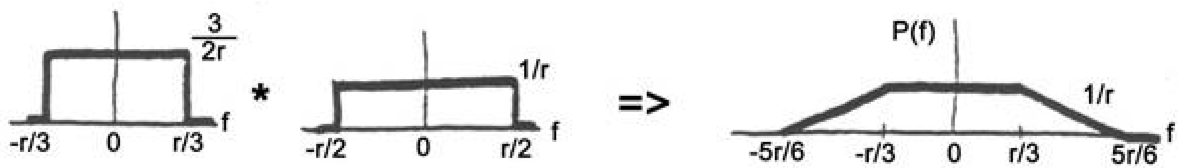
11.3-2

$p_\beta(t) = \ \operatorname{sinc}(2rt/3)$

$p(t) = \ \operatorname{sinc}\dfrac{2rt}{3}\ \operatorname{sinc} rt,\ \dfrac{1}{r}\ = \ D$

Additional zero crossings at $t = \pm 3D/2,\quad \pm 9D/2,\quad \pm 15D/2, \ldots$





11.3-3

Given B = 3 kHz

(a) $B = \dfrac{r}{2} + \beta,$ 100% $\Rightarrow \beta = r/2 \Rightarrow B = \dfrac{r}{2} + \dfrac{r}{2} = r \Rightarrow r = 3$ kbps.

(b) 50% $\Rightarrow \beta = \dfrac{r}{4} \Rightarrow B = \dfrac{r}{2} + \dfrac{r}{4} = \dfrac{3}{4} r \Rightarrow r = 4$ kpbs.

(c) 25% $\Rightarrow \beta = \dfrac{r}{8} \Rightarrow B = \dfrac{r}{2} + \dfrac{r}{8} = \dfrac{5}{8} r \Rightarrow r = 4.8$ kpbs

11.3-4

Figure P11.3-4 are baseband waveforms for 10110100 using Nyquist pulses with β=r/2 (dotted plot), β=r/4 (solid plot). Note that the plot with β=r/2 is the same as the plot of Figure 11.3-2.

In comparing the two waveforms, the signal with β=r/4 exhibits higher intersymbol interference (ISI) than the signal with β=r/4.

11.3-4 continued

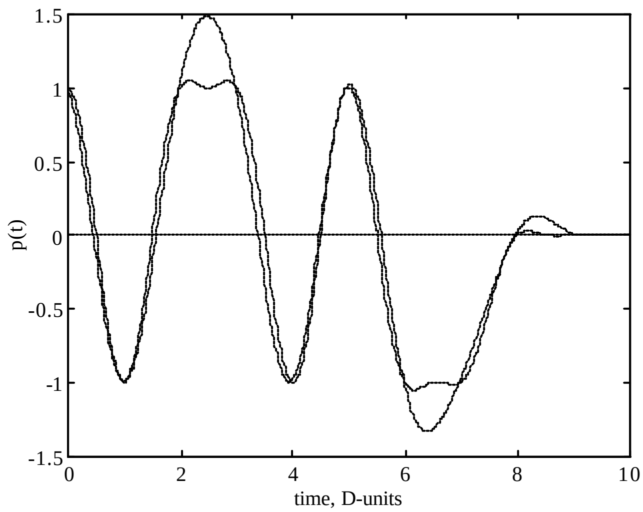


Figure P11.3-5

11.3-5

(a) Given $B = r/2 + \beta$ where β represents excess bandwidth.
With a data rate of r = 56 kpbs $\Rightarrow$ theoretical bandwidth of $B = r/2 =$ 28 kHz.
But with 100 rolloff $\Rightarrow$100% excess bandwidth $\Rightarrow$ $\beta = r/2 \Rightarrow B = r/2 + r/2 = r =$ 56 kHz.

(b) 50% excessive bandwidth $\Rightarrow \beta = r/4 \Rightarrow B = r/2 + r/4 = 3/4r \Rightarrow B = 42$ kHz.

(c) 25% excessive bandwidth $\Rightarrow \beta = r/8 \Rightarrow B = r/2 + r/8 = 5/8r \Rightarrow B = 35$ kHz.

11.3-6

$$p(t) = \frac{\cos \pi rt}{1-(2rt)^2} \quad \frac{\sin \pi rt}{\pi rt} = \frac{\frac{1}{2}\sin 2\pi rt}{[1-(2rt)^2]\pi rt} = \frac{\text{sinc } 2rt}{1\text{-}(2rt)^2}$$

At $t = \pm D/2 = \pm 1/2r$, sinc $2rt = 0$, and $1-(2rt)^2 = 0$

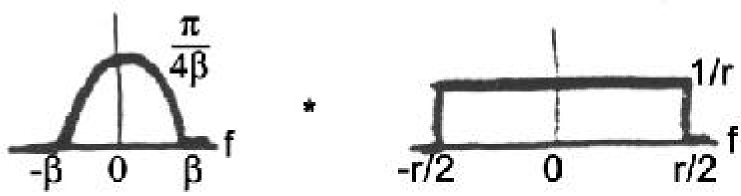
So use L'Hopital's rule with $p(t) = \dfrac{\sin 2\pi rt}{2\pi rt[1-(2rt)^2]}$

$$\frac{d}{dt}\sin 2\pi t = 2\pi r\cos 2\pi rt, \quad \frac{d}{dt}\left\{2\pi rt[1-(2rt)^2]\right\} = 2\pi r - 24\pi r^3 t^2$$

thus $p\left(\pm\dfrac{1}{2r}\right) = \dfrac{2\pi\cos(\pm\pi)}{2\pi r - 6\pi r} = \dfrac{1}{2}$

11.3-7

$$P_\beta(f)=\frac{\pi}{4\beta}\cos\frac{\pi f}{2\beta}\Pi\left(\frac{f}{2\beta}\right),\qquad P(f)=$$



$$|f|<\frac{r}{2}-\beta,\qquad P(f)=\int_{-\beta}^{\beta}\frac{\pi}{4\beta}\cos\frac{\pi\lambda}{2\beta}\frac{1}{r}d\lambda=\frac{1}{2r}\int_{-\pi/2}^{\pi/2}\cos\theta\,d\theta=1/r$$

$$\frac{r}{2}-\beta<f<\frac{r}{2}+\beta,\qquad P(f)=\int_{f-r/2}^{\beta}\frac{\pi}{4\beta}\cos\frac{\pi\lambda}{2\beta}\frac{1}{r}d\lambda=\frac{1}{2r}\int_{\frac{\pi}{2\beta}(f-r/2)}^{\pi/2}\cos\theta\,d\theta$$

$$=\frac{1}{2r}\left[1-\sin\frac{\pi}{2\beta}(f-r/2)\right]=\frac{1}{2r}\left[1+\cos\frac{\pi}{2\beta}(f-r/2+\beta)\right]$$

$$=\frac{1}{r}\cos^2\frac{\pi}{4\beta}(f-r/2+\beta)$$

$$f>\frac{r}{2}+\beta,\qquad P(f)=0$$

Since $P(f)$ has even symmetry,

$$P(f)=\begin{cases}1/r & |f|<r/2+\beta\\ \frac{1}{r}\cos^2\frac{\pi}{4\beta}(|f|-r/2+\beta) & r/2-\beta<|f|<r/2+\beta\\ 0 & |f|>r/2+\beta\end{cases}$$

$$p_\beta(t)=\mathfrak{I}^{-1}\left[P_\beta(f)\right]=\frac{\pi}{4\beta}\int_{-\beta}^{\beta}\cos\frac{\pi f}{2\beta}e^{j\omega t}df=\frac{\pi}{2\beta}\int_{0}^{\beta}\cos\frac{\pi f}{2\beta}\cos 2\pi ft\,df$$

$$=\frac{\pi}{2\beta}\left[\frac{\sin(2\pi rt-\pi/2\beta)\beta}{2(2\pi rt-\pi/2\beta)\beta}+\frac{\sin(2\pi rt+\pi/2\beta)\beta}{2(2\pi rt+\pi/2\beta)\beta}\right]=\frac{1}{2}\left[\frac{-\cos 2\pi\beta t}{4\beta t-1}+\frac{\cos 2\pi\beta t}{4\beta t+1}\right]$$

Thus, $p(t)=p_\beta(t)\operatorname{sinc} rt\;=\;\dfrac{\cos 2\pi\beta t}{1-(4\beta t)^2}\operatorname{sinc} rt$

11.3-8

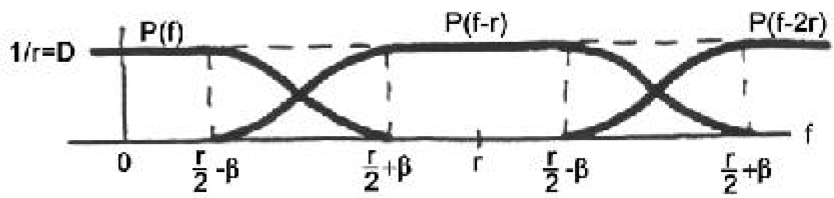
(a) $\Im\left[p(t)\sum_{\mathrm{k}}\delta(t-kD)\right]=P(f)*\left[\sum_k e^{-j2\pi fkD}\right]=P(f)*\left[\frac{1}{D}\sum_n \delta(f-n/D)\right]$

$= r\sum_n P(f-nr)=1$

$\Im\left[\sum_k p(kD)\delta(t-kD)\right]=\sum_k p(kD)\, e^{-j2\pi fkD}$

Thus, $\sum_k p(kD)\, e^{-j2\pi fkD} = 1$ for all f so $p(kD)=\begin{cases}1 & k=0\\ 0 & k\neq 0\end{cases}$

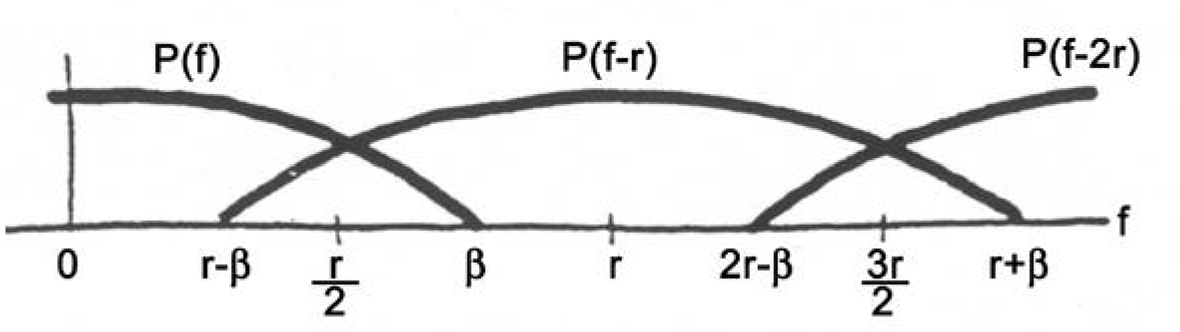
(b) $\frac{1}{r}=D$



Clearly, $\sum_{\mathrm{n}=-\infty}^{\infty} P(f-nr)=D$

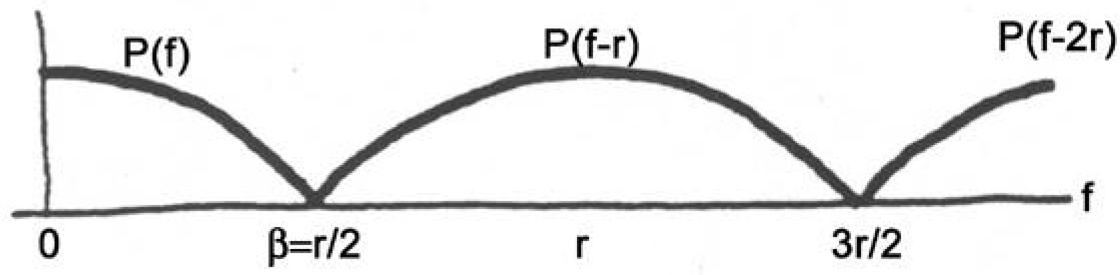
11.3-9

(a) Conditions: $P(f)=1/r \quad |f|<r-B$ and $P(f)+P(f-r)=1/r$
for $r-B<|f|<B$

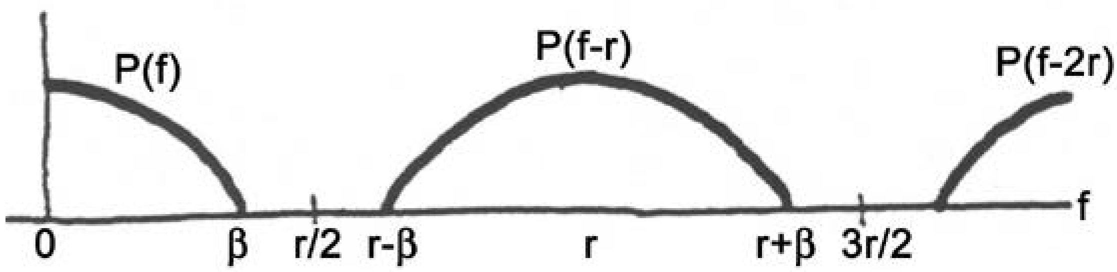


11.3-9 continued

(b) Condition: $P(f) = 1/r \qquad |f| \le B = r/2 \;\; so \;\; P(f) = \frac{1}{r}\Pi\left(\frac{f}{r}\right)$



(c) Can't satisfy the theorem when $B < r/2$.



11.3-10

We want smallest possible M to minimize S_T

$r = \frac{r_b}{\log_2 M} \le 2B \quad \Rightarrow \quad \log_2 M \ge \frac{r_b}{2B} = 1.5 \quad \Rightarrow \quad \text{take } M = 2^2 = 4$

$r = r_b/2 = 300$ kbaud, $\beta = B - r/2 = 50$ kHz

$P_{be} = 2\frac{3}{4 \times 2} Q\left(\frac{A}{2\sigma}\right) \le 10^{-5} \quad \Rightarrow \quad Q\left(\frac{A}{2\sigma}\right) \le \frac{4}{3} \times 10^{-5} \quad \Rightarrow \quad \frac{A}{2\sigma} \ge 4.23$

$\left(\frac{A}{2\sigma}\right)^2 = \frac{6 \times 2}{15} \frac{S_R}{N_0 r_b}$ so $S_T = LS_R \ge \frac{15}{12} 4.23^2 L N_0 r_b = 1.34$ W

11.3-11

We want smallest possible M to minimize S_T

$$r = \frac{r_b}{\log_2 M} \le 2B \quad \Rightarrow \quad \log_2 M \ge \frac{r_b}{2B} = 2.5 \quad \Rightarrow \quad \text{take } M = 2^3 = 8$$

$$r = r_b / 3 = 200 \text{ kbaud}, \ \beta = B - r/2 = 20 \ \text{kHz}$$

$$P_{be} = 2\frac{7}{8 \text{ x } 3} Q\left(\frac{A}{2\sigma}\right) \le 10^{-5} \quad \Rightarrow \quad Q\left(\frac{A}{2\sigma}\right) \le \frac{24}{14} \text{ x } 10^{-5} \quad \Rightarrow \quad \frac{A}{2\sigma} \ge 4.15$$

$$\left(\frac{A}{2\sigma}\right)^2 = \frac{6 \text{ x } 3}{63} \frac{S_R}{N_0 r_b} \quad \text{so } S_T = LS_R \ge \frac{63}{18} 4.15^2 LN_0 r_b = 3.61 \text{ W}$$

11.3-12

We want smallest possible M to minimize S_T

$$r = \frac{r_b}{\log_2 M} \le 2B \quad \Rightarrow \quad \log_2 M \ge \frac{r_b}{2B} = 3.75 \quad \Rightarrow \quad \text{take } M = 2^4 = 16$$

$$r = r_b / 4 = 150 \text{ kbaud}, \ \beta = B - r/2 = 5 \ \text{kHz}$$

$$P_{be} = 2\frac{15}{16 \text{ x } 4} Q\left(\frac{A}{2\sigma}\right) \le 10^{-5} \quad \Rightarrow \quad Q\left(\frac{A}{2\sigma}\right) \le \frac{64}{30} \text{ x } 10^{-5} \quad \Rightarrow \quad \frac{A}{2\sigma} \ge 4.10$$

$$\left(\frac{A}{2\sigma}\right)^2 = \frac{6 \text{ x } 4}{255} \frac{S_R}{N_0 r_b} \quad \text{so } S_T = LS_R \ge \frac{255}{24} 4.10^2 LN_0 r_b = 10.7 \text{ W}$$

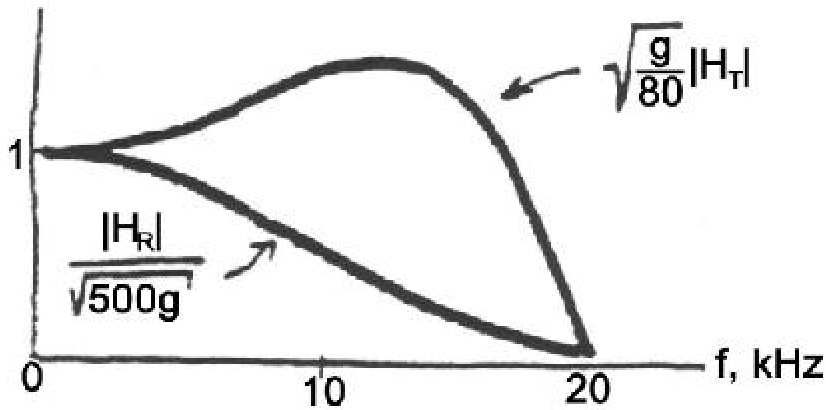
11.3-13

(a) $P(f) = 1/r \ \cos^2(\pi f / 2r)\Pi(f / 2r), \quad P_x(f) = \frac{1}{2r}\text{sinc}(f / 2r), \quad r = 20{,}000$

$$|H_R(f)|^2 = 500g \frac{\cos^2(\pi f / 2r)}{1 + 3 \text{ x } 10^{-4} |f|} \Pi(f / 2r)$$

$$|H_T(f)|^2 = \frac{80}{g} \frac{(1 + 3 \text{ x } 10^{-4} |f|)\cos^2(\pi f / 2r)}{\text{sinc}^2 (f / 2r)} \Pi(f / 2r)$$

11.3-13 continued



(b) $P_e = Q\left(\frac{A}{2\sigma}\right) = 10^{-6} \quad \Rightarrow \quad \frac{A}{2\sigma} = 4.75, \quad \left(\frac{A}{2\sigma}\right)^2_{max} = \frac{3S_T}{(4-1)r} I^{-2}$

where $I = \int_{-\infty}^{\infty} \frac{|P|\sqrt{G_n}}{|H_C|} df = \int_{-r}^{r} \frac{10^{-5}}{0.01r} \cos^2\left(\frac{\pi f}{2r}\right)(1 + 3 \text{ x } 10^{-4}|f|)\, df$

$$= 2\frac{10^{-3}}{r}\int_0^r (1 + 3 \text{ x } 10^{-4} f)\cos^2\left(\frac{\pi f}{2r}\right) df = 10^{-3}\left(4 - \frac{12}{\pi^2}\right) \approx \; 2.78 \text{ x } 10^{-3}$$

Thus, $S_T \geq rI^2\left(\frac{A}{2\sigma}\right)^2 = \; 2 \text{ x } 10^4 \text{ x } (2.78 \text{ x } 10^{-3})^2 \text{ x } 4.75^2 \approx 3.49 \text{ W}$

11.3-14

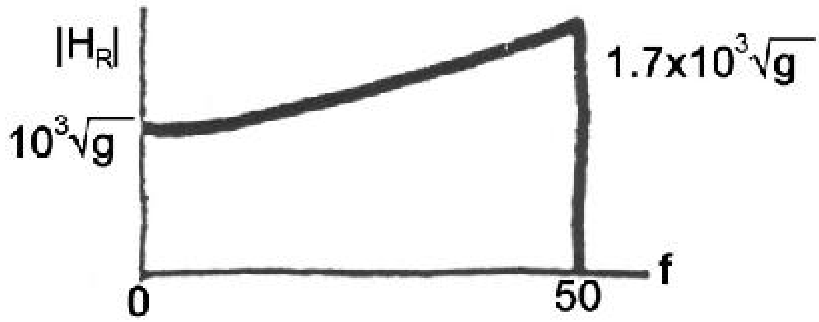
(a) $P(f) = \frac{1}{r}\Pi(f/r), \quad P_x(f) = \frac{1}{10r}\text{sinc}\left(\frac{f}{10r}\right), \quad r = 100$

$$|H_R(f)|^2 = 10^6 g\sqrt{1 + 32 \text{ x } 10^{-4} f^2}\,\Pi\left(\frac{f}{r}\right)$$

$$|H_T(f)|^2 = \frac{100}{g}\frac{\sqrt{1 + 32 \text{ x } 10^{-4} f^2}}{\text{sinc}^2(f/10r)}\Pi\left(\frac{f}{r}\right)$$

$$\approx \frac{1}{10^4 g^2}|H_R(f)|^2 \quad \text{since } \text{sinc}^2(f/10r) \approx 1 \text{ for } |f| \leq r/2$$

11.3-14 continued



(b) $P_e = 2\left(1-\frac{1}{4}\right)Q\left(\frac{A}{2\sigma}\right) = 10^{-6} \quad \Rightarrow \quad \frac{A}{2\sigma} = 4.85, \quad \left(\frac{A}{2\sigma}\right)^2_{\max} = \frac{3S_T}{(16-1)r} I^{-2}$

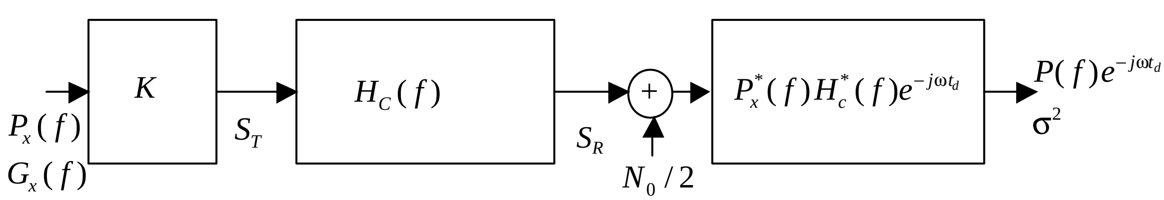
where $I = \int_{-\infty}^{\infty} \frac{|P|\sqrt{G_n}}{|H_C|} df = \int_{-r/2}^{r/2} \frac{10^{-5}}{r} \frac{1}{10^{-3}} \sqrt{1+32 \text{ x } 10^{-4} f^2}\, df$

$= \frac{2 \text{ x } 10^{-2}}{r} \int_0^{r/2} \sqrt{1+32 \text{ x } 10^{-4} f^2}\, df \approx 2 \text{ x } 10^{-4}\left[75 + \frac{100}{2\sqrt{32}} \ln\left(\frac{\sqrt{32}}{2}+3\right)\right] \approx 1.81 \text{ x } 10^{-2}$

Thus, $S_T \geq 5rI^2\left(\frac{A}{2\sigma}\right)^2 = 500(1.81 \text{ x } 10^{-2})^2 (4.85)^2 = 3.85 \text{ W}$

11.3-15

(a)



$P(f)e^{-j\omega t_d} = KH_C P_x^* H_c^* e^{-j\omega t_d} P_x \quad \Rightarrow \quad P(f) = K|P_x H_C|^2$

since $P(f)$ is real, even, and non-negative, $p(t)$ is even an maximum at $t = 0$.

$p(0) = \int_{-\infty}^{\infty} P(f)df = K\int_{-\infty}^{\infty} |P_x H_C|^2 df = 1 \quad \Rightarrow \quad K = \left[\int_{-\infty}^{\infty} |P_x H_C|^2 df\right]^{-1}$

$\sigma^2 = \frac{N_0}{2}\int_{-\infty}^{\infty} |P_x H_C|^2 df = \frac{N_0}{2K}$

$G_x(f) = \sigma_a^2 r|P_x H_C|^2, \quad \sigma_a^2 = (M^2-1)A^2/12$

11.3-15 continued

$$S_T = K^2 \int_{-\infty}^{\infty} G_x(f)df = K^2 \frac{M^2-1}{12} A^2 r \int_{-\infty}^{\infty} |P_x(f)|^2 df$$

Thus, $\left(\frac{A}{2\sigma}\right)^2 = \frac{1}{4} \frac{12 S_T}{(M^2-1)K^2 r \int_{-\infty}^{\infty} |P_x(f)|^2 df} \frac{2K}{N_0} = \frac{3S_T}{(M^2-1)r} I_{HR}^{-1}$

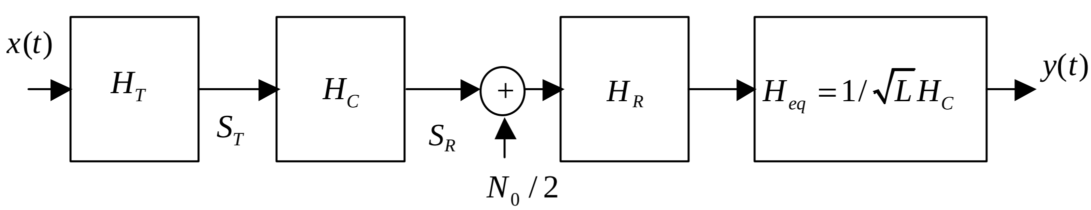
where $I_{HR} = \frac{N_0}{2} K \int_{-\infty}^{\infty} |P_x(f)|^2 df = \frac{N_0 \int_{-\infty}^{\infty} |P_x(f)|^2 df}{2\int_{-\infty}^{\infty} |P_x(f) H_C(f)|^2 df}$

(b) $S_R = \int_{-\infty}^{\infty} |H_C(f)|^2 K^2 G_x(f)df \quad \Rightarrow \quad \frac{S_T}{S_R} = \frac{\int_{-\infty}^{\infty} G_x df}{\int_{-\infty}^{\infty} |H_C(f)|^2 G_x df} = \frac{\int_{-\infty}^{\infty} |P_x|^2 df}{\int_{-\infty}^{\infty} |H_C|^2 |P_x|^2 df}$

Thus, $\left(\frac{A}{2\sigma}\right)^2 = \frac{3}{(M^2-1)r} \frac{\int_{-\infty}^{\infty} |P_x|^2 df}{\int_{-\infty}^{\infty} |H_C|^2 |P_x|^2 df} S_R \frac{2\int_{-\infty}^{\infty} |P_x H_C|^2 df}{N_0 \int_{-\infty}^{\infty} |P_x|^2 df} = \frac{6S_R}{(M^2-1)N_0 r}$

11.3-16

(a)



$|H_T|^2 = \sqrt{\frac{N_0 L}{2}} \frac{|P|}{g|P_x|^2} \qquad |H_R|^2 = \sqrt{\frac{2L}{N_0}} g|P|$

$S_T = \frac{M^2-1}{12} A^2 r \int_{-\infty}^{\infty} |H_T P_x|^2 df = \frac{M^2-1}{12} A^2 r \sqrt{\frac{N_0 L}{2}} \frac{1}{g} \int_{-\infty}^{\infty} |P| df$ where $\int_{-\infty}^{\infty} |P| df = 1$

11.3-16 continued

$$\sigma^2 = \frac{N_0}{2}\int_{-\infty}^{\infty}|H_R H_{eq}|^2\, df = \frac{N_0}{2}\sqrt{\frac{2L}{N_0}}\frac{g}{L}\int_{-\infty}^{\infty}\frac{|P|}{|H_C|^2}df$$

Thus, $$\left(\frac{A}{2\sigma}\right)^2 = \frac{1}{4}\ \frac{12gS_T}{(M^2-1)r}\ \sqrt{\frac{2}{N_0 L}}\ \frac{2}{N_0}\ \sqrt{\frac{N_0}{2L}}\ \frac{L}{g}\left[\int_{-\infty}^{\infty}\frac{|P|}{|H_C|^2}df\right]^{-1} = \frac{6S_T/L}{K(M^2-1)N_0 r}$$

where $K = \frac{1}{L}\int_{-\infty}^{\infty}\frac{|P(f)|}{|H_C(f)|^2}df$

(b) $$K = \frac{1}{L}\int_{-r}^{r}\frac{1}{r}\cos^2\left(\frac{\pi f}{2r}\right)L\left[1+\left(\frac{2f}{r}\right)^2\right]df = \frac{2}{r}\int_{0}^{r}\cos^2\left(\frac{\pi f}{2r}\right)\left[1+\left(\frac{2f}{r}\right)^2\right]df$$

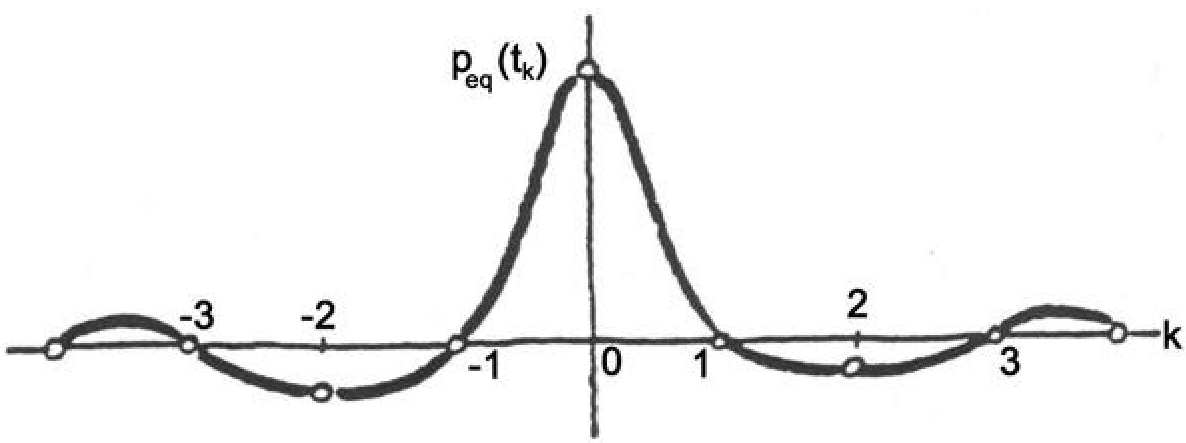
$$= \frac{4}{\pi}\int_0^{\pi/2}\cos^2\lambda d\lambda + \frac{64}{\pi^3}\int_0^{\pi/2}\lambda^2\cos^2\lambda d\lambda$$

$$= 1+\frac{64}{\pi^3}\left(\frac{\pi^3}{48}-\frac{\pi}{8}\right) = 1.52 = 1.83\text{ dB}$$

11.3-17

$$\begin{bmatrix}1.0 & 0.4 & 0.0\\ 0.2 & 1.0 & 0.4\\ 0.0 & 0.2 & 1.0\end{bmatrix}\begin{bmatrix}c_{-1}\\ c_0\\ c_1\end{bmatrix} = \begin{bmatrix}0\\1\\0\end{bmatrix} \Rightarrow\ c_{-1} = -10/21,\ c_0 = 25/21,\ c_1 = -5/21$$

$$p_{eq}(t_k) = -\frac{10}{21}\tilde{p}_{k+1} + \frac{25}{21}\tilde{p}_k - \frac{5}{21}\tilde{p}_{k-1} = \begin{cases}1.0 & k=0\\ -0.19 & k=-2\\ -0.15 & k=2\\ 0 & \text{otherwise}\end{cases}$$



11.3-18

$$\begin{bmatrix}1 & \varepsilon & 0\\ \delta & 1 & \varepsilon\\ 0 & \delta & 1\end{bmatrix}\begin{bmatrix}c_{-1}\\ c_0\\ c_1\end{bmatrix}=\begin{bmatrix}0\\1\\0\end{bmatrix} \quad \text{Let } \Delta=\begin{vmatrix}1 & \varepsilon & 0\\ \delta & 1 & \varepsilon\\ 0 & \delta & 1\end{vmatrix}=1-2\varepsilon\delta$$

$$c_{-1}=\frac{1}{\Delta}\begin{vmatrix}0 & \varepsilon & 0\\ 1 & 1 & \varepsilon\\ 0 & \delta & 1\end{vmatrix}=\frac{-\varepsilon}{1-2\varepsilon\delta},\quad c_0=\frac{1}{\Delta}\begin{vmatrix}1 & 0 & 0\\ \delta & 1 & \varepsilon\\ 0 & 0 & 1\end{vmatrix}=\frac{1}{1-2\varepsilon\delta},\quad c_1=\frac{1}{\Delta}\begin{vmatrix}1 & \varepsilon & 0\\ \delta & 1 & 1\\ 0 & \delta & 0\end{vmatrix}=\frac{-\delta}{1-2\varepsilon\delta}$$

11.3-19

$$\begin{bmatrix}1 & 0.1 & 0.0 & 0.0 & 0.0\\ 1.0 & 0.1 & 0.0 & 0.0 & 0.0\\ -0.2 & 1.0 & 0.1 & 0.0 & 0.0\\ 0.1 & -0.2 & 1.0 & 0.1 & 0.0\\ 0.0 & 0.0 & 0.1 & -0.2 & 1.0\end{bmatrix}\begin{bmatrix}c_{-2}\\ c_{-1}\\ c_0\\ c_1\\ c_2\end{bmatrix}=\begin{bmatrix}0\\0\\1\\0\\0\end{bmatrix}$$

Solving yields $c_{-2}=0.0094,\quad c_{-1}=-0.0941,\quad c_0=0.9596,\quad c_1=0.2068,\quad c_2=-0.0549$

$$p_{eq}(t_k)=c_{-2}\tilde{p}_{k+2}+c_{-1}\tilde{p}_{k+1}+c_0\tilde{p}_k+c_1\tilde{p}_{k-1}+c_2\tilde{p}_{k-2}$$

$$=\begin{cases}0 & k\le -4\\ 0.00094 & k=-3\\ 0 & k=-2,-1\\ 1 & k=0\\ 0 & k=1,2\\ -0.0468 & k=3\\ -0.0055 & k=4\\ 0 & k\ge 5\end{cases}$$

$t_k=k+2D$

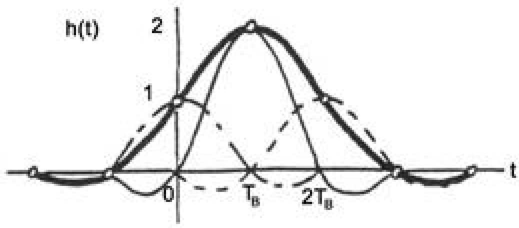
worst-case ISI now occurs at $k=3$ ($t = 5D$) but has not been reduced significantly in magnitude.

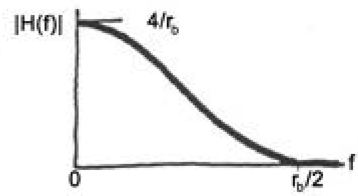
11.3-20

(a) $h(t) = \text{sinc } r_b t \ + \ 2 \text{ sinc } r_b(t - T_b) \ + \ \text{sinc } r_b(t - 2T_b)$

$$H(f) = \frac{1}{r_b}\Pi\left(\frac{f}{r_b}\right)\left(1 + 2e^{-j\omega T_b} + e^{-j\omega 2T_b}\right) = \frac{1}{r_b}\Pi\left(\frac{f}{r_b}\right)\left(2 + \cos\omega T_b\right)e^{-j\omega T_b}$$

$$= \frac{1}{r_b}\Pi\left(\frac{f}{r_b}\right)2\left(1 + \cos 2\frac{\pi f}{r_b}\right)e^{-j\omega T_b} = \frac{4}{r_b}\cos^2\frac{\pi f}{r_b}\Pi\left(\frac{f}{r_b}\right)e^{-j\omega T_b}$$





(b) $a_k' = a_k + 2a_{k-1} + a_{k-2}$

$$= \left(m_k' - \frac{1}{2} + 2m_{k-1}' - 1 + m_{k-2}' - \frac{1}{2}\right)A = \left(m_k' + 2m_{k-1}' + m_{k-2}' - 2\right)A$$

m_k'	m_{k-1}'	m_{k-2}'	a_k'/A
0	0	0	-2
0	0	1	-1
0	1	0	0
0	1	1	1
1	0	0	-1
1	0	1	0
1	1	0	1
1	1	1	2

Thus,

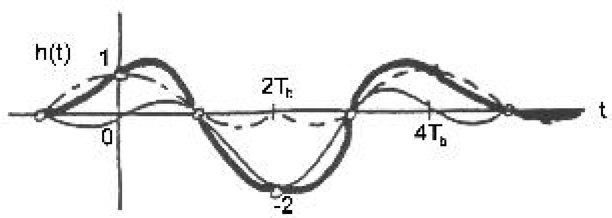
$$y(t_k) = \begin{cases} 2A & m_k' = m_{k-1}' = m_{k-2}' = 1 \\ A & m_{k-1}' = 1, \quad m_k' \neq m_{k-2}' \\ 0 & m_{k-1}' \neq m_k', \quad m_k' = m_{k-2}' \\ -A & m_{k-1}' = 0, \quad m_k' \neq m_{k-2}' \\ -2A & m_k' = m_{k-1}' = m_{k-2}' = 0 \end{cases}$$

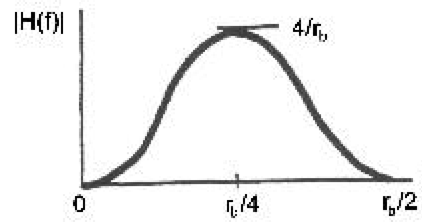
11.3-21

(a) $h(t) = \text{sinc}\, r_b t \;+\; 2\,\text{sinc}\, r_b\,(t-T_b) \;+\; \text{sinc}\, r_b\,(t-2T_b)$

$$H(f) = \frac{1}{r_b}\Pi\left(\frac{f}{r_b}\right)\left(1-2e^{-j\omega T_b} + e^{-j\omega 4T_b}\right) = \frac{1}{r_b}\Pi\left(\frac{f}{r_b}\right)\left(2\cos 2\omega T_b - 2\right)e^{-j\omega 2T_b}$$

$$= \frac{1}{r_b}\Pi\left(\frac{f}{r_b}\right)(-2)\left(1-\cos 2\frac{\pi f}{r_b}\right)e^{-j\omega 2T_b} = -\frac{4}{r_b}\sin^2\frac{2\pi f}{r_b}\Pi\left(\frac{f}{r_b}\right)e^{-j\omega 2T_b}$$





(b) $a_k' = a_k - 2\,a_{k-2} + a_{k-4}$

$$= \left(m_k' - \frac{1}{2} - 2m_{k-2}' + 1 + m_{k-4}' - \frac{1}{2}\right)A = \left(m_k' - 2m_{k-2}' + m_{k-4}'\right)A$$

m_k'	m_{k-1}'	m_{k-4}'	a_k' / A
0	0	0	0
0	0	1	1
0	1	0	-2
0	1	1	-1
1	0	0	1
1	0	1	2
1	1	0	-1
1	1	1	0

Thus,

$$y(t_k') = \begin{cases} 2A & m_{k-2}' = 0,\;\; m_k' = m_{k-4}' = 1 \\ A & m_{k-2}' = 0,\;\; m_k' \neq m_{k-4}' \\ 0 & m_k' = m_{k-2}' = m_{k-4}' \\ -A & m_{k-2}' = 1,\;\; m_k' \neq m_{k-4}' \\ -2A & m_{k-2}' = 1,\;\; m_k' = m_{k-4}' = 0 \end{cases}$$

11.3-22

$$\frac{1}{\sqrt{L}}\left|H_T(f)H_R(f)\right| = \left|H(f)\right| = \frac{2}{r_b}\cos\frac{\pi f}{r_b}\Pi\left(\frac{f}{r_b}\right)$$

$$\left|H_T\right|^2 = L\frac{|H|^2}{|H_R|^2}, \quad P_x(f)=1, \quad G_n(f)=N_0/2$$

$$S_T = \frac{A^2 r_b}{4}\int_{-\infty}^{\infty}\left|H_T P_x\right|^2 df \qquad \sigma^2 = \frac{N_0}{2}\int_{-\infty}^{\infty}\left|H_R\right|^2 df$$

Thus, $\left(\frac{A}{2\sigma}\right)^2 = \frac{2S_T}{N_0 r_b L}\left[\int_{-\infty}^{\infty}\frac{|H|^2}{|H_R|^2}df\int_{-\infty}^{\infty}\left|H_R\right|^2 df\right]^{-1}$

where $\frac{2S_T}{N_0 r_b L} = \frac{2S_R}{N_0 r_b} = 2\gamma_b$

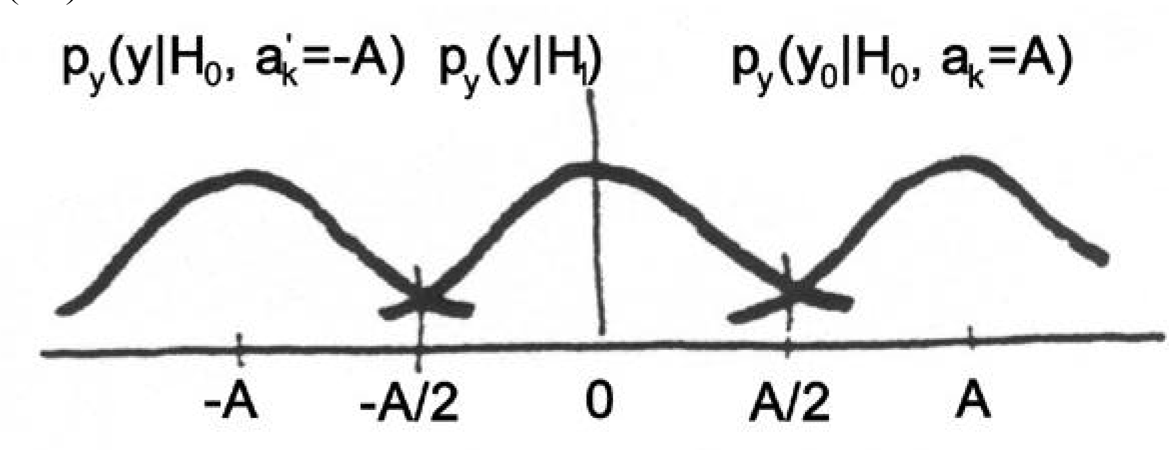
$$\int_{-\infty}^{\infty}\frac{|H|^2}{|H_R|^2}df\int_{-\infty}^{\infty}\left|H_R\right|^2 df \ge \left|\int_{-\infty}^{\infty}\frac{|H|}{|H_R|}\left|H_R\right|df\right|^2$$

with equality when $\frac{|H|}{|H_R|} = g\left|H_R\right|$

So take $\left|H_R(f)\right|^2 = \frac{1}{g}\left|H(f)\right|$ and $\left|H_T(f)\right|^2 = gL\left|H(f)\right|$

Then, since $\int_{-\infty}^{\infty}\left|H(f)\right|df = \frac{4}{r_b}\int_{0}^{r_b/2}\cos\frac{\pi f}{r_b}df = \frac{4}{\pi}$

$$\left(\frac{A}{2\sigma}\right)^2_{\max} = \frac{2\gamma_b}{(4/\pi)^2}$$



11.2-22 continued

$P_{e1} = 2Q\left(\frac{A}{2\sigma}\right)$ and

$P_{e0} = \frac{1}{2}Q\left(\frac{A}{2\sigma}\right) + \frac{1}{2}Q\left(\frac{A}{2\sigma}\right)$ since $P\left(a_k^{'} = \pm A \mid H_0\right) = \frac{1}{2}$

Hence, $P_e = \frac{1}{2}(P_{e1} + P_{e0}) = \frac{3}{2}Q\left(\frac{A}{2\sigma}\right) = \frac{3}{2}Q\left(\frac{\pi}{4}\sqrt{2\gamma_b}\right)$

11.3-23

(a) Assume $m_{-1} = \hat{m}_{-1} = 0.$

Use $y(t_k) = (m_k + m_{k-1} - 1)A$ to calculate $y(t_k)$ given m_k, and m_{k-1} and to calculate $\hat{m}_k$ given $y(t_k)$ and $\hat{m}_{-1}$.

k	0	1	2	3	4	5	6	7	8
m_k	1	0	1	0	1	1	1	0	1
m_{k-1}	0	1	0	1	0	1	1	1	0
$y(t_k)$	0	0	0	0	0	2	2	0	0
$\hat{m}_{k-1}$	0	1	0	1	0	1	1	1	0
$\hat{m}_k$	1	0	1	0	1	1	1	0	1

(b) dc value: $\overline{y(t_k)} = (2+2)/9 = 0.44$

(c) As the table below indicates, if bit $\hat{m}_2$ is received in error such that $\hat{m}_2 = 0 \Rightarrow y(t_2) = -2$ instead of 0.

Because $m_k = f(m_{k-1}) \Rightarrow$ errors in $\hat{m}_{k=3}$ will affect all subsequent values of $\hat{m}_k$ as indicated in the table below.

k	0	1	2	3	4	5	6	7	8	
$y(t_k)$	0	0	−2	0	0	2	2	0	0	
$\hat{m}_{k-1}$	0	1	0	0	1	0	2	0	1	
$\hat{m}_k$	1	0	0	1	0	2	0	1	0	
			x	x	x	x	x	x	x	= errors

11.3-24

(a) Use the precoder of Fig. 11.3-9 to convert $m_k \to m_k'$, Eq. (23) for $y(t_k)$, Eq. (24) to determine $\hat{m}_k$ from the received value of $y(t_k)$. Note that with precoding $\hat{m}_k$ is not a function of $\hat{m}_{k-1}'$. Also, assume $m_{-1}' = 0$.

k	0	1	2	3	4	5	6	7	8
m_k	1	0	1	0	1	1	1	0	1
m_{k-1}'	0	1	1	0	0	1	0	1	1
m_k'	1	1	0	0	1	0	1	1	0
$y(t_k)$	0	2	0	–2	0	0	0	2	0
$\hat{m}_k$	1	0	1	0	1	1	1	0	1

(b) dc value: $\overline{y(t_k)} = (2-2+2)/9 = 0.22$

(c) If bit $\hat{m}_2$ is received in error, only that bit is affected since with precoding $\hat{m}_k$ is not a function of $\hat{m}_{k-1}'$.

11.3-25

(a) Use the precoder of Fig. 11.3-9 to convert $m_k \to m_k'$ except use two stages of delay $\Rightarrow m_k' = m_k \oplus m_{k-2}'$.
Then use Eq. (27) to determine $y(t_k)$ from m_k' and m_{k-2}' and $\hat{m}_k$ from $y(t_k)$.
Assume $m_{-1}' = m_{-2}' = 0$.

k	0	1	2	3	4	5	6	7	8
m_k	1	0	1	0	1	1	1	0	1
m_{k-2}'	0	0	1	0	0	0	1	1	0
m_k'	1	0	0	0	1	1	0	1	1
$y(t_k)$	2	0	–2	0	2	2	–2	0	2
$\hat{m}_k$	1	0	1	0	1	1	1	0	1

(b) dc value: $\overline{y(t_k)} = (2-2+2+2-2+2)/9 = 0.44$

(c) If bit $\hat{m}_2$ is received in error, only that bit is affected since we can obtain $\hat{m}_k$ directly from $y(t_k)$.

11.4-1

(a) Using structure of Fig. 11.4-6a to scramble the input sequence we get:

m_k	0	1	1	1	1	1	1	0	1	1	1	0	1	1	1
m'_{k-1}	0	0	1	0	0	1	1	1	0	1	1	0	1	1	0
m'_{k-2}	0	0	0	1	0	0	1	1	1	0	1	1	0	1	1
m'_{k-3}	0	0	0	0	1	0	0	1	1	1	0	1	1	0	1
m'_{k-4}	0	0	0	0	0	1	0	0	1	1	1	0	1	1	0
m'_{k-5}	0	0	0	0	0	0	1	0	0	1	1	1	0	1	1
m''_k	0	0	1	1	0	0	0	0	0	0	1	1	0	1	1
m'_k	0	1	0	0	1	1	1	0	1	1	0	1	1	0	0

dc values of unscrambled and scrambled sequences:

$\overline{m}_k = (1+1+1+1+1+1+1+1+1+1+1+1)/15 = 12/15 = 0.80$

$\overline{m}'_k = (1+1+1+1+1+1+1+1)/15 = 8/15 = 0.53$

(b) Using the structure of Fig 11.4-6b to unscramble m'_k we get:

m'_k	0	1	0	0	1	1	1	0	1	1	0	1	1	0	0
m'_{k-1}	0	0	1	0	0	1	1	1	0	1	1	0	1	1	0
m'_{k-2}	0	0	0	1	0	0	1	1	1	0	1	1	0	1	1
m'_{k-3}	0	0	0	0	1	0	0	1	1	1	0	1	1	0	1
m'_{k-4}	0	0	0	0	0	1	0	0	1	1	1	0	1	1	0
m'_{k-5}	0	0	0	0	0	0	1	0	0	1	1	1	0	1	1
m''_k	0	0	1	1	0	0	0	0	0	0	1	1	0	1	1
m_k	0	1	1	1	1	1	1	0	1	1	1	0	1	1	1

11.4-2

Using the results from Exercise 11.4-1, we get the output sequence and its shifted versions to generate the following table used to calculate the autocorrelation function.

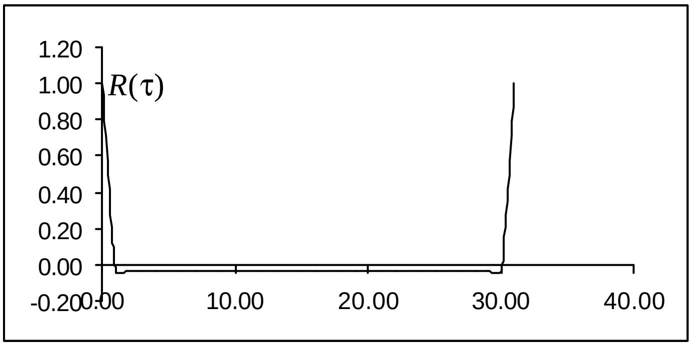
11.4-2 continued

τ	original/shifted	$v(\tau)$	$R(\tau) = v(\tau) / N$
0	1111100100110000101101010001110 1111100100110000101101010001110	31	1
1	1111100100110000101101010001110 0111110010011000010110101000111	-1	-0.032
2	1111100100110000101101010001110 1011111001001100001011010100011	-1	-0.032
3	1111100100110000101101010001110 1101111100100110000101101010001	-1	-0.032
4	1111100100110000101101010001110 1110111110010011000010110101000	-1	-0.32
5	1111100100110000101101010001110 0111011111001001100001011010100	-1	-0.032
6	1111100100110000101101010001110 0011101111100100110000101101010	-1	-0.032
7	1111100100110000101101010001110 0001110111110010011000010110101	-1	-0.032
8	1111100100110000101101010001110 1000111011111001001100001011010	-1	-0.032
9	1111100100110000101101010001110 0100011101111100100110000101101	-1	-0.032
10	1111100100110000101101010001110 1010001110111110010011000010110	-1	-0.032
11	1111100100110000101101010001110 0101000111011111001001100001011	-1	-0.032
12	1111100100110000101101010001110 1010100011101111100100110000101	-1	-0.032
13	1111100100110000101101010001110 1101010001110111110010011000010	-1	-0.032
14	1111100100110000101101010001110 0110101000111011111001001100001	-1	-0.032

11.4-2 continued

τ	original/shifted	$v(\tau)$	$R(\tau) = v(\tau) / N$
15	1111100100110000101101010001110	-1	-0.032
	1011010100011101111100100110000		
16	1111100100110000101101010001110	-1	-0.032
	0101101010001110111110010011000		
17	1111100100110000101101010001110	-1	-0.032
	0010110101000111011111001001100		
28	1111100100110000101101010001110	-1	-0.032
	0001011010100011101111100100110		
19	1111100100110000101101010001110	-1	-0.032
	0000101101010001110111110010011		
20	1111100100110000101101010001110	-1	-0.032
	1000010110101000111011111001001		
21	1111100100110000101101010001110	-1	-0.032
	1100001011010100011101111100100		
22	1111100100110000101101010001110	-1	-0.032
	0110000101101010001110111110010		
23	1111100100110000101101010001110	-1	-0.032
	0011000010110101000111011111001		
24	1111100100110000101101010001110	-1	-0.032
	1001100001011010100011101111100		
25	1111100100110000101101010001110	-1	-0.032
	0100110000101101010001110111110		
26	1111100100110000101101010001110	-1	-0.032
	0010011000010110101000111011111		
27	1111100100110000101101010001110	-1	-0.032
	1001001100001011010100011101111		
28	1111100100110000101101010001110	-1	-0.032
	1100100110000101101010001110111		
29	1111100100110000101101010001110	-1	-0.032
	1110010011000010110101000111011		
30	1111100100110000101101010001110	-1	-0.032
	1111001001100001011010100011101		
31	1111100100110000101101010001110	31	1
	1111100100110000101101010001110		

11.4-2 continued



$R(\tau)$ is periodic with period = 31.

Is the output a ml sequence? Apply ML rules: (1) #1s =16, #0s = 15⟹ obeys balance property; (2) obeys the run property; (3) has a single autocorrelation peak; (4) obeys Mod-2 property, and (5) all 32 states exist during sequence generation.

11.4-3

$m_1 = m_2 + m_4$ and the output sequence = m_4

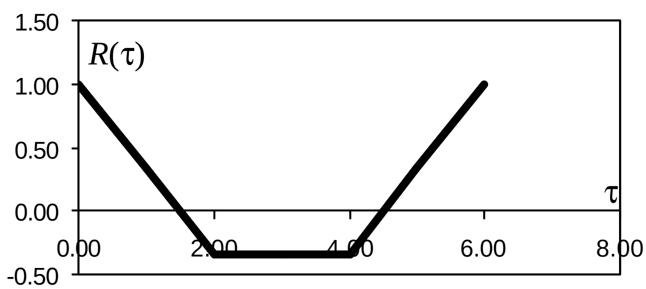
shift	$m_1 m_2 m_3 m_4$
0	1 1 1 1
1	0 1 1 1
2	0 0 1 1
3	1 0 0 1
4	1 1 0 0
5	1 1 1 0
6	1 1 1 1

11.4-3 continued

The autocorrelation function is calculated as follows:

τ	original/shifted	$v(\tau)$	$R(\tau) = v(\tau)/N$
0	1 1 1 1 0 0	6	6/6=1
	1 1 1 1 0 0		
1	1 1 1 1 0 0	2	0.333
	0 1 1 1 1 0		
2	1 1 1 1 0 0	-2	-0.333
	0 0 1 1 1 1		
3	1 1 1 1 0 0	-2	-0.333
	1 0 0 1 1 1		
4	1 1 1 1 0 0	-2	-0.333
	1 1 0 0 1 1		
5	1 1 1 1 0 0	2	0.333
	1 1 1 0 0 1		
6	1 1 1 1 0 0	2	1.
	1 1 1 1 0 0		

$R(\tau)$ is periodic with period = 6.



The [4,2] register does not produce a ml sequence since there are only 5/16 possible states exist in the output sequence and the period is not 2^4-1.

Chapter 12

12.1-1

$$\left.\begin{aligned}\frac{1}{2}vf_s &\le B_T \\ f_s &\ge 2W\end{aligned}\right\} \; v \le \frac{2B_T}{2W} = 3.33 \;\;\Rightarrow\;\; v = 3,\; f_s \le \frac{2B_T}{v} = 33.3 \text{ kHz}$$

$$q = M^3 \ge 200 \;\;\Rightarrow\;\; M = 2^n \ge 200^{1/3} = 5.85 \;\;\Rightarrow\;\; n = 3$$

12.1-2

$$\left.\begin{aligned}\frac{1}{2}vf_s &\le B_T \\ f_s &\ge 2W\end{aligned}\right\} \; v \le \frac{2B_T}{2W} = 5.33 \;\;\Rightarrow\;\; v = 5,\; f_s \le \frac{2B_T}{v} = 32 \text{ kHz}$$

$$q = M^5 \ge 200 \;\;\Rightarrow\;\; M = 2^n \ge 200^{1/5} = 2.88 \;\;\Rightarrow\;\; n = 2$$

12.1-3

Transmit one quantized pulse, having q^N possible values, for every N successive quantized samples. The output pulse rate is f_s / N so $B_T \ge \frac{1}{2}\left(f_s / N\right) \ge W / N$, allowing $B_T < W$.

12.1-4

$$\left.\begin{aligned}|\varepsilon_k| &\le 1/q \\ 2|x(t)| &= 2\end{aligned}\right\} \;\; \frac{1/q}{2} \le \frac{P}{100} \;\;\Rightarrow\;\; q = M^v \ge \frac{50}{P}$$

Thus, $v \ge \log_M (50/P)$

12.1-5

$$(S/N)_D = 3q^2 \text{ x } 0.25 \ge 10^4 \;\;\Rightarrow\;\; q \ge 116,\; v \le B_T/W = 5.3,\; q = M^v$$

M	v	q	comment
2	5	32	q too small, try $M > 2$
3	5	243	ok
3	4	81	q too small

$\Rightarrow$ Take $M = 3$, $v = 5$, $f_s \le 2B_T / v = 6.4$ kHz

12.1-6

$(S/N)_D = 3q^2$ x 0.25 $\geq 4000 \;\Rightarrow\; q \geq 73,\; v \leq B_T/W = 6.7,\; q = M^v$

M	v	q	comment
2	6	64	q too small, try $M > 2$
3	6	729	q excessively large
3	5	81	ok

$\Rightarrow$ Take $M = 3,\; v = 5,\; f_s \leq 2B_T/v = 10$ kHz

12.1-7

$W = 5$ kHz, $(S/N)_D = 40 - 50$ dB $= 10^4 - 10^5$

(a) $(S/N)_D = 0.9q^2 \geq 10^4 - 10^5 \;\Rightarrow\; q = 2^v = 105 - 333$ so v = 7 or 8

$B_T \geq vW = 35 - 40$ kHz

(b) $v \leq 4,\; q = M^4 = 105 - 333 \;\Rightarrow\; M \geq 4$

12.1-8

$W = 20$ kHz, $(S/N)_D = 55 - 65$ dB $= 3.2$ x $10^5 - 3.2$ x 10^6

(a) $(S/N)_D = 0.9q^2 \geq 3.2$ x $10^5 - 3.2$ x $10^6 \Rightarrow q = 2^v = 596 - 1886$ so $v = 10 \Rightarrow B_T \geq vW = 200$ kHz

(b) $v \leq 4,\; q = M^4 = 596 - 1886 \;\Rightarrow\; M \geq 5$

12.1-9

$x_{min} = 5$ x 10^{-6} V and $x_{max} = 200$ x 10^{-3} V $\Rightarrow$ normalize $\Rightarrow x_{min} = 25$ x 10^{-6} V and $x_{max} = 1$ V

Assume a sinusoidal input, the power of the smallest signal is $\Rightarrow S_x = (25$ x $10^{-6})^2/2 = 3.12$ x 10^{-10}

Using Eq. (7) $\Rightarrow$ 40 dB $= 4.8 + 10\log(2^{2v}$ x 3.12 x $10^{-10}) \Rightarrow v = 21.7 \Rightarrow v = 22$.

12.1-10

Scale +/- 10V input by a factor of 10 to make input +/- 1V, then because its sinusoidal $\Rightarrow S_x = 0.5$

$q = 2^{12} \Rightarrow$ quantum size $= 2/q = 2/4096 = 0.488$ mV.

$(S/N)_D = 3q^2 S_x = 3(2^{12})^2$ x 0.5=2.52 x $10^7 \Rightarrow$ 74 dB.

12.1-11

Scale +/- 10V input by a factor of 10 to make input +/- 1V, then because its sinusoidal $\Rightarrow S_x = 0.5$

$q = 2^{16} \Rightarrow$ quantum size = $2/q = 2/65{,}536 = 30.5$ uV.

$(\text{S/N})_D = 3q^2 S_x = 3(2^{16})^2$ x 0.5=6.44 x $10^9 \Rightarrow 98$ dB.

12.1-12

Let v_{max}, v_{min} = maximum and minimum input voltages

$\Rightarrow$ Dynamic range = 20log(v_{max}/v_{min})

Assuming the largest signal = 1 volt $\Rightarrow$ the smallest signal = $1/q$ volts

$\Rightarrow$ Dynamic range = 20log(q)=20log(2^v)

Dynamic range = $20\log(2^v) = 120$ dB $\Rightarrow v = 20$ bits.

12.1-13

If $(S/N)_D = 35$ dB and assuming $S_x = 1$

$\Rightarrow 35 = 4.8 + 6v \Rightarrow v = 5.03$

Memory must hold:

10 min x 60 secs/min x 8000 samples/sec = 4.8 Msamples @ 5 bits/sample = 24 Mbits

12.1-14

(a) $v = 12$ bits $\Rightarrow q = 2^{12}$ =4096. With $|x|_{max} = 10$ V $\Rightarrow$ each step size = 10 x $2/q$ = 4.88 x 10^{-3} V

Maximum input = $x(kT_s)_{max} = (q-1)/q$ = 4095/4096 x 10 = 9.9976 V

For positive inputs from 0 to 10 V, $x_q(kT_s) = 0.00244$, 0.00732, 0.01221, 0.01709, 0.02197......

$\Rightarrow x(t) = 0.02$ V $\Rightarrow x_q(kT_s) = 0.02197 \Rightarrow |\varepsilon_k| = |0.02 - 0.02197| = 0.00197$

Quantization error% = 0.00197/0.02 x 100% = 9.85%

(b) For $x(t) = 0.2$ V $\Rightarrow x_q(kT_s) = 0.19775 \Rightarrow |\varepsilon_k| = |0.2 - 0.19775| = 0.00225$

Quantization error% = 0.00225/0.2 x 100% =1.125%

12.1-15

Let $v = 2 + n,\ \ n \geq 1$ then $h_i = 2/q = \dfrac{2}{2}v = \dfrac{1}{2^n}$ x $\dfrac{1}{2} \leq \dfrac{1}{4}$

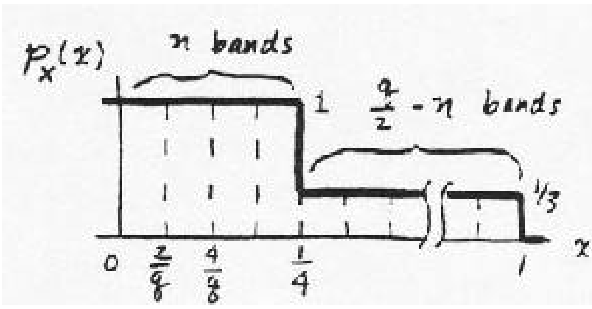
so $p_x(\text{x})$ is constant over each band.

$a_i = x_i - 1/q,\ b_i = x_i + 1/q$

12.1-15 continued

$$n\frac{2}{q} = \frac{1}{4} \quad \Rightarrow \quad n = q/8$$

$$q/2 - n = 3q/8$$



$$\overline{\varepsilon_i^2} = p_x(x_i)\int_{x_i-1/q}^{x_i+1/q}(x_i - x)^2\,dx = \frac{2p_x(x_i)}{3q^3}$$

$$\sigma_q^2 = 2\frac{2}{3q^3}\left[n \text{ x } 1 + \left(\frac{q}{2} - n\right) \text{ x } \frac{1}{3}\right] = \frac{1}{3q^2}$$

$$S_x = 2\left[\int_0^{\frac{1}{4}} x^2\,dx + \int_{\frac{1}{4}}^{1} x^2 \text{ x } \frac{1}{3}\,dx\right] = \frac{22}{96} = 0.229$$

Thus, $(S/N)_D = 3q^2 \text{ x } 0.229 \approx 0.7\,q^2$

12.1-16

$h_i = 2/q,\;\; a_i = x_i - 1/q,\;\; b_i = x_i + 1/q,\;\; p_x(x) \approx p_x(x_i)$ for $a_i < x < b_i$

Thus, $\overline{\varepsilon_i^2} \approx p_x(x_i)\int_{x_i-1/q}^{x_i+1/q}(x_i - x)^2\,dx = \frac{2p_x(x_i)}{3q^3}$

so $\sigma_q^2 \approx \frac{2}{3q^3}\sum_{i=-q/2}^{q/2} p_x(x_i)$

But $\int_{-1}^{1} p_x(x)dx = \sum_{i=-q/2}^{q/2}\frac{2}{q}p_x(x_i)$ and $\int_{-1}^{1} p_x(x)dx = P\left[|x| < 1\right] \approx 1$

Hence, $\sum_{i=-q/2}^{q/2} p_x(x_i) \approx \frac{q}{2}$ so $\sigma_q^2 \approx \frac{2}{3q^3}\frac{q}{2} = \frac{1}{3q^2}$

12.1-17

$$\left.\begin{array}{l}\overline{x}=0 \quad\Rightarrow\quad \sigma_x=\sqrt{S_x} \\ P\left[|x|>1\right]=2Q(1/\sigma_x)\le 0.01 \quad\Rightarrow\quad \dfrac{1}{\sigma_x}\ge 2.6\end{array}\right\}\ S_x\le\frac{1}{2.6^2}\approx 0.148$$

Thus, $(\mathrm{S/N})_D \le 10\log_{10}(3 \text{ x } 2^{2v} \text{ x } 0.148) = -3.5+6v$ dB

12.1-18

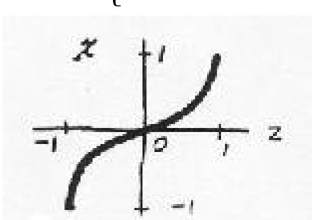
$$P\left[|x|>1\right]=2\int_1^{\infty}\frac{\alpha}{2}e^{-\alpha x}dx=e^{-\alpha}\le 0.01 \quad\Rightarrow\quad \alpha\ge -\ln 0.01$$

so $S_x = 2/\alpha^2 \le 2/(\ln 100)^2 = 0.0943$

and $(\mathrm{S/N})_D \le 10\log_{10}(3 \text{ x } 2^{2v} \text{ x } 0.0943) = -5.5 + 6v$ dB

12.1-19

(a) $x=\begin{cases} z^2 & z>0 \\ -z^2 & z<0\end{cases} \quad\Rightarrow\quad x(z)=(\operatorname{sgn} z)z^2$



(b) $z'(x)=\frac{1}{2}|x|^{-3/2}$,

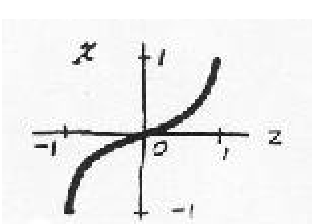
$$K_z = 2\int_0^1 4x^3 p_x(x)dx = 8\left[\int_0^{1/4} x^3 dx + \frac{1}{3}\int_{1/4}^{1} x^3 dx\right] = 0.672$$

12.1-20

(a) $z>0:\quad \ln(1+\mu x) = z\ln(1+\mu) = \ln(1+\mu)^2 \quad\Rightarrow\quad x=[(1+\mu)^2-1]/\mu$

$z<0:\quad \ln(1-\mu x) = -z\ln(1+\mu) = \ln(1+\mu)^{-2} \quad\Rightarrow\quad x=-[(1+\mu)^{-2}-1]/\mu$

Thus, $x(z)=(\operatorname{sgn} z)\dfrac{(1+\mu)^{|z|}-1}{\mu}$



12.1-20 continued

(b) $K_z = 2\left[\dfrac{\ln(1+\mu)}{\mu}\right]^2 \int_0^1 (1+\mu x)^2 p_x(x)dx$

$$=2\frac{\ln^2(1+\mu)}{\mu^2}\left[\int_0^1 p_x(x)dx + 2\mu\int_0^1 x p_x(x)dx + \mu^2\int_0^1 x^2 p_x(x)dx\right]$$

where $\int_0^1 p_x(x)dx = 1/2,\quad 2\int_0^1 xp_x(x)dx = \int_{-1}^1 |x| p_x(x)dx = \overline{|x|}$

$$\int_0^1 x^2 p_x(x)dx = \frac{1}{2}\int_{-1}^1 x^2 p_x(x)dx = \frac{1}{2}\overline{x^2} = \frac{1}{2}S_x$$

Thus, $K_z = \dfrac{\ln^2(1+\mu)}{\mu^2}\left(1+2\mu\overline{|x|}+\mu^2 S_x\right)$

12.1-21

(a) With $x(t) = 0.02$ V $\Rightarrow$ with companding using Eq. (12) we have

$$z(x) = 9.9976\left[\frac{\ln(1+ 255 \text{ x } 0.02/9.9976)}{\ln(256)}\right] = 0.7432$$

$z(x)$ is fed to the quantizer giving $z(kT_s) = 0.74463$

Using Eq. (13) with $x_q(kT_s) = z(kT_s)$ gives

$$\hat{x} = \frac{9.9976}{255}\left[(1+255)^{0.74463/9.9976} - 1\right] = 0.02005 \Rightarrow |\varepsilon_k| = |0.02 - 0.02005| \approx 0 \Rightarrow 0\% \text{ quantization error.}$$

(b) With $x(t) = 0.2$V $\Rightarrow$ with companding using Eq. (12) we have

$$z(x) = 9.9976\left[\frac{\ln(1+ 255 \text{ x } 0.2/9.9976)}{\ln(256)}\right] = 3.26059$$

$z(x)$ is fed to the quantizer giving $z(kT_s) = 3.25928$

Using Eq. (13) with $x_q = z(kT_s)$ gives

$$\hat{x} = \frac{9.9976}{255}\left[(1+ 255)^{3.25928/9.9976} - 1\right] = 0.19983 \Rightarrow |\varepsilon_k| = |0.2 - 0.19983| \approx 0 \Rightarrow 0\% \text{ quantization error.}$$

12.1-22

(a) $z'(x) = 3e^{-3x}, \quad x > 0$

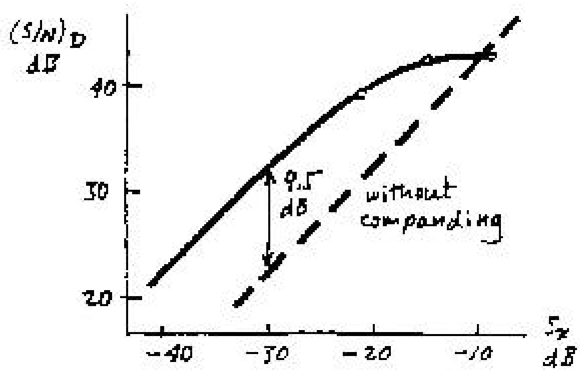
$$K_z = 2\int_0^1 \frac{1}{9}e^{6x}\frac{\alpha}{2}e^{-\alpha x}dx = \frac{\alpha}{9(6-\alpha)}\left(e^{6-\alpha} - 1\right) \approx \frac{\alpha}{9(-\alpha)}(0-1) = \frac{1}{9} \text{ for } \alpha \gg 1$$

$$\left(S/N\right)_D \approx 10\log_{10}(9 \text{ x } 3 \text{ x } 2^{2v}S_x) = 14.3 + 6.0v + S_x \text{ dB}$$

12.1-22 continued

(b) $\left(S/N\right)_D = 52.9 + S_x - K_z \quad \text{dB}$

α	S_x (dB)	K_z (dB)	$\left(S/N\right)_D$
4	-9	1.5	42.4
8	-15	-4.2	42.1
16	-21	-7.5	39.4
$\gg 1$		-9.5	$62.4 + S_x$



12.1-23

(a) $$z'(x) = \begin{cases} \dfrac{A}{1+\ln A} & 0 \le x \le 1/A \\ \dfrac{A}{1+\ln A}\dfrac{1}{x} & 1/A \le x \le 1 \end{cases}$$

$$K_z = (1+\ln A)^2 \left[2\int_0^{1/A} \frac{1}{A^2} p_x(x)dx + 2\int_{1/A}^{1} x^2 p_x(x)dx \right]$$

$$= (1+\ln A)^2 \left[2\int_0^{1} x^2 p_x(x)dx + 2\int_0^{1/A} \left(\frac{1}{A^2} - x^2 \right) p_x(x)dx \right]$$

where $2\int_0^1 x^2 p_x(x)dx = \int_{-1}^{1} x^2 p_x(x)dx = S_x$

12.1-23 continued

(b) $2\int_0^{1/A}\left(\frac{1}{A^2}-x^2\right)p_x(x)dx=\frac{\alpha}{A^2}\int_0^{1/A}e^{-\alpha}dx-\alpha\int_0^{1/A}x^2e^{-\alpha x}dx$

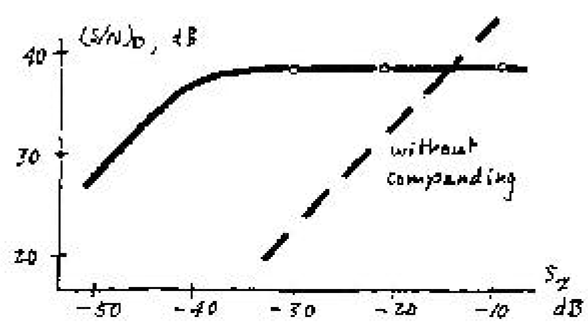
$=\frac{2}{\alpha^2}\left(\frac{\alpha}{A}+1\right)e^{-\alpha/A}+\frac{1}{A^2}-\frac{2}{\alpha^2},\quad S_x=\frac{2}{\alpha^2}$

Thus, $K_z=(1+\ln A)^2\left[\frac{2}{\alpha^2}\left(\frac{\alpha}{A}+1\right)e^{-\alpha/A}+\frac{1}{A^2}\right]$

$=\left(\frac{1+\ln A}{A}\right)^2\left[1+2\frac{A}{\alpha}\left(\frac{A}{\alpha}+1\right)e^{-\alpha/A}\right]\approx\left(\frac{1+\ln A}{A}\right)^2\quad$ if $\alpha \gg A$

(c) $(S/N)_D=52.9+S_x-K_z$ dB

α	S_x(dB)	K_z(dB)	$(S/N)_D$
4	-9	5.9	38
16	- 21	-6.1	38
64	-33	-17.9	37.8
$\gg$ 100		-25	$77.9+S_x$



12.2-1

$(S/N)_D=3q^2$ x $1/2\geq$ 4 x 10^3 $\Rightarrow$ $q>51,\ v\leq B_T/W=2.5$

so $q\leq M^2>51$ $\Rightarrow$ $M>7$. Thus, take $M=8,\ v=2,\ q=64$

$\gamma=\frac{S_R}{N_0W}\geq\gamma_{th}=6(B_T/W)(M^2-1)=945\ \Rightarrow\ S_R\geq 945N_0W=56.7$ mW

12.2-2

$\left(S/N\right)_D = 3q^2$ x $1/2 \geq$ 4 x $10^3 \quad \Rightarrow \quad q > 51, \; v \leq B_T/W = 3.33$

so $q \leq M^3 > 51 \quad \Rightarrow \quad M > 3.7$ Thus, take $M = 4, \; v = 3, \; q = 64$

$\gamma = \dfrac{S_R}{N_0 W} \geq \gamma_{th} = 6\left(B_T/W\right)\left(M^2 - 1\right) = 300 \quad \Rightarrow \quad S_R \geq 300 N_0 W = 18$ mW

12.2-3

$\left(S/N\right)_D = 3q^2$ x $1/2 \geq$ 4 x $10^3 \quad \Rightarrow \quad q > 51, \; v \leq B_T/W = 8.33$

so $q \leq M^8 > 51 \quad \Rightarrow \quad M > 1.2$ Thus, take $M = 2, \; v = 6, \; q = 64$

$\gamma = \dfrac{S_R}{N_0 W} \geq \gamma_{th} = 6\left(B_T/W\right)\left(M^2 - 1\right) = 150 \quad \Rightarrow \quad S_R \geq 150 N_0 W = 9$ mW

12.2-4

PCM: $P_e = 20Q\left[\sqrt{(S/N)_1}\right] \leq 10^{-5} \quad \Rightarrow \quad (S/N)_1 \geq 4.9^2$

B_T/W $\geq v = 8, \; \gamma = \left(B_T/\text{W}\right) \; (S/N)_1 \geq 192 \approx 22.8$ dB

Analog: $(S/N)_R = \dfrac{1}{20}(S/N)_1 = 37$ dB $\approx 5000, \; \gamma = (S/N)_1 = 10^5 = 50$ dB

PCM advantage: 50-22.8 = 27.2 dB

12.2-5

PCM: $P_e = 100Q\left[\sqrt{(S/N)_1}\right] \leq 10^{-5} \quad \Rightarrow \quad (S/N)_1 \geq 5.2^2$

B_T/W $\geq v = 8, \; \gamma = \left(B_T/\text{W}\right) \; (S/N)_1 \geq 216 = 23.4$ dB

Analog: $(S/N)_R = \dfrac{1}{100}(S/N)_1 = 37$ dB $\approx 5000, \; \gamma = (S/N)_1 =$ 5 x $10^5 = 57$ dB

PCM advantage: 57 - 23.4 = 33.6 dB

12.2-6

$10\log_{10}(1+4q^2P_e) = 1\text{ dB} \Rightarrow 1+4q^2P_e = 10^{0.1} = 1.259$

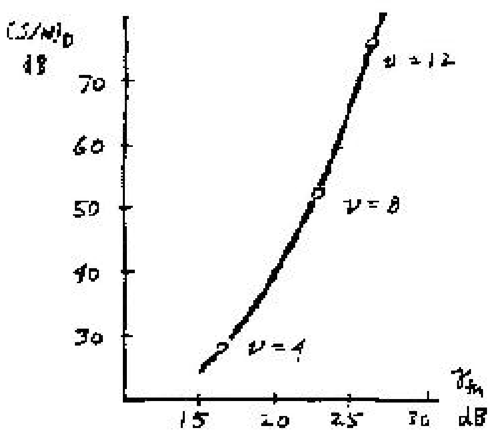
so $P_e = 0.259/4q^2 \approx 1/15q^2$

$$P_e = Q\left[\sqrt{(S/N)_R}\right] \approx \frac{1}{15 \text{ x } 2^{2v}}$$

$$(S/N)_R = \frac{W}{B_T}\gamma \le \frac{1}{v}\gamma \quad \Rightarrow \quad \gamma_{\text{th}} \approx v(S/N)_R$$

$(S/N)_D = 4.8 + 6.0v$ dB

v	P_e	$\sqrt{(S/N)_R}$	γ_{th} (dB)	$(S/N)_D$, dB
4	$2.6 \text{ x } 10^{-4}$	3.5	16.9	28.8
8	$1.0 \text{ x } 10^{-6}$	4.8	22.7	52.8
12	$4.0 \text{ x } 10^{-9}$	5.8	26.1	76.8



12.2-7

Errors in magnitude bits have same effect as before, and there are $q/2$ equiprobable values for i. Thus

$$\overline{\varepsilon_m^2} = \frac{1}{v}\left[\sum_{m=0}^{v-2}\left(\frac{2}{q}2^m\right)^2 + \frac{1}{q/2}\sum_{i=0}^{q/2}\left(\frac{2}{q}\right)^2(2i-1)^2\right] = \frac{4}{vq^2}\left[\sum_{m=0}^{v-2}4^m + \frac{2}{q}\sum_{i=0}^{q/2}(4i^2-4i+1)\right]$$

$$= \frac{4}{vq^2}\left\{\frac{4^{v-1}-1}{3} + \frac{2}{q}\left[4\frac{q/2(q/2+1)(q+1)}{6} - 4\frac{q/2(q/2+1)}{2} + \frac{q}{2}\right]\right\}, \quad 4^v = q^2$$

$$= \frac{4}{vq^2}\left(\frac{5q^2}{12} - \frac{2}{3}\right) = \frac{5q^2-8}{3vq^2} \approx \frac{5}{3v} \quad \text{if } 5q^2 \gg 8$$

12.2-8

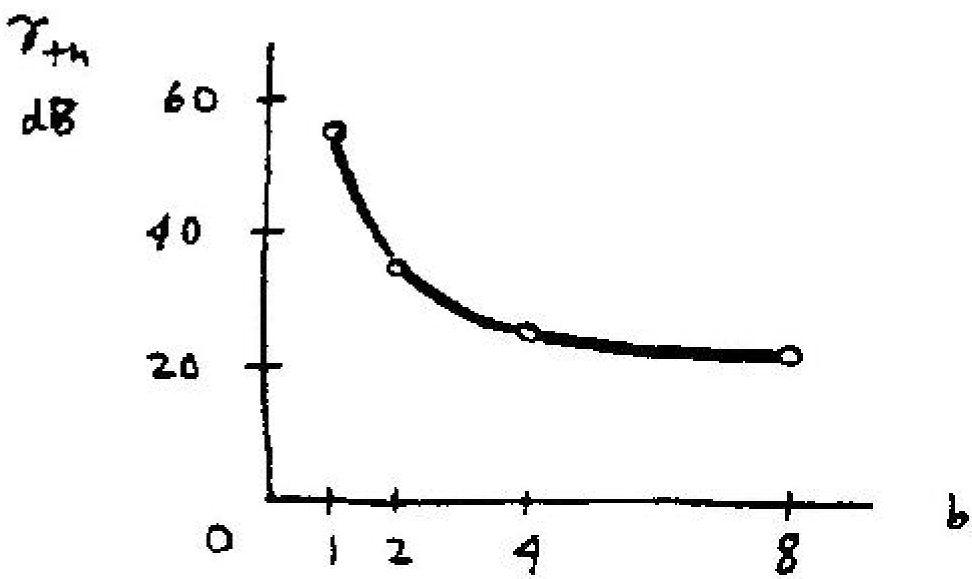
$$\gamma \geq \gamma_{th} \approx 6b(M^2 - 1) \;\;\Rightarrow\;\; M^2 \leq \frac{\gamma}{6b} + 1,\;\; v \leq b$$

Thus, $q_{\max} = M_{\max}^{v_{\max}} = \left(\frac{\gamma}{6b} + 1\right)^{v/2}$

12.2-9

$M^b = 256, \quad \gamma_{th} = 6b(M^2 - 1)$

M	b	γ_{th}	dB
2	8	144	21.6
4	4	360	25.6
16	2	3060	34.9
256	1	3.93×10^5	55.9



12.3-1

Using Eq. (5) and a sine wave $\Rightarrow f_s\Delta \geq |\dot{x}(t)|_{\max} = 2\pi f_m A_m \Rightarrow \Delta \geq \dfrac{2\pi f_m A_m}{f_s}$

If $W = 3$ kHz and normalized input with $A_m = 1 \Rightarrow f_m = 3$ kHz $\Rightarrow \Delta \geq \dfrac{2\pi 3}{30} = 0.628$

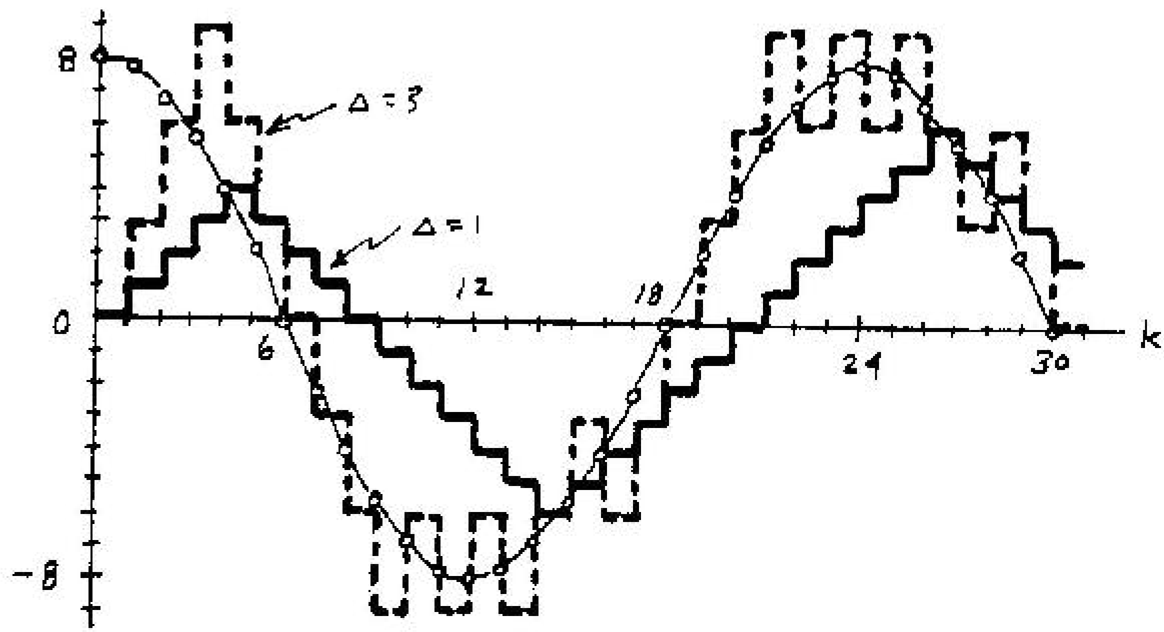
12.3-2

Using Eq. (5) and a sine wave $\Rightarrow f_s\Delta \geq |\dot{x}(t)|_{\max} = 2\pi f_m A_m \Rightarrow A_m \leq \dfrac{f_s\Delta}{2\pi f_m}$

If $W = 1$ kHz and Nyquist sampling $\Rightarrow f_s =$ 20 kHz

$$\Rightarrow A_m \leq \frac{20 \text{ x } 10^3 \text{ x } 0.117}{2\pi \text{ x } 1000} = 0.372$$

12.3-3



Note slope overload when Δ=1.

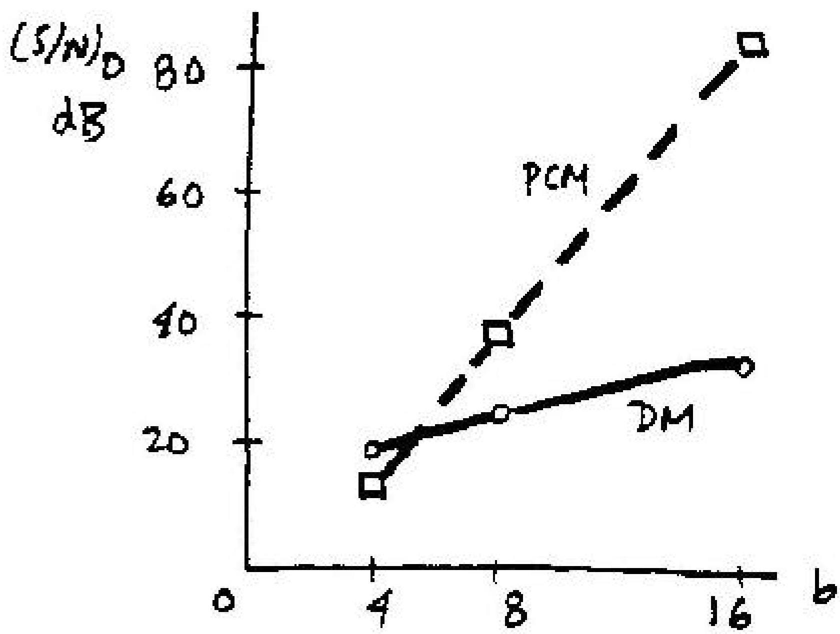
12.3-4

DM: $(S/N)_D = 5.8b^3/(\ln 2b)^2 = 7.6 + 10\log_{10}\dfrac{b^3}{(\ln 2b)^2}$ dB

PCM: $(S/N)_D = 3 \text{ x } 2^{2b} \text{ x } \dfrac{1}{30} = 6.0b - 10$ dB

b	DM	PCM
4	19.3	14
8	25.8	38
16	32.9	86

12.3-4 continued



12.3-5

$f_s = 2Wb \approx 8b$ kHz, $\sigma = \sqrt{S_x} = 1/3$

$$s_{opt} = \frac{f_s \Delta_{opt}}{2\pi\sigma W_{rms}} \approx \ln 2b$$

$$\Delta_{opt} \approx \frac{1.3\pi}{12}\frac{\ln 2b}{b}$$

b	f_s (kHz)	Δ_{opt}
4	32	0.177
8	64	0.118
16	128	0.0737

12.3-6

$f_s\Delta \geq 2\pi f_0 \quad \Rightarrow \quad \Delta \geq 2\pi f_0 / f_s, \;\; f_s = 2Wb$

$$(S/N)_D = \frac{3 f_s}{\Delta^2 W} S_x \leq \frac{3 f_s^2}{4\pi^2 f_0^2 W} S_x = \frac{3}{4\pi^2}\frac{8W^3 b^3}{f_0^2 W} S_x = \frac{6}{\pi^2}\left(\frac{W}{f_0}\right)^2 b^3 S_x$$

12.3-7

$$S_x = \int_{-\infty}^{\infty} G_x(f)df = 2K\int_0^W \frac{df}{f_0^2 + f^2} = \frac{2K}{f_0}\arctan\frac{W}{f_0} \Rightarrow K = \frac{f_0 S_x}{2\arctan(W/f_0)}$$

$$W_{rms}^2 = \frac{1}{S_x}\int_{-\infty}^{\infty} f^2 G_x(f)df = \frac{2K}{S_x}\int_0^W \frac{f^2}{f_0^2 + f^2}df$$

$$= \frac{2K}{S_x}\left[\int_0^W \frac{f_0^2 + f^2}{f_0^2 + f^2}df - \int_0^W \frac{f_0^2}{f_0^2 + f^2}df\right] = \frac{2K}{S_x}\left[W - f_0 \arctan\frac{W}{f_0}\right]$$

$$\text{Thus, } W_{rms} = \left\{\frac{f_0}{\arctan(W/f_0)}\left[W - f_0\arctan\frac{W}{f_0}\right]\right\}^{1/2} = 1.3 \text{ kHz}$$

when $W = 4$ kHz, $f_0 = 0.8$ kHz

12.3-8

$$\Delta = 2\pi\sigma W_{rms} s / f_s, \quad b = f_s / 2W, \quad \sigma = \sqrt{S_x}$$

$$N_g = \frac{W}{f_s}\frac{\Delta^2}{3} = \frac{\pi^2}{6b^3}\left(\frac{W_{rms}}{W}\right)^2 s^2 S_x$$

$$N_g + N_{so} = K\left[s^2 + a(3s+1)e^{-3s}\right] \text{ where } K = \frac{\pi^2}{6b^3}\left(\frac{W_{rms}}{W}\right)^2 S_x, \; a = \frac{16}{9}b^3$$

$$\frac{d}{ds}\left(N_g + N_{so}\right) = K\left[2s + 3ae^{-3s} - 3a(3s+1)e^{-3s}\right] = 0 \Rightarrow 2 - 9ae^{-3s} = 0$$

$$\text{Thus, } s_{opt} = \frac{1}{3}\ln\frac{9a}{2} = \frac{1}{3}\ln 8b^3 = \ln 2b$$

12.3-9

$$\rho_0 = R_x(0)/S_x = 1$$

$$n = 1: \; \rho_0 c_1 = \rho_1 \Rightarrow c_1 = \rho_1, \; G_\rho = [1 - \rho_1^2]^{-1} = 2.77 = 4.4 \text{ dB}$$

$$n = 2: \; \begin{bmatrix} 1 & 0.8 \\ 0.8 & 1 \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 0.8 \\ 0.6 \end{bmatrix} \Rightarrow c_1 = 8/9, \; c_2 = -1/9$$

$$G_p = \left[1 - \frac{8}{9} \text{ x } 0.8 + \frac{1}{9} \text{ x } 0.6\right]^{-1} = 2.81 = 4.5 \text{ dB}$$

12.3-10

$\rho_0 = R_x(0)/S_x = 1$

$n = 1: \ \rho_0 c_1 = \rho_1 \ \Rightarrow \ c_1 = \rho_1, \ G_\rho = [1 - \rho_1^2]^{-1} = 10.26 = 10.1 \text{ dB}$

$$n = 2: \begin{bmatrix} 1 & 0.95 \\ 0.95 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 0.95 \\ 0.90 \end{bmatrix} \ \Rightarrow \ c_1 = 0.9744, \ c_2 = -0.0256$$

$G_p = [1 - 0.9744 \text{ x } 0.95 + 0.0256 \text{ x } 0.9]^{-1} = 10.27 = 10.1 \text{ dB}$

12.3-11

$x[(k-1)T_s] \approx x_q(k-1)$

$$\frac{dx(t)}{dt} \approx \frac{1}{T_s}\left[x(t) - x(t - T_s)\right] \ \Rightarrow \ T_s \left.\frac{dx(t)}{dt}\right|_{(k-1)T_s} \approx x_q(k-1) - x_q(k-2)$$

Thus, take $\tilde{x}_q(k) = x_q(k-1) + [x_q(k-1) - x_q(k-2)] = 2x_q(k-1) - x_q(k-2)$

so $c_1 = 2, \ c_2 = -1$

12.3-12

$\tilde{x}_q(k) = cx_q(k-1) \approx cx(k-1)$ so $\epsilon_q(k) \approx x(k) - cx(k-1)$

Then $\overline{\epsilon^2} = E[x^2(k) - 2cx(k)x(k-1) + c^2x^2(k-1)]$

where $E\left[x^2(k)\right] = E\left[x^2(k-1)\right] = E\left[x^2(t)\right] = S_x$

$E\left[x(k)x(k-1)\right] = E\left[x(t)x(t - T_s)\right] = R_x(T_s)$

so $\overline{\epsilon^2} = S_x - 2cR_x(T_s) + c^2 S_x = \left(1 + c^2\right)S_x - 2cR_x(T_s)$

$$\frac{d\overline{\epsilon^2}}{dc} = 2cS_x - 2R_x(T_s) = 0 \ \Rightarrow \ c = R_x(T_s)/S_x = \rho_1$$

12.3-13

$\tilde{x}_q(k) = c_1 x_q(k-1) + c_2 x_q(k-2) \approx c_1 x(k-1) + c_2 x(k-2)$

so $\epsilon_q(k) \approx x(k) - c_1(k-1) - c_2(k-2)$

$$\overline{\epsilon^2} = E\begin{bmatrix} x^2(k) + c_1^2 x^2(k-1) + c_2^2 x^2(k-2) - 2c_1 x(k)x(k-1) \\ -2c_2 x(k)x(k-2) + 2c_1 c_2 x(k-1)x(k-2) \end{bmatrix}$$

where $E\left[x^2(k-n)\right] = E\left[x^2(t)\right] = S_x$

$E\left[x(k-n)x(k-m)\right] = E\left[x(t - nT_s)x(t - mT_s)\right] = R_x[(m-n)T_s]$

Hence, $\overline{\epsilon^2} = (1 + c_1^2 + c_2^2)S_x + 2c_1(c_2 - 1)R_x(T_s) - 2c_2 R_x(2T_s)$

12.3-13 continued

We want

$$\partial \overline{\varepsilon^2}/\partial c_1 = 2c_1 S_x + 2(c_2 - 1)R_x(T_s) = 0$$

$$\partial \overline{\varepsilon^2}/\partial c_2 = 2c_2 S_x + 2c_1 R_x(T_s) - 2R_x(2T_s) = 0$$

so

$$\left.\begin{aligned} c_1 + \frac{R_x(T_s)}{S_x}c_2 &= \frac{R_x(T_s)}{S_x} \\ \frac{R_x(T_s)}{S_x}c_1 + c_2 &= \frac{R_x(2T_s)}{S_x} \end{aligned}\right\} \Rightarrow \begin{bmatrix} 1 & \rho_1 \\ \rho_1 & 1 \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} \rho_1 \\ \rho_2 \end{bmatrix}$$

Same result as Eq. (16b) with $n = 2$ since $\rho_0 = R_x(0)/S_x = 1$

12.4-1

Assume just music samples and no parity or control information

70 min/CD x 1.4112 Mbits/sec x 60 sec/min = 5.927 Gbits.

12.4-2

981 pages x 2 columns/page x 57 lines/column x 45 characters/line x 7 bits/character =35 Mbits.

Based on problem 12.4-1, a CD can store 5.9 Gbits=5900 Mbytes $\Rightarrow$ 35/5900 x 100% = 0.59%

12.4-3

Assume with the 2 Gbyte hard drive there is no need to store extra control or parity bits.

Music $\Rightarrow$ 1.4112 Mbits/sec x 1 byte/8bits

$$\frac{1\text{ sec}}{1.4112 \times 10^6 \text{bits}} \times 8 \text{ bits/byte} \times 1 \text{ min/60secs} \times 2 \times 10^9 \text{bytes/hard drive} = 189 \text{ minutes}$$

If we do incorporate the same error control used on the CD, the recording time is:

$$\frac{1\text{ sec}}{7350\text{ frames}} \times 1 \text{ min/60 secs} \times 1 \text{ frame/561 bits} \times 8 \text{ bits/byte} \times 2 \times 10^9 \text{ bytes/hard drive} = 65 \text{ minutes.}$$

12.5-1

$r_b = vf_s \geq$ 12 x 2 x 15 kHz = 360 kbps

$N \leq \dfrac{1.544 \text{ Mbps}}{r_b} = 4.2 \quad \Rightarrow \quad N = 4$

$$\left.\begin{array}{l} \text{Digital } B_T \geq \frac{1}{2} \text{ x 1.544 Mbps = 772 kHz} \\ \text{Analog } B_T \geq NW = \text{60 kHz} \end{array}\right\} Eff = \frac{60}{772} = 7.8\%$$

12.5-2

$r_b = vf_s \geq$ 12 x 2 x 15 kHz = 360 kbps

$N \leq \dfrac{2.048 \text{ Mbps}}{r_b} = 5.6 \quad \Rightarrow \quad N = 5$

$$\left.\begin{array}{l} \text{Digital } B_T \geq \frac{1}{2} \text{ x 2.048 Mbps = 1.024 MHz} \\ \text{Analog } B_T \geq NW = \text{75 kHz} \end{array}\right\} Eff = \frac{75}{1024} = 7.3\%$$

12.5-3

From Fig. 12.5-8, if we subtract Transport and Path overhead,
a SONET frame has 9 rows x 86 bytes/row =774 bytes of user data.
Thus a STS-1 has a capacity of 774 bytes/frame x 8 bits/byte x 8000 bits/frame = 49.536 Mbps.
A DS0 line is 64 kpbs $\Rightarrow$ and STS-1 can handle 49536/64 = 774 DS0 lines.

However, in practice, a VT is used to interface DS0 and DS1 lines to a STS-1.
The additional overhead of the VT reduces the number of DS0 inputs to 672 and the
number of DS1 inputs to 28. See Bellamy (1991) for more information.

12.5-4

(600 dots/inch)^2 x (8 inches x 11 inches)/page = 31,680 kbits/page.
2 BRI channel $\Rightarrow$ 128 kpbs $\Rightarrow$ 31,680 kbits/128 kbps = 247 seconds/page.
Obviously, some image compression is needed for this to be practical!

12.5-5

(600 dots/inch)^2 x (8 inches x 11 inches)/page = 31,680 kbits/page.
1-56 kbps channel $\Rightarrow$ 31,680 kbits/56 kbps = 566 seconds/page.

Chapter 13

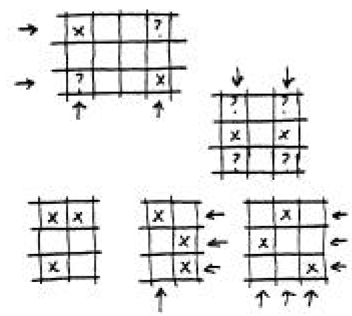
13.1-1

$$P(\text{no errors}) = P(0,4) = (1-0.1)^4 = 0.6561$$
$$P(\text{detected errors}) = P(1,4) = 4\times 0.1\times 0.9^3 = 0.2916$$
$$P(\text{undetedcte d errors}) = 1 - P(0,4) - P(1,4) = 0.0523$$

13.1-2

$$P(\text{no errors}) = P(0,9) = (1-0.05)^9 = 0.5971$$
$$P(\text{detected errors}) = P(1,9) = 9\times 0.05\times 0.95^8 = 0.2985$$
$$P(\text{undetedcte d errors}) = 1 - P(0,9) - P(1,9) = 0.0712$$

13.1-3



(a) Two errors not in the same row or column yields 4 intersections as possible error locations. Two errors in the same row (or column) yields two columns (or rows) as possible error locations.

(b) L shaped error pattern yields no parity failures and is undetectable. Other patterns yield 4 or 6 parity failures and are detectable.

13.1-4

$$(31,26)\ \ t=1:\ \ Q\left(\sqrt{\frac{52}{31}g_b}\right) \le \left(\frac{1}{30}\times 10^{-4}\right)^{1/2} \Rightarrow g_b \ge \frac{31}{52}\times 2.9^2 = 7.0\text{dB}$$

$$(31,21)\ \ t=2:\ \ Q\left(\sqrt{\frac{42}{31}g_b}\right) \le \left(\frac{2}{30\times 29}\times 10^{-4}\right)^{1/3} \Rightarrow g_b \ge \frac{31}{42}\times 2.53^2 = 6.7\text{dB}$$

$$(31,16)\ \ t=3:\ \ Q\left(\sqrt{\frac{32}{31}g_b}\right) \le \left(\frac{3\times 2}{30\times 29\times 28}\times 10^{-4}\right)^{1/4} \Rightarrow g_b \ge \frac{31}{32}\times 2.25^2 = 6.9\text{dB}$$

$$\text{Uncoded}\quad :\ Q\left(\sqrt{2g_b}\right) \le 10^{-4} \Rightarrow g_b \ge \frac{1}{2}\times 3.73^2 = 8.4\text{dB}$$

Thus, use (31, 21) code to save 1.7 dB.

13.1-5

$$(31,26)\ \ t=1:\ \ Q\left(\sqrt{\frac{52}{31}\gamma_b}\right)\le\left(\frac{1}{30}\times10^{-6}\right)^{1/2}\Rightarrow\gamma_b\ge\frac{31}{52}\times3.6^2=8.9\text{dB}$$

$$(31,21)\ \ t=2:\ \ Q\left(\sqrt{\frac{42}{31}\gamma_b}\right)\le\left(\frac{2}{30\times29}\times10^{-6}\right)^{1/3}\Rightarrow\gamma_b\ge\frac{31}{42}\times3.0^2=8.2\text{dB}$$

$$(31,16)\ \ t=3:\ \ Q\left(\sqrt{\frac{32}{31}\gamma_b}\right)\le\left(\frac{3\times2}{30\times29\times28}\times10^{-6}\right)^{1/4}\Rightarrow\gamma_b\ge\frac{31}{32}\times2.65^2=8.3\text{dB}$$

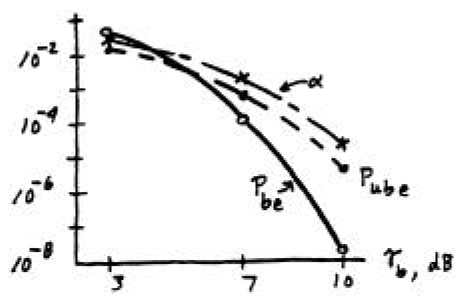
$$\text{Uncoded}\quad:\ Q\left(\sqrt{2\gamma_b}\right)\le10^{-6}\Rightarrow\gamma_b\ge\frac{1}{2}\times4.76^2=10.5\text{dB}$$

Thus, use (31, 21) code to save 2.3 dB.

13.1-6

$(31,26)\ \ t=1:$

$$P_{ube}=Q\left(\sqrt{2\gamma_b}\right),\alpha=Q\left(\sqrt{\frac{52}{31}\gamma_b}\right),P_{be}=30\alpha^2$$

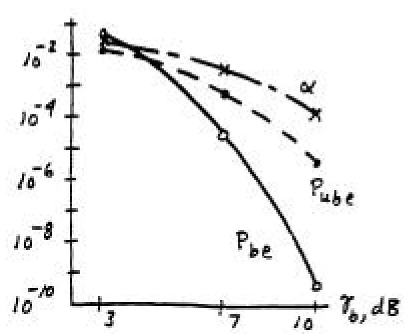


γ_b	dB	P_{ube}	α	P_{be}
2	3	2.3×10^{-2}	3.5×10^{-2}	3.7×10^{-2}
5	7	8.5×10^{-4}	2×10^{-3}	1.2×10^{-4}
10	10	4×10^{-6}	2.2×10^{-5}	1.5×10^{-8}

13.1-7

$(31,21)\ \ t=2:$

$$P_{ube}=Q\left(\sqrt{2\gamma_b}\right),\alpha=Q\left(\sqrt{\frac{42}{31}\gamma_b}\right),P_{be}=\frac{30\times29}{2}\alpha^3$$



γ_b	DB	P_{ube}	α	P_{be}
2	3	2.3×10^{-2}	5×10^{-2}	5.4×10^{-2}
5	7	8.5×10^{-4}	4.7×10^{-3}	4.5×10^{-5}
10	10	4×10^{-6}	1.2×10^{-4}	7.5×10^{-10}

13.1-8

Coded transmission has $r_b / r = R_c'$ and $Q(\sqrt{2R_c' g_b}) = a$

(12,11) $l = 1$:

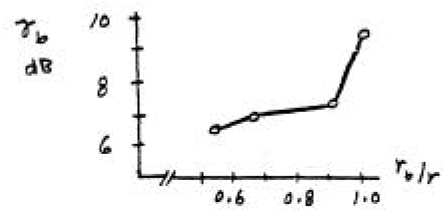
$$a = \left(\frac{1}{11}\times 10^{-5}\right)^{1/2} = 9.5\times 10^{-4}, R_c' = \frac{11}{12}(1-12a) = 0.906, g_b = 3.12^2 / 2R_c' = 7.3\text{dB}$$

(15,11) $l = 2$:

$$a = \left(\frac{2}{14\times 13}\times 10^{-5}\right)^{1/3} = 4.8\times 10^{-3}, R_c' = \frac{11}{15}(1-15a) = 0.681, g_b = 2.6^2 / 2R_c' = 7.0\text{dB}$$

(16,11) $l = 3$:

$$\alpha = \left(\frac{3\times 2}{15\times 14\times 13}\times 10^{-5}\right)^{1/4} = 1.22\times 10^{-2}, R_c' = \frac{11}{16}(1-16\alpha) = 0.554, \gamma_b = 2.25^2 / 2R_c' = 6.6\text{dB}$$



Uncoded transmission $r_b / r = 1$

$$Q\left(\sqrt{2g_b}\right) = 10^{-5} \Rightarrow g_b \approx \frac{1}{2}\times 4.27^2 = 9.6\text{dB}$$

13.1-9

Coded transmission has $r_b / r = R_c'$ and $Q(\sqrt{2R_c' g_b}) = a$

(12,11) $l = 1$:

$$a = \left(\frac{1}{11}\times 10^{-6}\right)^{1/2} = 3.0\times 10^{-4}, R_c' = \frac{11}{12}(1-12a) = 0.913, g_b = 3.45^2 / 2R_c' = 8.1\text{dB}$$

(15,11) $l = 2$:

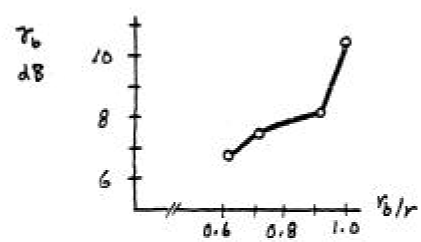
$$a = \left(\frac{2}{14\times 13}\times 10^{-6}\right)^{1/3} = 2.2\times 10^{-3}, R_c' = \frac{11}{15}(1-15a) = 0.709, g_b = 2.85^2 / 2R_c' = 7.6\text{dB}$$

(16,12) $l = 3$:

$$a = \left(\frac{3\times 2}{15\times 14\times 13}\times 10^{-6}\right)^{1/4} = 6.9\times 10^{-3}, R_c' = \frac{11}{16}(1-16a) = 0.612, g_b = 2.45^2 / 2R_c' = 6.9\text{dB}$$

Uncoded transmission $r_b / r = 1$

$$Q\left(\sqrt{2g_b}\right) = 10^{-6} \Rightarrow g_b \approx \frac{1}{2}\times 4.77^2 = 10.6\text{dB}$$



13.1-10

$$\alpha = Q(\sqrt{2\times 4}) = 2.3\times 10^{-3}, l = 2, P_{be} = \frac{14\times 13}{2}\alpha^3 \approx 10^{-6}$$

$$\left.\begin{aligned} T_w &= n/r = 30\ \mu\text{s}, \\ t_d &\geq 45\text{km}/3\times 10^5\text{km/s} = 150\ \mu\text{s} \end{aligned}\right\} \Rightarrow N \geq \frac{2t_d}{T_w} = 10$$

$$p = n\alpha \approx 0.035, R_c' = \frac{11}{15}\frac{1-0.035}{1+9\times 0.035} = 0.538 \Rightarrow r_b = 269\text{kbps}$$

13.1-11

$$\alpha = Q(\sqrt{2\times 4}) = 2.3\times 10^{-3}, l = 3, P_{be} = \frac{15\times 14\times 13}{3\times 2}\alpha^4 \approx 10^{-8}$$

$$\left.\begin{aligned} T_w &= n/r = 32\ \mu\text{s}, \\ t_d &\geq 45\text{km}/3\times 10^5\text{km/s} = 150\ \mu\text{s} \end{aligned}\right\} \Rightarrow N \geq \frac{2t_d}{T_w} = 9.38 \Rightarrow N = 10$$

$$p = n\alpha \approx 0.037, R_c' = \frac{11}{16}\frac{1-0.037}{1+9\times 0.037} = 0.497 \Rightarrow r_b = 248\text{kbps}$$

13.1-12

$$P_{be} = k\alpha^2 \leq 10^{-6} \Rightarrow \alpha < 10^{-3} \text{ and } p = (k+1)\alpha << 1 \quad \text{if } k < 100$$

$$\left.\begin{aligned} D &\geq 2\times 18\text{km}/3\times 10^5\text{km/s} = 120\ \mu\text{s} \\ T_w &= (k+1)/r = 100(k+1)\ \mu\text{s} \end{aligned}\right\} \Rightarrow D/T_w \geq \frac{1.2}{k+1}$$

$$R_c' \leq \frac{k}{k+1}\times\frac{1-p}{1+D/T_w} \approx \frac{k}{k+1+1.2} \geq \frac{7200}{10{,}000} \Rightarrow k \geq 5.66 \Rightarrow k = 6$$

Then,

$$R_c' \approx 0.732, \alpha = Q(\sqrt{2R_c'\gamma_b}) \leq (\frac{1}{6}\times 10^{-6})^{1/2} = 4.1\times 10^{-4}$$

So, $\gamma_b \geq 3.35^2/2R_c' = 8.8\text{dB}$

13.1-13

$$P_{be} = k\alpha^2 \leq 10^{-6} \Rightarrow \alpha < 10^{-3} \text{ and } p = (k+1)\alpha << 1 \quad \text{if } k < 100$$

$$\left.\begin{aligned} D &\geq 2\times 60\text{km}/3\times 10^5\text{km/s} = 400\ \mu\text{s} \\ T_w &= (k+1)/r = 100(k+1)\ \mu\text{s} \end{aligned}\right\} \Rightarrow D/T_w \geq \frac{4}{k+1}$$

$$R_c' \leq \frac{k}{k+1}\times\frac{1-p}{1+D/T_w} \approx \frac{k}{k+1+4} \geq \frac{7200}{10{,}000} \Rightarrow k \geq 12.9 \Rightarrow k = 13$$

Then, $R_c' \approx 0.737, \alpha = Q(\sqrt{2R_c'\gamma_b}) \leq (\frac{1}{13}\times 10^{-6})^{1/2} = 2.8\times 10^{-4}$

So, $\gamma_b \geq 3.45^2/2R_c' = 9.1\text{dB}$

13.1-14

$$\overline{m} = 1(1-p) + (1+N)p(1-p) + (1+2N)p^2(1-p) + (1+3N)p^3(1-p) + \cdots\cdots$$
$$= (1-p)\left[1 + (1+N)p + (1+2N)p^2 + (1+3N)p^3 + \cdots\cdots\right]$$
$$= (1-p)\left[(1 + p + p^2 + p^3 + \cdots) + Np(1 + 2p + 3p^2 + \cdots)\right]$$

But $(1 + p + p^2 + p^3 + \cdots) = (1-p)^{-1}$,

and $(1 + 2p + 3p^2 + \cdots) = (1-p)^{-2}$

Thus, $\overline{m} = (1-p)\left[\frac{1}{1-p} + \frac{Np}{(1-p)^2}\right] = 1 + \frac{Np}{1-p}$

13.1-15

A word has a detected error and must be retransmitted if the number of bit error is $t < i \le l$, so

$$p = \sum_{i=t+1}^{l} P(i,n) \approx P(t+1,n) = \binom{n}{t+1}\alpha^{t+1}(1-\alpha)^{n-t-1} \approx \binom{n}{t+1}\alpha^{t+1}$$

With $d_{\min} = 4, l = 2$ and $t = 1$ so $p \approx \frac{n(n-1)}{2}\alpha^2$

13.1-16

(a) $p = P(t+1,n) = \binom{n}{t+1}\alpha^{t+1}(1-\alpha)^{n-t-1} \approx \binom{n}{t+1}\alpha^{t+1}$

$$P_{we} = \sum_{i=t+2}^{n} P(i,n) \approx P(t+2,n) = \binom{n}{t+2}\alpha^{t+2}(1-\alpha)^{n-t-2} \approx \binom{n}{t+2}\alpha^{t+2}$$

$$\dot{P}_{be} = \frac{t+2}{n}P_{we} \approx \frac{t+2}{n}\frac{n(n-1)\cdots(n-t-1)}{(t+2)!}\alpha^{t+2} = \binom{n-1}{t+1}\alpha^{t+2}$$

(b) $d_{\min} = 2t + 2 = 8 \Rightarrow t = 3, R_c' \approx \frac{12}{24} = \frac{1}{2}$, assuming $p \ll 1$

$$P_{be} \approx \frac{23\times22\times21\times20}{4\times3\times2}\alpha^5 = 10^{-5} \Rightarrow \alpha = 1.62\times10^{-2}$$

$$p \approx \frac{24\times23\times22\times21}{4\times3\times2}\alpha^4 = 7.3\times10^{-4} \ll 1 \quad \text{as assumed}$$

$$\alpha = Q\left(\sqrt{2\times\frac{1}{2}\gamma_b}\right) = 1.62\times10^{-2} \Rightarrow \gamma_b = 2.15^2 = 4.62$$

Uncoded: $P_{be} = Q(\sqrt{2\gamma_b}) \approx 1.3\times10^{-3}$

13.2-1

(a) Let n_u = number of 1s in U, n_v = number of 1s in V, and
n_{uv} = number of 1s position in U and V. Then,

$$W(U)+W(V)=n_u+n_v$$

$$d(U,V)=W(U+V)=n_u+n_v-n_{uv}\le W(U)+W(V)$$

(b) $U+V=X+Y+Y+Z$

$$\begin{aligned}&=(x_1\oplus y_1\oplus y_1\oplus z_1,\cdots),\text{ where } y_1\oplus y_1=0,\text{ etc.}\\&=(x_1\oplus z_1,\cdots)\\&=X+Z\end{aligned}$$

Thus,

$$d(U,V)=W(U+V)=W(X+Z)=d(X,Z)$$

and $\quad W(U)+W(V)=W(X+Y)+W(Y+Z)=d(X,Y)+d(Y,Z)$

so $\quad d(X,Z)\le W(U)+W(V)=d(X,Y)+d(Y,Z)$

13.2-2

$d(X,Y)=i\le l,$

$d(X,Y)+d(Y,Z)\ge d(X,Z)\ge d_{\min}\ge l+1$

so,

$l+1\le d_{\min}\le i+d(Y,Z)\le l+d(Y,Z)\Rightarrow d(Y,Z)\ge 1$

Thus, Y cannot be a vector in the code, and the errors are detectable.

13.2-3

$d(X,Y)=i\le t,$

$d(X,Y)+d(Y,Z)\ge d(X,Z)\ge d_{\min}\ge 2t+1$

so,

$2t+1\le d_{\min}\le i+d(Y,Z)\le t+d(Y,Z)$ and $d(Y,Z)\ge t+1>d(X,Y)$

Thus, Y is closer to X than to any other valid code vector, and the errors can be corrected.

13.2-4

$n = 3$, $k = 1$, $q = 2$

$$H^T=\begin{bmatrix}1 & 1\\ \hline 1 & 0\\ 0 & 1\end{bmatrix},\quad S=EH^T\Rightarrow$$

E	S
1 0 0	1 1
0 1 0	1 0
0 0 1	0 1

13.2-4 continued

Y	S	$\hat{E}$	$Y+\hat{E}$
0 0 0	0 0	0 0 0	0 0 0
0 0 1	0 1	0 0 1	0 0 0
0 1 0	1 0	0 1 0	0 0 0
0 1 1	1 1	1 0 0	1 1 1
1 0 0	1 1	1 0 0	0 0 0
1 0 1	1 0	0 1 0	1 1 1
1 1 0	0 1	0 0 1	1 1 1
1 1 1	0 0	0 0 0	1 1 1

13.2-5

$$H^T = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ \hline 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad S = EH^T \Rightarrow$$

$\hat{E}$	S
1 0 0 0 0 0	1 1 0
0 1 0 0 0 0	0 1 1
0 0 1 0 0 0	1 0 1
0 0 0 1 0 0	1 0 0
0 0 0 0 1 0	0 1 0
0 0 0 0 0 1	0 0 1

13.2-6

$$H^T = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \\ \hline 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad S = EH^T \Rightarrow$$

E	S	
0 0 0 0 0	0 0	⇒ Correctly indicates no errors.
1 0 0 0 0	1 1	
0 1 0 0 0	0 1	Different Es yield
0 0 1 0 0	1 0	} same S, so error
0 0 0 1 0	1 0	} correction is impossible.
0 0 0 0 1	0 1	

13.2-7

$$P = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix},$$

$$c_1 = m_5 \oplus m_6 \oplus m_7 \oplus m_8 \oplus m_9 \oplus m_{10} \oplus m_{11}$$
$$c_2 = m_2 \oplus m_3 \oplus m_4 \oplus m_8 \oplus m_9 \oplus m_{10} \oplus m_{11}$$
$$c_3 = m_1 \oplus m_3 \oplus m_4 \oplus m_6 \oplus m_7 \oplus m_{10} \oplus m_{11}$$
$$c_4 = m_1 \oplus m_2 \oplus m_4 \oplus m_5 \oplus m_7 \oplus m_9 \oplus m_{11}$$

	S	Error bit
	0 0 0 0	None
P {	0 0 1 1	1
	0 1 0 1	2
	0 1 1 0	3
	0 1 1 1	4
	1 0 0 1	5
	1 0 1 0	6
	1 0 1 1	7
	1 1 0 0	8
	1 1 0 1	9
	1 1 1 0	10
	1 1 1 1	11
	1 0 0 0	12
	0 1 0 0	13
	0 0 1 0	14
	0 0 0 1	15

13.2-8

(a) $\left.\begin{array}{l} z^q - 1 \ge n \\ q = n - k \end{array}\right\} \quad \dfrac{z^n}{n+1} \ge z^k = 64 \quad \Rightarrow n \ge 10$ (by trial and error)

Take n = 10, so q = 4 and $(q+n) \times z^q = 224$ bits

(b) Any 6×4 matrix whose rows are all different and contain at least 2 ones.
Example:

$$P = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

13.2-9

(a) $\left.\begin{array}{l} z^q - 1 \ge n \\ q = n - k \end{array}\right\} \quad \dfrac{z^n}{n+1} \ge z^k = 256 \quad \Rightarrow n \ge 12$ (by trial and error)

Take n = 12, so q = 4 and $(q+n) \times z^q = 256$ bits

(b) Any 8×4 matrix whose rows are all different and contain at least 2 ones.
Example:

$$P = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

13.2-10

$$a_{ij} = \sum_{m=1}^{n} g_{im} h_{mj}{}^T \quad (\text{mod } 2), \quad \text{where}$$

$g_{im} = m^{th}$ element of i^{th} row of G $\qquad h_{mj}{}^T = m^{th}$ element of j^{th} row of H^T

$$= \begin{cases} 0 & 1 \le m \le i-1 \\ 1 & m = i \\ 0 & i+1 \le m \le k \\ p_{i(m-k)} & k+1 \le m \le n \end{cases} \qquad = \begin{cases} p_{mj} & 1 \le m \le n-k \\ 0 & n-k+1 \le m \le j+k-1 \\ 1 & m = j+k \\ 0 & j+k+1 \le m \le n \end{cases}$$

13.2-10 continued

Thus, $a_{ij} = 1 \cdot p_{ij} \oplus p_{i(j+k-k)} \cdot 1 = p_{ij} \oplus p_{ij} = 0$

13.2-11

(a) The binary number $s_1 s_2 s_3$ equals the error location,
i.e.,
$000 \Rightarrow$ no error
$001 \Rightarrow 1^{st}$ bit
$010 \Rightarrow 2^{nd}$ bit
etc.

	s_1	s_2	s_3	E						
	0	0	0	0	0	0	0	0	0	0
	0	0	1	1	0	0	0	0	0	0
	0	1	0	0	1	0	0	0	0	0
	0	1	1	0	0	1	0	0	0	0
H^T	1	0	0	0	0	0	1	0	0	0
	1	0	1	0	0	0	0	1	0	0
	1	1	0	0	0	0	0	0	1	0
	1	1	1	0	0	0	0	0	0	1

(b)

$$s_1 = x_4 \oplus x_5 \oplus x_6 \oplus x_7 = 0$$
$$s_2 = x_2 \oplus x_3 \oplus x_6 \oplus x_7 = 0$$
$$s_3 = x_1 \oplus x_3 \oplus x_5 \oplus x_7 = 0$$

Since $x_1, x_2,$ and x_4 appear only once, they must be the check bits.
Thus,

$$X = (c_1 \quad c_2 \quad m_1 \quad c_3 \quad m_2 \quad m_3 \quad m_4)$$

$$\Rightarrow \begin{cases} c_3 = m_2 \oplus m_3 \oplus m_4 \\ c_2 = m_1 \oplus m_3 \oplus m_4 \\ c_1 = m_1 \oplus m_2 \oplus m_4 \end{cases}$$

13.2-12

(a)

M	C	w	M	C	w
0 0 0 0	0 0 0 0	0	1 0 0 0	1 0 1 1	4
0 0 0 1	0 1 1 1	4	1 0 0 1	1 1 0 0	4
0 0 1 0	1 1 0 1	4	1 0 1 0	0 1 1 0	4
0 0 1 1	1 0 1 0	4	1 0 1 1	0 0 0 1	4
0 1 0 0	1 1 1 0	4	1 1 0 0	0 1 0 1	4
0 1 0 1	1 0 0 1	4	1 1 0 1	0 0 1 0	4
0 1 1 0	0 0 1 1	4	1 1 1 0	1 0 0 0	4
0 1 1 1	0 1 0 0	4	1 1 1 1	1 1 1 1	8

(b) Note from check-bit equations in Example 13.2-1 that
$c_1 \oplus c_2 \oplus c_3 = m_2$, so $m_1 \oplus m_2 \oplus m_3 \oplus m_4 \oplus c_1 \oplus c_2 \oplus c_3 = m_1 \oplus m_3 \oplus m_4$
Thus, $c_4 = m_1 \oplus m_3 \oplus m_4$

(c) Form $d = \sum_{i=1}^{n} y_i \pmod 2$ so $\begin{cases} d = 0 \Rightarrow \text{no errors or even number of errors} \\ d = 1 \Rightarrow \text{odd number of errors} \end{cases}$

13.2-12 continued

If $S = (000)$ and $d = 0$, assume no errors so $X = Y$,

If $S \neq (000)$ and $d = 1$, assume single error so $X = Y + \hat{E}$,
If $S \neq (000)$ and $d = 0$, assume detectedbut uncor rectable errors.

13.2-13

```
   G        111
11101 ) 1010000
         11101
          10010
          11101
           11110
           11101
            0011
```

$\Rightarrow \quad Q_M(p) = p^2 + p + 1,\ \ C(p) = 0 + 0 + p + 1 \quad \Rightarrow \quad X = (101 \vdots 0011)$

$X' = (0\ \ 1\ \ 0\ \ 0\ \ 1\ \ 1\ \ 1) \qquad Y(p) = 0 + p^5 + 0 + 0 + p^2 + p + 1$

```
            011
11101 ) 0100111
         11101
          11101
          11101
           0000
```

$\Rightarrow S(p) = 0$

13.2-14

```
   G        100
10111 ) 1010000
         10111
           1100
```

$\Rightarrow \quad Q_M(p) = p^2 + 0 + 0,\ \ C(p) = p^3 + p^2 + 0 + 0 \quad \Rightarrow \quad X = (101 \vdots 1100)$

$X' = (0\ \ 1\ \ 1\ \ 1\ \ 0\ \ 0\ \ 1) \qquad Y(p) = 0 + p^5 + p^4 + p^3 + 0 + 0 + 1$

13.2-14 continued

$$
\begin{array}{r}
011 \\
10111 \overline{)\,0111001} \\
\underline{10111} \\
10111 \\
\underline{10111} \\
0000
\end{array}
$$

$\Rightarrow S(p) = 0$

13.2-15

$$
\begin{array}{r}
1011 \\
\overbrace{1011}^{G} \overline{)\,1000000} = p^6 \\
\underline{1011} \\
1100 \\
\underline{1011} \\
1110 \\
\underline{1011} \\
101 = R_1
\end{array}
$$

$$
\begin{array}{r}
101 \\
1011 \overline{)\,0100000} = p^5 \\
\underline{1011} \\
1100 \\
\underline{1011} \\
111 = R_2
\end{array}
$$

Similarly, $R_3 = 110$ and $R_4 = 011$

$$
P = \begin{bmatrix} \underline{R_1} \\ \underline{R_2} \\ \underline{R_3} \\ R_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}
$$
, which is the P matrix in Example 13.2-1.

13.2-16

$$\begin{array}{r} 1110 \\ \overbrace{1101}^{G})\overline{1000000} = p^6 \\ \underline{1101} \\ 1010 \\ \underline{1101} \\ 1110 \\ \underline{1101} \\ 110 = R_1 \end{array}$$

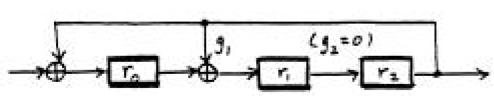
$$\begin{array}{r} 0111 \\ 1101)\overline{0100000} = p^5 \\ \underline{1101} \\ 1010 \\ \underline{1101} \\ 1110 \\ \underline{1101} \\ 011 = R_2 \end{array}$$

Similarly, $R_3 = 111$ and $R_4 = 101$

$$P = \begin{bmatrix} R_1 \\ \hline R_2 \\ \hline R_3 \\ \hline R_4 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$, same rows as P matrix in Example 13.2-1, but different order.

13.2-17

$p^3 M(p) = 1100000$

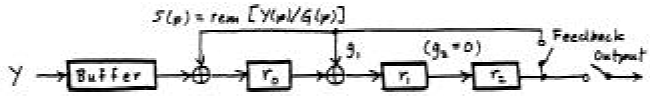


Input bit m	Register before shift r_0	r_1	r_2	$r_0' = m \oplus r_2$	$r_1' = r_0 \oplus r_2$	$r_2' = r_1$	Output r_2	$Q_M(p)$
1	0	0	0	1	0	0	0	0
1	1	0	0	1	1	0	0	0
0	1	1	0	0	1	1	0	0
0	0	1	1	1	1	1	1	p^3
0	1	1	1	1	0	1	1	p^2
0	1	0	1	1	0	0	1	p
0	1	0	0	0	1	0	0	0

$C(p) = 0 + p + 0$

13.2-18

Initialize register to (00……0), close feedback, open output switch, and shift y into register. After 7 shift cycles, open feedback switch and close output shift, shift out S(p) from register in 3 shift cycles.



Input bit y	Register before shift r_0	r_1	r_2	$r_0^1 = y \oplus r_2$	$r_1' = r_0 \oplus r_2$	$r_2' = r_1$
1	0	0	0	1	0	0
1	1	0	0	1	1	0
0	1	1	0	0	1	1
0	0	1	1	1	1	1
0	1	1	1	1	0	1
1	1	0	1	1	0	1
0	0	0	0	0	0	0 $S(p) = 0 + 0 + 0$

13.2-19

$P_{be} = 1 \text{ x } 10^{-5} = Q(\sqrt{2\gamma_b})$ and $d_{\min} = 3$

From Table T.6 $\Rightarrow \sqrt{2\gamma_b} = 4.3 \Rightarrow \gamma_b = 9.2$

(a) For (7,4) code $\Rightarrow n = 7, k = 4,\ R_c = 4/7,\ t = 1$

$\binom{n}{i} = \dfrac{n!}{i!(n-i)!} \Rightarrow \binom{n-1}{t} = \dfrac{6!}{1!(6-1)!} = 6$

$P_{be} = \binom{n-1}{t}[Q(\sqrt{2R_c\gamma_b})]^{t+1} = 6[Q(\sqrt{8/7 \text{ x } 9.68})]^2 = 2.2 \text{ x } 10^{-6}$

$\alpha = Q(\sqrt{2R_c\gamma_b}) = Q(\sqrt{8/7 \text{ x } 9.2}) = 6 \text{ x } 10^{-4}$

(b) (15,11) code $\Rightarrow d_{\min} = 3 \geq 2t + 1 \Rightarrow t = 1$ and $R_c = 11/15$

$\binom{n-1}{t} = \dfrac{14!}{1!(14-1)!} = 14 \Rightarrow P_{be} = 14Q(\sqrt{22/15 \text{ x } 9.2})]^2 = 3.2 \text{ x } 10^{-7}$

$\alpha = Q(\sqrt{2R_c\gamma_b}) = Q(\sqrt{22/15 \text{ x } 9.68}) = 1.5 \text{ x } 10^{-4}$

13.2-19 continued

(c) (31,26) code $\Rightarrow R_c = 26/31,\ t = 1$

$\binom{n-1}{t} = \dfrac{30!}{1!(30-1)!} = 30 \Rightarrow P_{be} = 30Q(\sqrt{52/31 \text{ x } 9.2})]^2 = 6.1 \text{ x } 10^{-8}$

$\alpha = Q(\sqrt{2R_c\gamma_b}) = Q(\sqrt{52/31 \text{ x } 9.2}) = 4.5 \text{ x } 10^{-5}$

13.2-20

With $\gamma_c = R_c\gamma_b$, α=Q($\sqrt{2\gamma_c}$) =Q($\sqrt{2R_c\gamma_b}$) $\Rightarrow$ if γ_c fixed $\Rightarrow R_c\gamma_b$ fixed $\Rightarrow \alpha$ fixed

Let's assume α=1 x 10^{-2}

With a (7,4) code and $t = 1 \Rightarrow P_{be} = 6Q^2 = 6 \text{ x } 10^{-4}$

The percent reduncancy of the (7,4) code is 3/7 x 100%=43%

With a (31,26) code and $t = 1 \Rightarrow P_{be} = 30Q^2 = 3 \text{ x } 10^{-3}$

The percent reduncancy of the (31,26) code is 5/31 x 100%=16%

$\Rightarrow P_{be}(7,4) < P_{be}(31,26)$

Because the (31,26) code has less redundancy than the (7,4) code, we would therefore expect $P_{be(7,4)} < P_{be(31,26)}$.

13.2-21

Uncoded system: $P_{ube} = 1 \text{ x } 10^{-5} = Q(\sqrt{2\gamma_b}) \Rightarrow \gamma_b = 9.2$

If we transmit at 3 times data rate $\Rightarrow R_c = 1/3 \Rightarrow P_e = Q(\sqrt{2 \text{ x } (1/3) \text{ x } 9.2}) = 7 \text{ x } 10^{-3}$

An error occurs if 2 or 3 transmissions are received in error $\Rightarrow P_{be} = \sum_{j=2}^{3} \binom{3}{j} p^j (1-p)^{3-j}$

with $p = 7 \text{ x } 10^{-3}$

$\binom{3}{j} = \dfrac{3!}{j!(3-j)!} \Rightarrow \binom{3}{j}_{j=2} = 3$ and $\binom{3}{j}_{j=3} = 1$

$\Rightarrow P_e = 3(7 \text{ x } 10^{-3})^2(1-7 \text{ x } 10^{-3})^1 + (7 \text{ x } 10^{-3})^3(1-7 \text{ x } 10^{-3})^0$

$= 1.46 \text{ x } 10^{-4} = P_e$ (triple reduncancy)

versus the case from problem 13.2-19 where $P_e(7,4) = 2.2 \text{ x } 10^{-6}$ and $P_e(15,11) = 3.2 \text{ x } 10^{-7}$

13.2-22

$$Y=[0100101] \text{ and with a (7,4) code } H=\begin{bmatrix}1110100\\0111010\\1101001\end{bmatrix} \Rightarrow S=YH^T=[0100101]\begin{bmatrix}101\\111\\110\\011\\100\\010\\001\end{bmatrix}=[010]$$

Using Table 13.2-2 $S=[010] \Rightarrow$ error in 6th bit $\Rightarrow$ X=$[0100111]$

(b) $$Y=[0111111] \Rightarrow S=YH^T=[0111111]\begin{bmatrix}101\\111\\110\\011\\100\\010\\001\end{bmatrix}=[101] \Rightarrow X=[1111111]$$

(c) $$Y=[1010111] \Rightarrow S=YH^T=[1010111]\begin{bmatrix}101\\111\\110\\011\\100\\010\\001\end{bmatrix}=[100] \Rightarrow X=[1010011]$$

(d) $$Y=[1101000] \Rightarrow S=YH^T=[1101000]\begin{bmatrix}101\\111\\110\\011\\100\\010\\001\end{bmatrix}=[001] \Rightarrow X=[1101001]$$

13.2-23

$$G(p) = p^{12} + p^{11} + p^3 + p^2 + p + 1$$

ASCII character "E" $= 1000101 \Rightarrow M(p) = p^6 + p^2 + 1 \Rightarrow p^{12}M(p) = p^{18} + p^{14} + p^{12}$

$$C(p) = \text{rem}\left[\frac{p^q M(p)}{G(p)}\right] \Rightarrow$$

$$\begin{array}{r|l} & p^6 + p^5 + p^4 + p^3 + 1 \\ \hline p^{12} + p^{11} + p^3 + p^2 + p + 1 & p^{18} + p^{14} + p^{12} \\ & p^{18} + p^{17} + p^9 + p^8 + p^7 + p^6 \\ \cline{2-2} & p^{17} + p^{14} + p^{12} + p^9 + p^8 + p^7 + p^6 \\ & p^{17} + p^{16} + \quad + p^8 + p^7 + p^6 + p^5 \\ \cline{2-2} & p^{16} + p^{14} + p^{12} + p^9 + \quad p^5 \\ & p^{16} + p^{15} + \quad p^7 + p^6 + p^5 + p^4 \\ \cline{2-2} & p^{15} + p^{14} + p^{12} + p^9 + p^7 + p^6 + \quad p^4 \\ & p^{15} + p^{14} + \quad p^6 + p^5 + p^4 + p^3 \\ \cline{2-2} & p^{12} + p^9 + p^7 + p^5 + p^3 \\ & p^{12} + p^{11} + \quad p^3 + p^2 + p + 1 \\ \cline{2-2} & p^{11} + p^9 + p^7 + p^5 + p^2 + p + 1 = C(p) \end{array}$$

$$X(p) = p^{12}M(p) + C(p) = p^{18} + p^{14} + p^{12} + p^{11} + p^9 + p^7 + p^5 + p^2 + p + 1$$

$$X = [1000101101010100111]$$

13.2-24

X is transmitted $\Rightarrow X \Rightarrow Y$.

$$\text{rem}\left[\frac{Y(p)}{G(p)}\right] = 0 \Rightarrow \text{no errors}$$

From problem 13.2-23, errors in received vector $\Rightarrow$ Y=1000101101010100100

$$\Rightarrow Y(p) = p^{18} + p^{14} + p^{12} + p^{11} + p^9 + p^7 + p^5 + p^2$$

13.2-24 continued

$Y(p)/G(p) =$

$$
\begin{array}{r l}
 & p^6 + p^5 + p^4 + p^3 + 1 \\
p^{12} + p^{11} + p^3 + p^2 + p + 1 \,\big) & p^{18} + p^{14} + p^{12} + p^{11} + p^7 + p^5 + p^2 \\
 & p^{18} + p^{17} + p^9 + p^8 + p^7 + p^6 \\
\hline
 & p^{17} + p^{14} + p^{12} + p^{11} + p^8 + p^6 + p^5 + p^2 \\
 & p^{17} + p^{16} + \qquad + p^8 + p^7 + p^6 + p^5 \\
\hline
 & p^{16} + p^{14} + p^{12} + + p^{11} + p^7 + \qquad p^2 \\
 & p^{16} + p^{15} + \qquad p^7 + p^6 + p^5 + p^4 \\
\hline
 & p^{15} + p^{14} + p^{12} + p^{11} + p^6 + p^5 + p^4 + p^2 \\
 & p^{15} + p^{14} + \qquad p^6 + p^5 + p^4 + p^3 \\
\hline
 & p^{12} + p^{11} + p^3 + p^2 \\
 & p^{12} + p^{11} + \qquad p^3 + p^2 + p + 1 \\
\hline
 & p + 1 \Rightarrow
\end{array}
$$

$\Rightarrow$ remainder $\neq 0 \Rightarrow$ an error has occured.

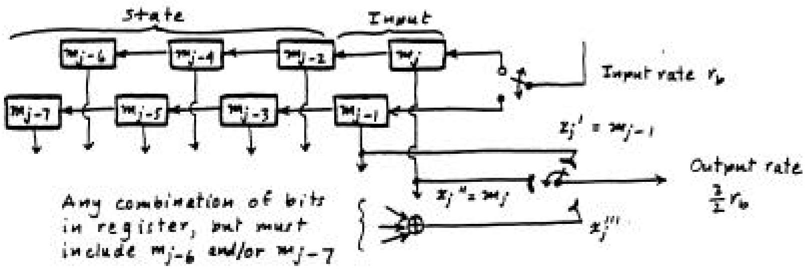
13.2-25

9600 bps $\Rightarrow$ 1/9600 $\Rightarrow$ 0.1 ms/bit $\Rightarrow$ 125 ms noise burst $\Rightarrow$ 125 ms x 10 bit/ms=1250 bits in error for each burst.

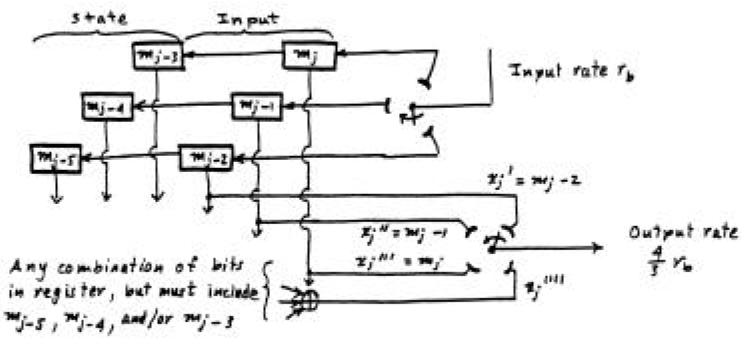
A (63,45) code can correct for 3 errors $\Rightarrow$ distribute 1200 errors over 400 blocks $\Rightarrow$ 3 errors/block. We interleave the bits so we have 63+3=66 bits between errors, $\Rightarrow$ each block=66 bits long

latency delay between interleaving and deinterleaving $\Rightarrow$
2 x (400 blocks x 66 bits/block x 0.1 ms/bit)=2 x 2.64 seconds = 5.28 seconds

13.3-1

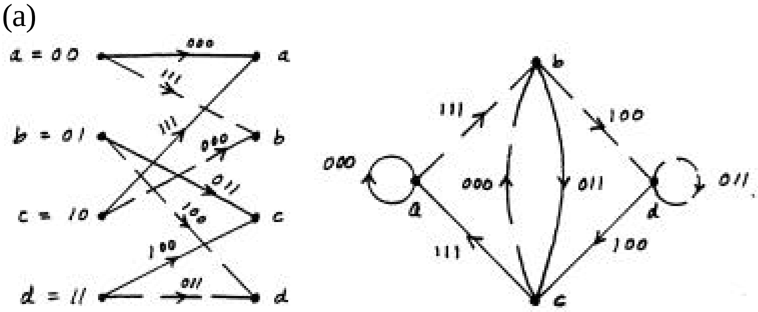


13.3-2



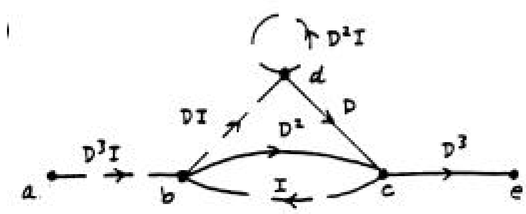
13.3-3

(a)



Input		1		0		1		1		0		0		1		1		1		1	
State	a		b		c		b		d		c		a		b		d		d		d
Output		111		011		000		100		100		111		111		100		011		011	

(b)

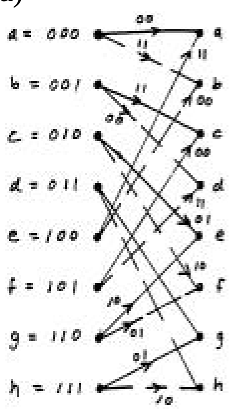


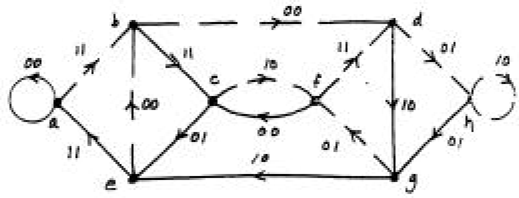
Minimum-weight paths: abce = $D^8 I^1$ and abcde = $D^8 I^2$

Thus, $d_f = 8$, $M(d_f) = 1 + 2 = 3$

13.3-4

(a)





Input	1	0	1	1	0	0	1	1	1	1	
State	a	b	c	f	d	g	e	b	d	h	h
Output	11	00	10	11	10	10	00	00	01	10	

(b)

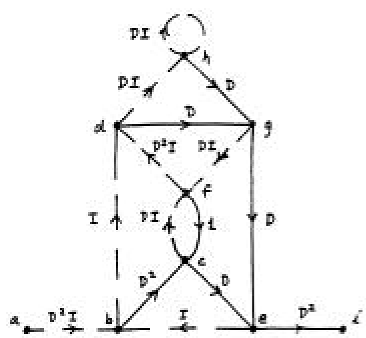
Minimum-weight paths

abdgei = $D^6 I^2$

abdce = $D^8 I^2$

Thus,

$d_f = 6$, $M(d_f) = 2$



13.3-5

$M = 110101110111000000 \Rightarrow 1 + D + D^3 + D^5 + D^6 + D^7 + D^9 + D^{10} + D^{11}$

$G_1 = 1 + D + D^3$ and $G_2 = D + D^2$ and $G_3 = 1$

$X_j' = G_1 M = (1 + D + D^3)(1 + D + D^3 + D^5 + D^6 + D^7 + D^9 + D^{10} + D^{11})$

$\quad = 1 + D^2 + D^5 + D^6 + D^{10} + D^{13} + D^{14}$

$X_j'' = G_2 M = (D + D^2)(1 + D + D^3 + D^5 + D^6 + D^7 + D^9 + D^{10} + D^{11})$

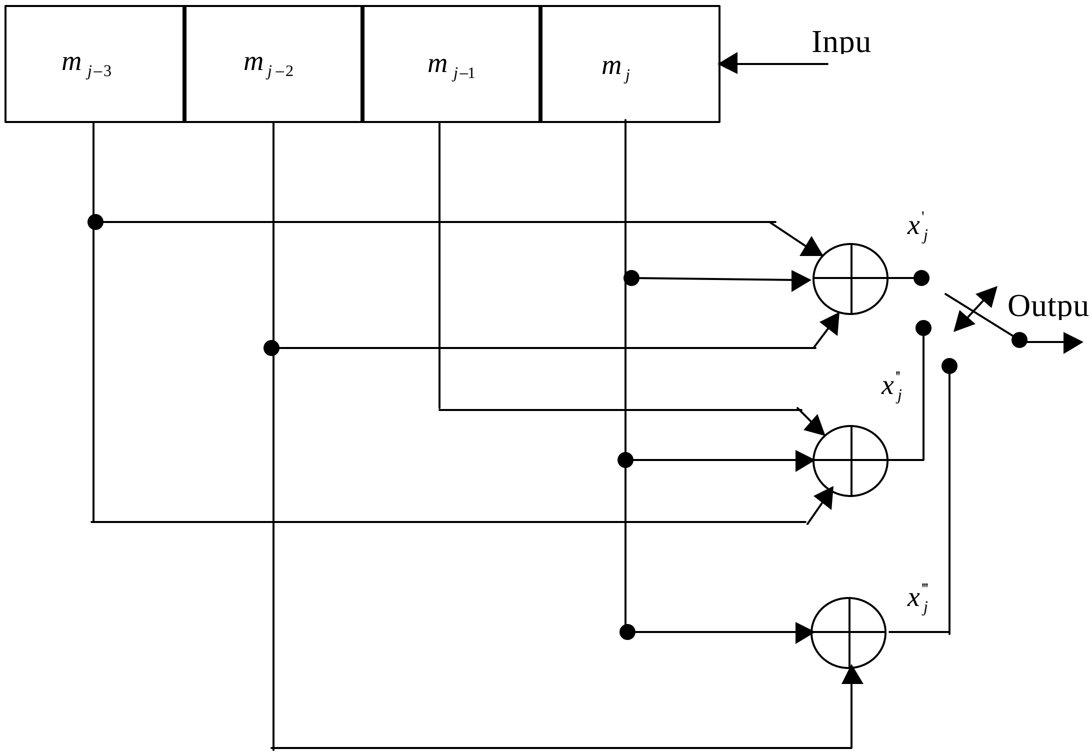
$\quad = D + D^3 + D^4 + D^5 + D^6 + D^9 + D^{10} + D^{13}$

$X_j''' = M = (1 + D + D^3 + D^5 + D^6 + D^7 + D^9 + D^{10} + D^{11})$

$j =$	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$x_j =$	011	101	010	101	100	111	111	001	000	101	111	001	000	110	010	000	000

13.3-6

Redrawing Figure 13.3-5 to eliminate the input distributor, we have the following equivalent convolutional encoder:



With $M = 110101110111000000 \Rightarrow 1 + D + D^3 + D^5 + D^6 + D^7 + D^9 + D^{10} + D^{11}$

and $G_1 = 1 + D^2 + D^3$, $G_2 = 1 + D + D^3$ and $G_3 = 1 + D^2$

$X'_j = G_1 M = (1 + D^2 + D^3)(1 + D + D^3 + D^5 + D^6 + D^7 + D^9 + D^{10} + D^{11})$

$= 1 + D + +D^2 + D^3 + D^4 + D^8 + D^9 + D^{12} + D^{14} + 0$

Because we eliminated the input distributor $\Rightarrow$ we will partition the output bits in groups of 2 and select the second bit for the ouput $\Rightarrow$ 11 11 10 00 11 00 10 10

$\Rightarrow x'_j =$ 1 1 0 0 0 0 0 0

$X''_j = G_2 M = (1 + D + D^3)(1 + D + D^3 + D^5 + D^6 + D^7 + D^9 + D^{10} + D^{11})$

$= 1 + D^5 + D^6 + D^{10} + D^{12} + D^{13} + D^{14}$

$\Rightarrow$ 10 00 01 10 00 10 11 10

$\Rightarrow x''_j =$ 0 0 1 0 0 0 1 0

13.3-6 continued

$$X_j''' = G_3 M = (1+D^2)(1+D+D^3+D^5+D^6+D^7+D^9+D^{10}+D^{11})$$

$$= 1+D+D^2+D^6+D^8+D^{10}+D^{12}+D^{13}$$

$\Rightarrow$ 11 10 00 10 10 10 11 00

$\Rightarrow x_j''' =$ 1 0 0 0 0 0 1 0

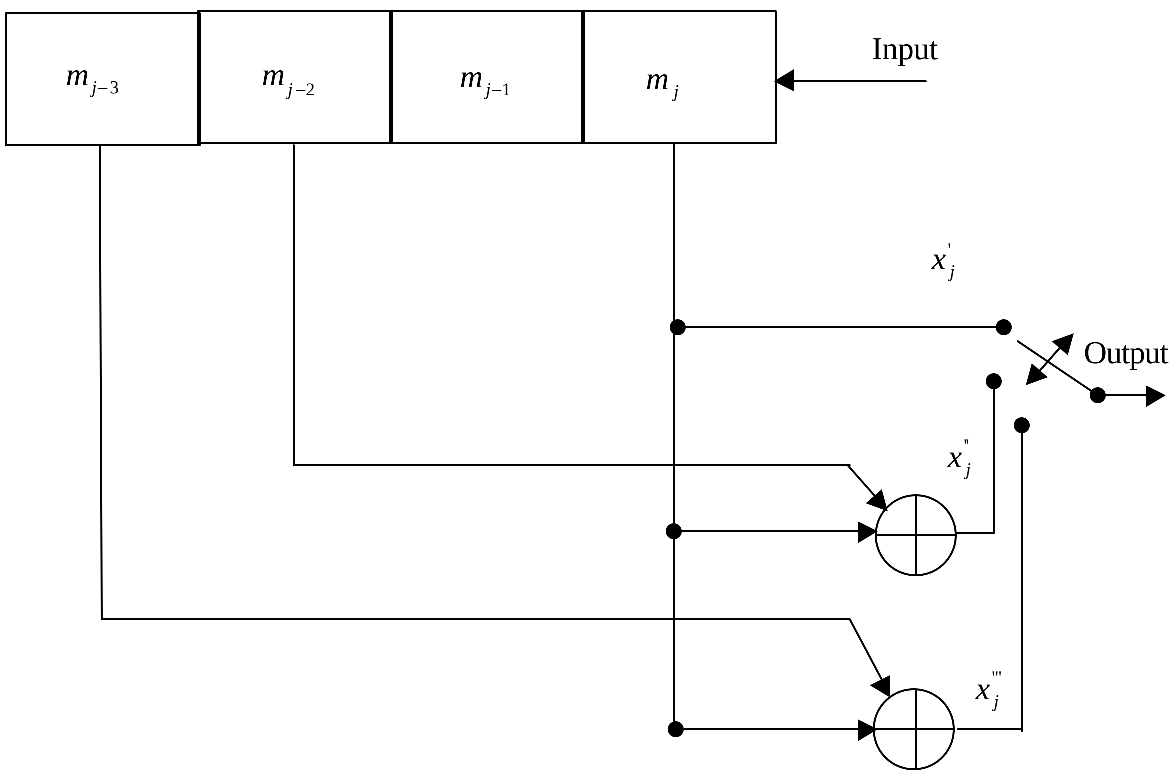
Interleaving x_j', x_j'' *and* x_j''' we get

Input message of 11 01 01 11 01 11 00 00

$\Rightarrow x_j =$ 101 100 010 000 100 000 011 000

13.3-7

Redrawing Figure 13.3-5 to eliminate the input distributor, we have the following equivalent convolutional encoder:



13.3-7 continued

With $M = 110101110111000000 \Rightarrow 1 + D + D^3 + D^5 + D^6 + D^7 + D^9 + D^{10} + D^{11}$

and $G_1 = 1,\ G_2 = 1 + D^2$ and $G_3 = D^2 + D^3$

$X_j^{'} = G_1 M = (1)(1 + D + D^3 + D^5 + D^6 + D^7 + D^9 + D^{10} + D^{11})$

$= 1 + D + +D^3 + D^5 + D^6 + D^7 + D^9 + D^{10} + D^{11} + 0 + 0$

Because we eliminated the input distributor $\Rightarrow$ we will partition the output bits in groups of 2 and select the second bit for the ouput

$\Rightarrow$ 11 01 01 11 01 11 00 00

$\Rightarrow x_j^{'} =$ 1 1 1 1 1 1 0 0

$X_j^{''} = G_2 M = (1 + D^2)(1 + D + D^3 + D^5 + D^6 + D^7 + D^9 + D^{10} + D^{11})$

$= 1 + D + D^2 + D^6 + D^8 + D^{10} + D^{12} + D^{13}$

$\Rightarrow$ 11 10 00 10 10 10 10 11

$\Rightarrow x_j^{''} =$ 1 0 0 0 0 0 0 1

$X_j^{'''} = G_3 M = (D^2 + D^3)(1 + D + D^3 + D^5 + D^6 + D^7 + D^9 + D^{10} + D^{11})$

$= D^2 + D^4 + D^5 + D^6 + +D^7 + D^{10} + D^{11} + D^{12} + D^{14} + 0$

$\Rightarrow$ 00 10 11 11 00 11 10 10

$\Rightarrow x_j^{'''} =$ 0 0 1 1 0 1 0 0

Interleaving $x_j^{'}$, $x_j^{''}$ *and* $x_j^{'''}$ we get

Input message of 11 01 01 11 01 11 00 00

$\Rightarrow x_j =$ 110 100 101 101 100 101 010 000

13.3-8

(a) The state transition diagram does not have a transition with a nonzero input that has a zero output $\Rightarrow$ noncatastrophic.

Alternatively, factoring $G_2(D) = 1 + D^2 = (1 + D)(1 + D)$ and dividing

$G_1(D) = D^2 + D + 1$ by $(D + 1)$ we get

13.3-8 continued

$$\begin{array}{r} D \\ D+1\overline{)D^2+D+1} \\ D^2+D \\ \hline 1 \end{array}$$

$\Rightarrow$ there are no common factors to $G_1(D)$ and $G_2(D)$ $\Rightarrow$ noncatastrophic

(b) The state transition diagram of Fig 13.3-5 does not have a transition with a nonzero input that has a ze
output $\Rightarrow$ noncatastrophic.

Alternatively, with $G_1(D)=D^3+D^2+1$, $G_2(D)=D^3+D+1$, and $G_3(D)=D^2+1=(D+1)(D+1)$

Dividing $G_1(D)$ and $G_2(D)$ by $D+1$ we get

$$\begin{array}{r} D^2 \\ D+1\overline{)D^3+D^2+1} \\ D^3+D^2 \\ \hline 1 \end{array} \quad \text{and} \quad \begin{array}{r} D^2+D \\ D+1\overline{)D^3+D+1} \\ D^3+D^2 \\ \hline D^2+D+1 \\ D^2+D \\ \hline 1 \end{array}$$

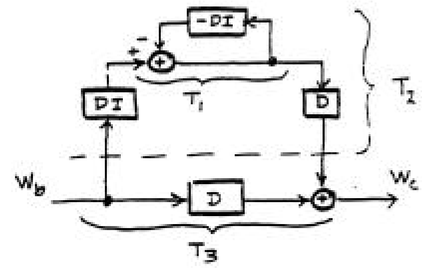
$\Rightarrow$ there are no common factors to $G_1(D), G_2$(D) and $G_3(D) \Rightarrow$ noncatastrophic

(c) $G_1(D)=D+D^2=D(D+1)$ and $G_2(D)=D^2+1=(D+1)(D+1)$

$\Rightarrow (D+1)$ is common to both $G_1(D)$ and $G_2(D) \Rightarrow$ catastrophic

13.3-9

First, we consider $W_b \to W_c$ section where

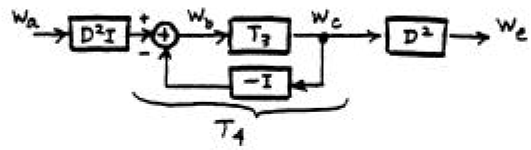


$$T_1 = \frac{1}{1-DI},$$

$$T_2 = DI \times T_1 \times D = \frac{D^2 I}{1-DI},$$

$$T_3 = D + T_2 = \frac{D}{1-DI}$$

Then,



$$T_4 = \frac{T_3}{1-IT_3} = \frac{D}{1-2DI},$$

and $T(D,I) = D^2 I \times T_4 \times D^2 = \dfrac{D^5 I}{1-2DI}$

13.3-10

$$\frac{\partial T(D,I)}{\partial I} = \sum_{d=d_f}^{\infty} \sum_{i=1}^{\infty} A(d,i) D^d{}_i I^{i-1} \Rightarrow \left.\frac{\partial T(D,I)}{\partial I}\right|_{I=1} = \sum_{d=d_f}^{\infty} \left[\sum_{i=1}^{\infty} iA(d,i) \right] D^d$$

Thus,

$$P_{be} \le \frac{1}{k} \sum_{d=d_f}^{\infty} M(d) \left[2\sqrt{a(1-a)} \right]^d \qquad \text{where } M(d) = \sum_{i=1}^{\infty} iA(d,i)$$

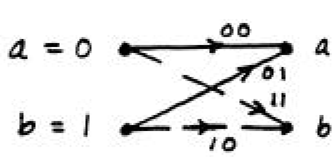
$$\le \frac{1}{k} \left\{ M(d_f) 2^{d_f} \left[a(1-a) \right]^{d_f/2} + \sum_{d=d_f+1}^{\infty} M(d) 2^d \left[a(1-a) \right]^{d/2} \right\}$$

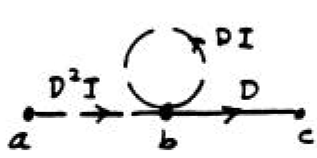
If $\sqrt{a} \ll 1$, then $1-a \approx 1$ and $M(d) 2^d \left[\alpha(1-\alpha) \right]^{d_f/2} \ll M(d_f) 2^{d_f} \left[\alpha(1-\alpha) \right]^{d/2}$ for $d \ge d_f + 1$, so

$$P_{be} \approx \frac{1}{K} M(d_f) 2^{d_f} a^{d_f/2}$$

13.3-11

(a)

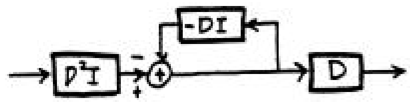




13.3-11 continued

Minimum-weight paths: $abc = D^3 I^1$ so, $d_f = 3$, $M(d_f) = 1$

(b)



$$T(D,I) = D^2 I \times \frac{1}{1-DI} \times D = \frac{D^3 I}{1-DI}$$

(c)

$$\frac{\partial T(D,I)}{\partial I} = \frac{D^3}{(1-DI)^2}$$

Eq.(9) yields

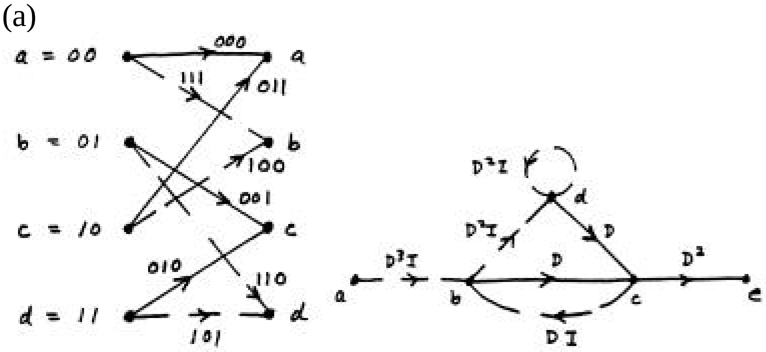
$$P_{be} \le \frac{1}{1} \times \frac{2^3 [a(1-a)]^{3/2}}{\left[1 - 2\sqrt{a(1-a)}\right]^2} \approx 8a^{3/2}, a \ll 1$$

Eq.(10) yields

$$P_{be} \approx 2^3 a^{3/2} = 8a^{3/2}$$

13.3-12

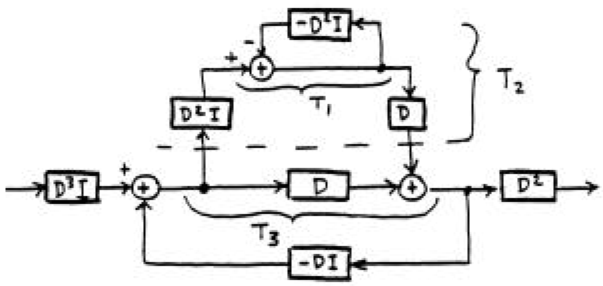
(a)



Minimum-weight paths: abce $= D^6 I^1$ so, $d_f = 6$, $M(d_f) = 1$

13.3-12 continued

(b)



$$T_1 = \frac{1}{1-D^2 I}$$

$$T_2 = D^2 I \times T_1 \times D = \frac{D^3 I}{1-D^2 I}$$

$$T_3 = D + T_2 = \frac{D}{1-D^2 I}$$

$$T(D,I) = D^3 I \times \frac{T_3}{1-DIT_3} \times D^2 = \frac{D^6 I}{1-2D^2 I}$$

(c)

$$\frac{\partial T(D,I)}{\partial I} = \frac{D^6}{(1-2D^2 I)^2}$$

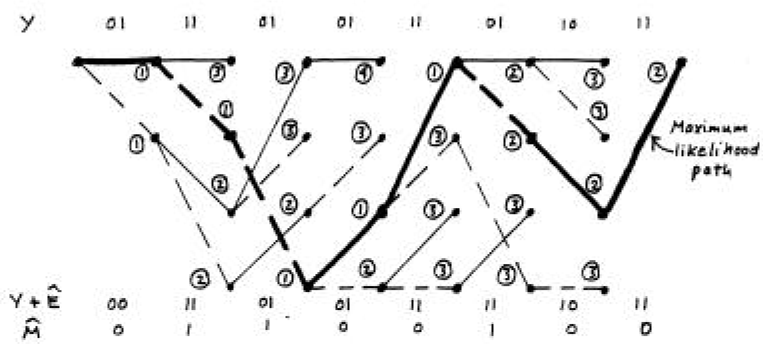
Eq.(9) yields

$$P_{be} \le \frac{1}{1} \times \frac{4^3 a^3 (1-a^3)}{[1-8a(1-a)]^2} \approx 64a^3, a \ll 1$$

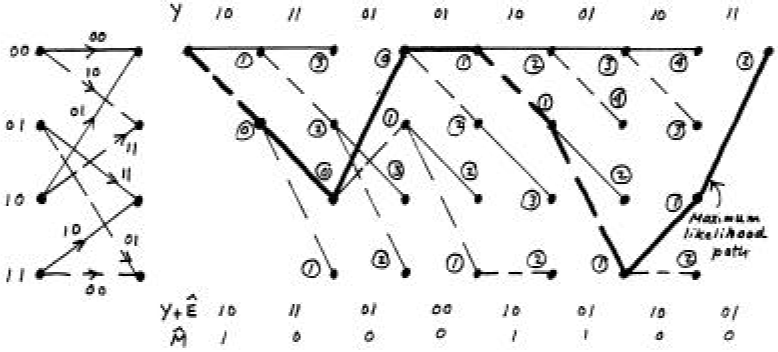
Eq.(10) yields

$$P_{be} \approx 2^6 a^{6/2} = 64a^3$$

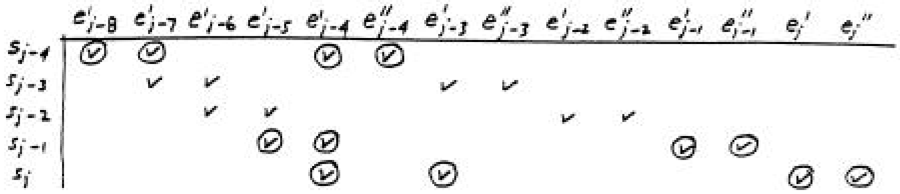
13.3-13



13.3-14



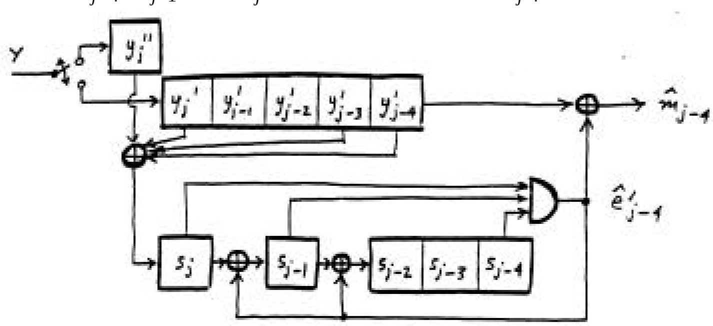
13.3-15



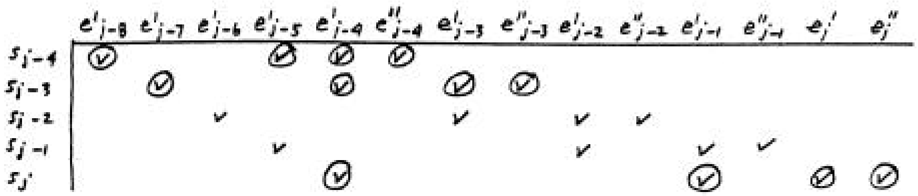
From $g_0 = g_3 = g_4 = 1$ and $g_1 = g_2 = 0$, We can get:

$s_j = e'_{j-4} \oplus e'_{j-3} \oplus e'_j \oplus e''_j$

Thus, s_{j-4}, s_{j-1} and s_j are orthogonal on $e'_{j\text{-}4}$



13.3-16

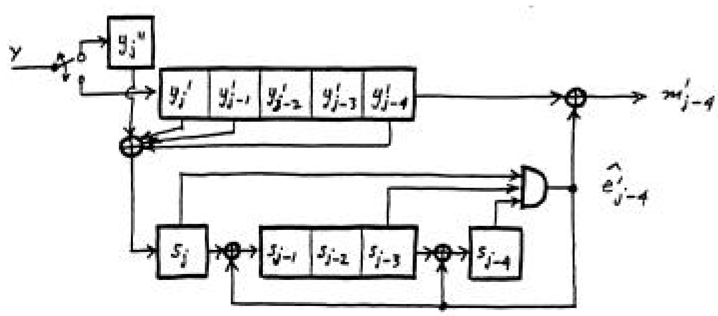


From $g_0 = g_1 = g_4 = 1$ and $g_2 = g_3 = 0$, We can get:

$s_j = e'_{j-4} \oplus e'_{j-1} \oplus e'_j \oplus e''_j$

Thus, s_{j-4}, s_{j-3} and s_j are orthogonal on $e'_{j\text{-}4}$

13.3-16 continued

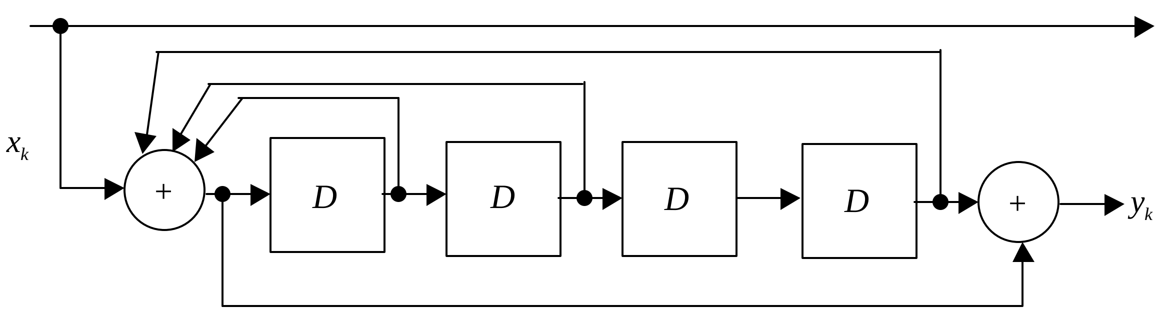


13.3-17

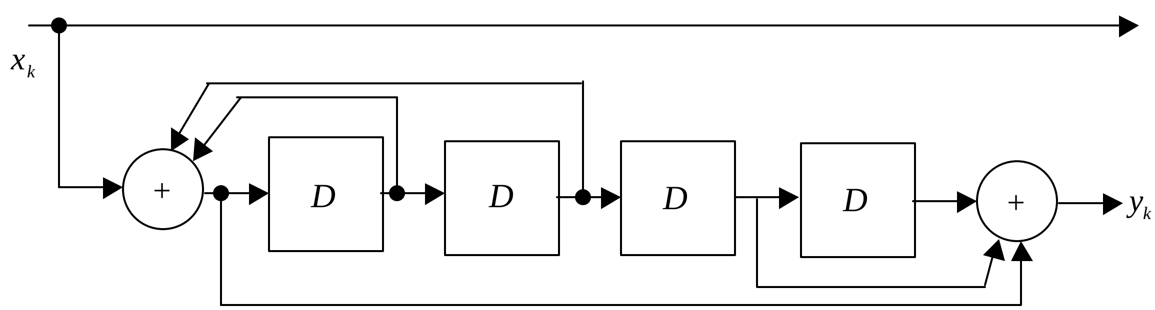
(a) $(37,21) \Rightarrow (35_8, 21_8) \Rightarrow (11\ 101_2, 10\ 001_2)$ where the 1s and 0s define the feedback and output connections respectively.

(b) $(34,23) \Rightarrow (34_8, 23_8) \Rightarrow (11\ 100_2, 10\ 011_2)$

The corresponding RSC block diagrams are shown as follows:



(35,21) RSC encoder



(34,23) RSC encoder

13.4-1

(a) $p = 5,\ q = 11 \Rightarrow\ pq = 55$ and $\phi(n)$=4 x 10 = 40=2 x 2 x 2 x 5 $\Rightarrow$ pick e = 7 (since 7 is relatively prime to 40.

$7d = 40Q + 1 \Rightarrow$ if $Q = 4 \Rightarrow 7d = 160 + 1 \Rightarrow d = 23$

$y = x^e \bmod n$ and $x = y^d \bmod n$

$x = 8 \Rightarrow y = 8^7 \bmod 55 = 2$

To check $\Rightarrow x = 2^{23} \bmod 55 = 8$

$x = 27 \Rightarrow y = 27^7 \bmod 55 = 3$

To check $\Rightarrow x = 3^{23} \bmod 55 = 27$

$x = 51 \Rightarrow y = 51^7 \bmod 55 = 6$

To check $\Rightarrow x = 6^{23} \bmod 55 = 51$

(b) $p = 11,\ q = 37 \Rightarrow pq = 407$ and $\phi(n)$=10 x 36 = 360=2 x 2 x 2x 3 x 3 x 5 $\Rightarrow$ pick e = 7 (since 7 is relatively prime to 360

$7d = 360Q + 1 \Rightarrow$ if $Q = 2 \Rightarrow 7d = 720 + 1 \Rightarrow d = 103$

$y = x^e \bmod n$ and $x = y^d \bmod n$

$x = 8 \Rightarrow y = 8^7 \bmod 407 = 288$

To check $\Rightarrow x = 288^{103} \bmod 407 = 8$

$x = 27 \Rightarrow y = 27^7 \bmod 407 = 212$

To check $\Rightarrow x = 212^{103} \bmod 407 = 27$

$x = 51 \Rightarrow y = 51^7 \bmod 407 = 171$

To check $\Rightarrow x = 171^{23} \bmod 407 = 51$

(c) $p = 13,\ q = 37 \Rightarrow pq = 481$ and $\phi(n)$=12 x 36 = 432=2^4 x 3^3 $\Rightarrow$ pick e =5 (since 5 is relatively prime to 432)

$5d = 432Q + 1 \Rightarrow$ if $Q = 2 \Rightarrow 5d = 864 + 1 \Rightarrow d = 173$

$y = x^e \bmod n$ and $x = y^d \bmod n$

$x = 8 \Rightarrow y = 8^5 \bmod 481 = 60$

To check $\Rightarrow x = 60^{173} \bmod 481 = 8$

$x = 27 \Rightarrow y = 27^5 \bmod 481 = 196$

To check $\Rightarrow x = 196^{173} \bmod 481 = 27$

$x = 51 \Rightarrow y = 51^5 \bmod 481 = 103$

To check $\Rightarrow x = 103^{173} \bmod 481$

$= \{[((103^2 \bmod 481)^6 \bmod 481)^{14} \bmod 481][103^5 \bmod 481] \bmod 481\} = 51$

13.4-2

$p = 11$, q=31 $\Rightarrow$ n=341 and $\phi(n) = 10 \text{ x } 30 = 300 = 2^2 \text{x3x5}$
$\Rightarrow$ select d to be any prime number greater than 5
$de = Q\phi(n) + 1 \Rightarrow$ if $e = 7$ and $Q = 1 \Rightarrow d = 43$

There are as many values of e as there are prime numbers greater than 5.
if $e = 7$ and Q=8 $\Rightarrow de = Q\phi(n) + 1 \Rightarrow d = 343$
$\Rightarrow$ pattern is such that for a given value of e and d,
successive values of d can be found by adding $\phi(n)$ to previous values of d

It is also observed that the values of p and q must be such that the product of p, q must be greater than the number you are trying to encrypt.

Chapter 14

14.1-1

$$\overline{x_c^2} = \int_{-\infty}^{\infty} G_c(f)df = \frac{A_c^2}{4}\int_{-\infty}^{\infty}[G_{lp}(f-f_c)+G_{lp}(f+f_c)]df$$

$$= \frac{A_c^2}{4} 2\int_{-\infty}^{\infty} G_{lp}(\lambda)df \quad \text{since } G_{lp}(f-f_c) \text{ and } G_{lp}(f+f_c) \text{ don't overlap if } f_c \gg r$$

$$= \frac{A_c^2}{2}\left[\frac{M^2-1}{12r}\int_{-\infty}^{\infty}\text{sinc}^2(f/r)df + \frac{(M-1)^2}{4}\int_{-\infty}^{\infty}\delta(f)df\right]$$

$$= \frac{A_c^2}{2}\left[\frac{M^2-1}{12r}r + \frac{(M-1)^2}{4}\right] = \frac{A_c^2}{12}(M-1)(2M-1)$$

$$P_c = 2 \text{ x } \frac{A_c^2}{4}\frac{(M-1)^2}{4}, \qquad \frac{P_c}{\overline{x_c^2}} = \frac{3M-3}{4M-2} = \begin{cases}1/2 & M=2\\ 3/4 & M \gg 1\end{cases}$$

14.1-2

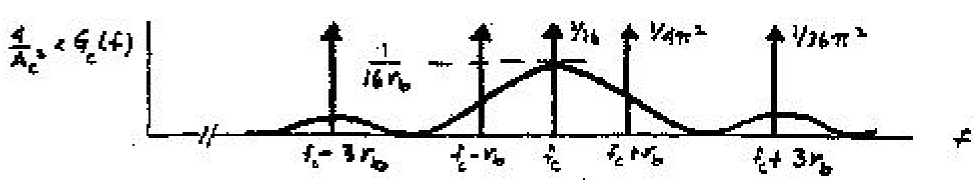
(a) $x_i(t) = \sum_k a_k\, p(t)$ where $p(t) = u(t) - u(t - T_b/2)$, where $x_q(t) = 0$

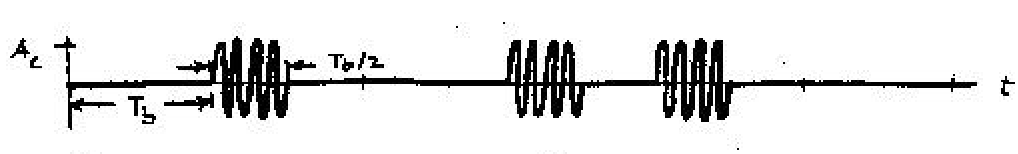
$$a_k = 0, 1 \;\Rightarrow\; m_a = 1/2, \;\; \sigma_a^2 = 1/4$$

$$|P(f)|^2 = \left(\frac{T_b}{2}\right)^2 \text{sinc}^2\left(\frac{fT_b}{2}\right) = \frac{1}{4r_b^2}\text{sinc}^2\left(\frac{f}{2r_b}\right)$$

$$\Rightarrow \; (m_a r_b)^2\,|P(nr_b)|^2 = \begin{cases}1/16 & n=0\\ 0 & n = \pm 2, \; \pm 4, \ldots\\ 1/(2\pi n)^2 & n \text{ odd}\end{cases}$$

$$G_{lp}(f) = G_i(f) = \frac{1}{16r_b}\text{sinc}^2\frac{f}{2r_b} + \frac{1}{16}\delta(f) + \sum_{\pm n \text{ odd}}\frac{1}{(2\pi n)^2}\delta(f - nr_b)$$





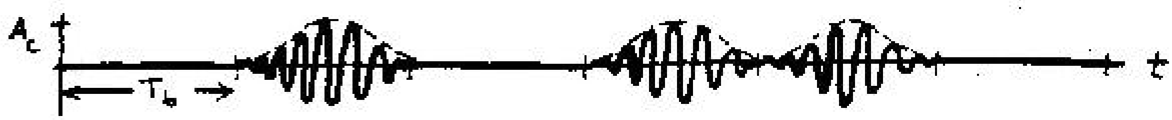
14.1-3

$$m_a = 1/2, \quad \sigma_a^2 = 1/4, \quad r = r_b$$

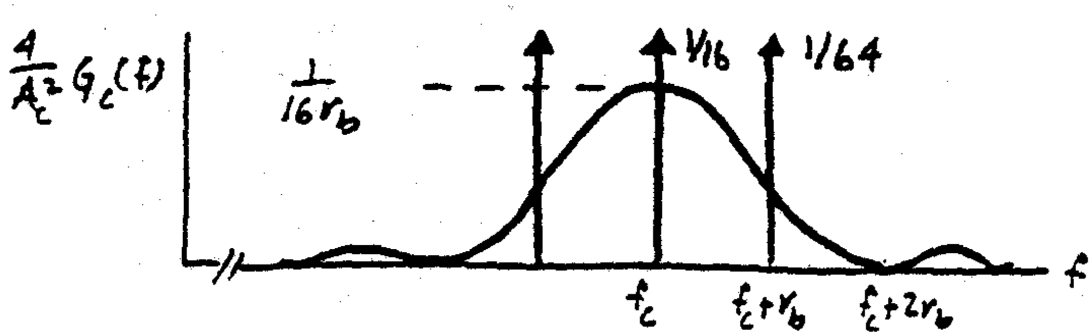
$$G_{lp}(f) = \frac{r_b}{4} |P(f)|^2 + \frac{r_b^2}{4} \sum_n |P(nr_b)|^2 \, \delta(f - nr_b),$$

$$P(f) = \begin{cases} \tau & f = 0 \\ \tau/2 & f = \pm 1/2T \\ 0 & f = \pm \dfrac{m}{2\tau}, \; m \geq 0 \end{cases}$$

(a) $2\tau = T_b = 1/r_b$

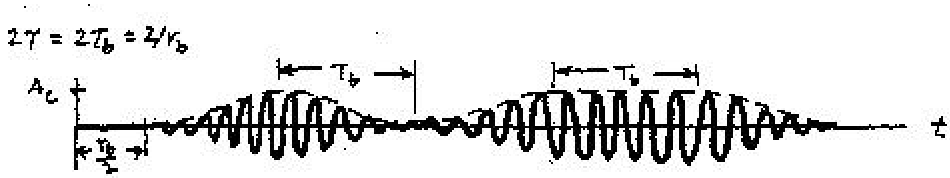


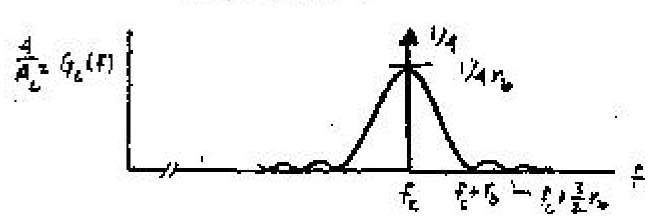
$$G_{lp}(f) = \frac{1}{16r_b} \frac{\text{sinc}^2(f/r_b)}{|1-(f/r_b)|^2} + \frac{1}{16}\delta(f) + \frac{1}{64}\left[\delta(f - r_b) + \delta(f + r_b)\right]$$



(b) $2\tau = 2T_b = 2/r_b$

$$G_{lp}(f) = \frac{1}{4r_b} \frac{\text{sinc}^2(2f/r_b)}{|1-(2f/r_b)|^2} + \frac{1}{4}\delta(f)$$





14.1-4

(a) For $kD < t < (k+1)D$, $x_i(t) = a_{2k}$ and $x_q(t) = a_{2k+1}$ with $a_k = \pm 1$

Thus $x_i^2(t) + x_q^2(t) = 2$ for all t so $A(t) = \sqrt{2}A_c$ for all t

whereas $\phi(t) = \sum_k \arctan\left(\frac{(a_{2k+1})}{a_{2k}}\right) p_D(t-kD)$

(b) For $kD < t < (k+1)D$, $x_i(t) = a_{2k}\, p(t\text{-}kD)$

and $x_q(t) = a_{2k+1} p(t\text{-}kD)$, $a_k = \pm 1$

Thus $x_i^2(t) + x_q^2(t) = 2p^2(t\text{-}kD)$ so $A(t) = \sqrt{2}A_c \sum_k |p(t-kD)|$

and $\phi(t) = \sum_k \arctan\left[\frac{x_q(t)}{x_i(t)}\right] p_D(t-kD) = \sum_k \arctan\left(\frac{a_{2k+1}}{a_{2k}}\right) p_D(t-kD)$

14.1-5

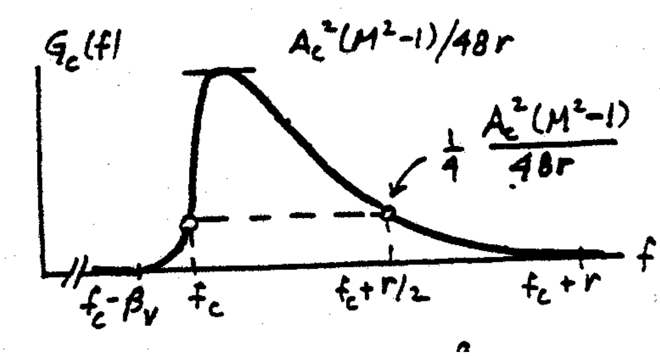
$m_a = \overline{a}_k = 0$, $\sigma_a^2 = \overline{a_k^2} = (m^2\text{-}1)/12$ from Eq. (21), Sect 11.2, with $A = 1$

$P(f) = \frac{1}{r}\cos^2\left(\frac{\pi f}{2r}\right)\Pi\left(\frac{f}{2r}\right)$ so $P(f) = 0$ for $|f| \geq r$

$G_{lp}(f) = G_i(f) = \frac{M^2 - 1}{12r}\cos^4\left(\frac{\pi f}{2r}\right)\Pi\left(\frac{f}{2r}\right)$

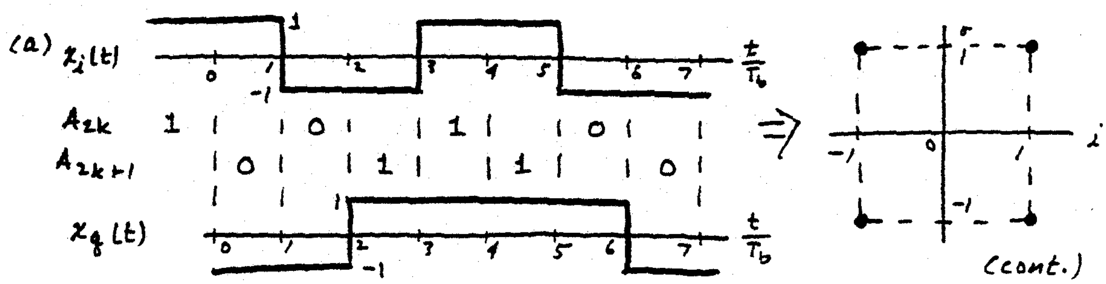
$G_c(f) = |H_{VSB}(f)|^2 \frac{A_c^2}{4}\left[G_{lp}(f - f_c) + G_{lp}(f + f_c)\right]$

where $|H_{VSB}(f)| = \begin{cases} 1 & f > f_c + \beta_v \\ 1/2 & f = f_c \\ 0 & 0 < f < f_c - \beta_v \end{cases}$



14.1-6

(a)



Since x_q changes from ± 1 to ∓ 1 while x_i stays constant at ± 1, and vice versa, it follows that $\Delta\phi = \pm\pi/2$

(b) a_{2k} and a_{2k+1}, are independent sequences with $m_a = 0$, $\sigma_a^2 = 1$, $r = r_b/2$.
Time delay of $x_q(t)$ affects only the phase of $P(f)$, so
$G_q(f) = G_i(f)$ with $|P(f)| = |P_D(f)|$ where $D = 2T_b$

Hence, $G_{lp}(f) = 2 \text{ x } r|P_D(f)|^2 = \dfrac{4}{r_b}\text{sinc}^2(2f/r_b)$, same as QPSK.

14.1-7

A_k	B_k	C_k	I_k	Q_k
0	0	0	α	β
0	0	1	β	α
0	1	0	$-\beta$	α
0	1	1	$-\alpha$	β
1	0	0	α	$-\beta$
1	0	1	β	$-\alpha$
1	1	0	$-\beta$	$-\alpha$
1	1	1	$-\alpha$	$-\beta$

14.1-7 continued

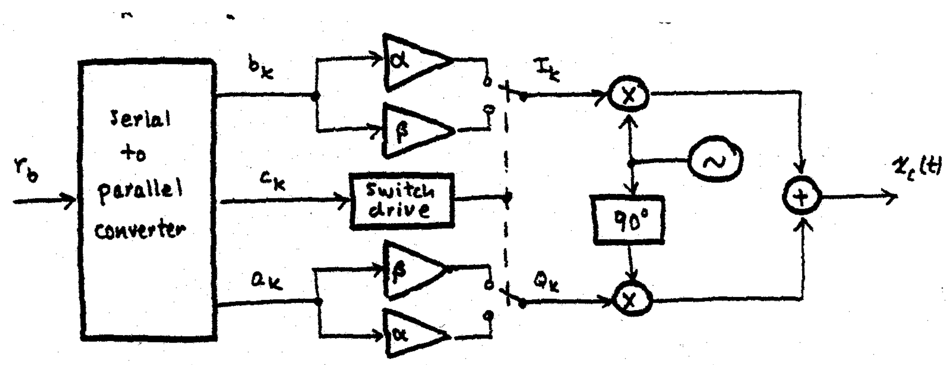
Let $I_{ko} = (1-C_k)\alpha + C_k\beta$ and $Q_{ko} = C_k\alpha + (1-C_k)\beta$
and construct reduced table

A_k	B_k	I_k	Q_k
0	0	I_{k0}	Q_{ko}
0	1	$-I_{k0}$	Q_{ko}
1	0	I_{k0}	$-Q_{ko}$
1	1	$-I_{k0}$	$-Q_{ko}$

Thus, $I_k = (1-2B_k)I_{ko}$ and $Q_k = (1-2A_k)Q_{ko}$
where $(1-2A_K) = a_k$, $(1-2B_k) = b_k$, and $C_k = (1-c_k)/2$, so

$$I_k = b_k\left[\frac{1+c_k}{2}\alpha + \frac{1-c_k}{2}\beta\right] = \begin{cases} b_k\alpha & c_k = +1 \\ b_k\beta & c_k = -1 \end{cases}$$

$$Q_k = a_k\left[\frac{1-c_k}{2}\alpha + \frac{1+c_k}{2}\beta\right] = \begin{cases} a_k\beta & c_k = +1 \\ a_k\alpha & c_k = -1 \end{cases}$$



14.1-8

$x_c(t) = A_c\left[x_1(t) + x_0(t)\right]$ where, with $a_k = 0,1$

$$x_1(t) = \left[\sum_k a_k\, p_{T_b}(t-kT_b)\right]\cos(\omega_1 t + \theta_1),$$

$$x_0(t) = \left[\sum_k (1-a_k)\, p_{T_b}(t-kT_b)\right]\cos(\omega_0 t + \theta_0)$$

14.1-8 continued

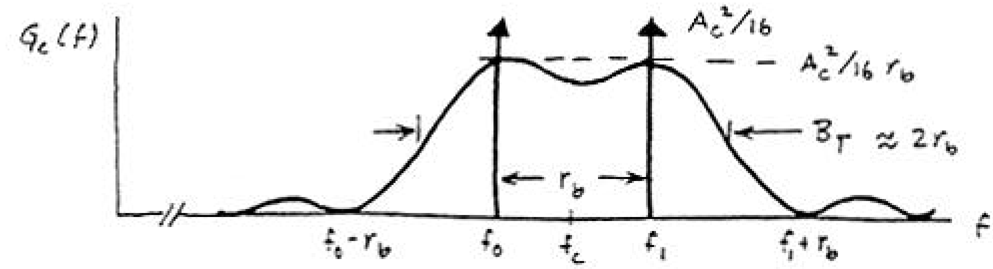
Then, from Eq. (7) with $M = 2$ and $r = r_b$,

$$G_{1lp}(f) = G_{0lp}(f) = \frac{1}{4r_b}\,\text{sinc}^2\,\frac{f}{r_b} + \frac{1}{4}\delta(f)$$

and $G_c(f) = \dfrac{A_c^2}{4}\left[G_{1lp}(f - f_1) + G_{0lp}(f - f_0) + G_{1lp}(f + f_1) + G_{0lp}(f + f_0)\right]$

Since $f_c \gg r_b$, for $f > 0$ we have

$$G_c(f) = \frac{A_c^2}{16}\left[\frac{1}{r_b}\,\text{sinc}^2\,\frac{f - f_1}{r_b} + \frac{1}{r_b}\,\text{sinc}^2\,\frac{f - f_0}{r_b} + \delta(f - f_1) + \delta(f - f_0)\right]$$



14.1-9

$$p(t) = \cos\left(2\pi\frac{r_b}{2}t - \frac{\pi}{2}\right)\Pi\left(\frac{t - T_b}{T_b}\right) \quad \text{so modulation theorem yields}$$

$$P(t) = \frac{T_b}{2}\left[\text{sinc}\left(f - \frac{r_b}{2}\right)T_b e^{-j\pi(f - r_b/2)T_b} e^{-j\pi/2} + \text{sinc}\left(f + \frac{r_b}{2}\right)T_b e^{-j\pi(f + r_b/2)T_b} e^{+j\pi/2}\right]$$

$$= \frac{1}{2r_b}\left[\text{sinc}\left(\frac{(f - (r_b/2))}{r_b}\right) + \text{sinc}\left(\frac{(f + (r_b/2))}{r_b}\right)\right]e^{-j\pi f T_b}$$

Thus, $|P(f)|^2 = \dfrac{1}{4r_b^2}\left[\text{sinc}\left(\dfrac{(f - (r_b/2))}{r_b}\right) + \text{sinc}\left(\dfrac{(f + (r_b/2))}{r_b}\right)\right]^2$

But $\text{sinc}\left(\dfrac{f}{r_b} \pm \dfrac{1}{2}\right) = \dfrac{1}{\pi\left(\dfrac{f}{r_b} \pm \dfrac{1}{2}\right)}\sin\left(\dfrac{\pi f}{r_b} \pm \dfrac{\pi}{2}\right) = \pm\dfrac{2}{\pi}\dfrac{\cos(\pi f / r_b)}{(2f/r_b)^2 \pm 1}$

so $|P(f)|^2 = \dfrac{1}{4r_b^2}\left(\dfrac{2}{\pi}\right)^2\left[\dfrac{-(\cos(\pi f / r_b))}{(2f/r_b) - 1} + \dfrac{(\cos(\pi f / r_b))}{(2f/r_b) + 1}\right]^2 = \dfrac{4}{\pi^2 r_b^2}\left[\dfrac{(\cos(\pi f / r_b))}{(2f/r_b)^2 - 1}\right]^2$

14.1-10

$$x_c(t) = A_c \sum_k [\cos(\omega_d a_k t)\cos(\omega_c t + \theta) - \sin(\omega_d a_k t)\sin(\omega_c t + \theta)] p_{T_b}(t - kT_b)$$

with $a_k = \pm 1$ and $\omega_d = \pi N / T_b = \pi N r_b$

$$x_i(t) = \sum_k \cos(\omega_d a_k t) p_{T_b}(t - kT_b) = \sum_k \cos \omega_d t p_{T_b}(t - kT_b)$$

$$= \cos \omega_d t \sum_k p_{T_b}(t - kT_b) = \cos \omega_d t = \cos 2\pi \frac{N r_b}{2} t \quad \text{for all } t$$

Thus, $G_i(f) = \frac{1}{4}[\delta(f - Nr_b/2) + \delta(f + Nr_b/2)]$

$$x_q(t) = \sum_k \sin(\omega_d a_k t) p_{T_b}(t - kT_b) = \sum_k a_k \sin(\omega_d a_k t\, p_{T_b}(t - kT_b) = \sum_k a_k \sin \omega_d t\ p_{T_b}(t - kT_b)$$

where $\sin \omega_d a_k t = \sin\left[\frac{\pi N}{T_b}(t - kT_b) + \pi N k\right]$

$$= \cos \pi N k \ \sin\left[\frac{\pi N}{T_b}(t - kT_b)\right] = (-1)^{Nk} \sin[\pi N r_b (t - kT_b)]$$

so $x_q(t) = \sum_k Q_k\, p(t - kT_b)$ with $Q_k = (-1)^{Nk}$ and

$$p(t) = \sin\left(2\pi \frac{Nr_b}{2} t\right)[u(t) - u(t - T_b)]$$

$\overline{Q_k} = (-1)^{Nk}\overline{a}_k = 0$ and $\overline{Q_k^2} = \overline{a_k^2} = 1$ so $G_q(f) = r_b |P(f)|^2$

$$P(f) = \Im[p(t)] = \Im\left[\cos\left(2\pi \frac{Nr_b}{2} t - \frac{\pi}{2}\right)\Pi\left(\frac{t - T_b/2}{T_b}\right)\right]$$

$$= \frac{T_b}{2}\left[\text{sinc}\left(f - \frac{Nr_b}{2}\right) T_b e^{-j\pi(f - Nr_b/2)T_b} e^{-j\pi/2} + \text{sinc}\left(f + \frac{Nr_b}{2}\right) T_b e^{-j\pi(f + Nr_b/2)T_b} e^{+j\pi/2}\right]$$

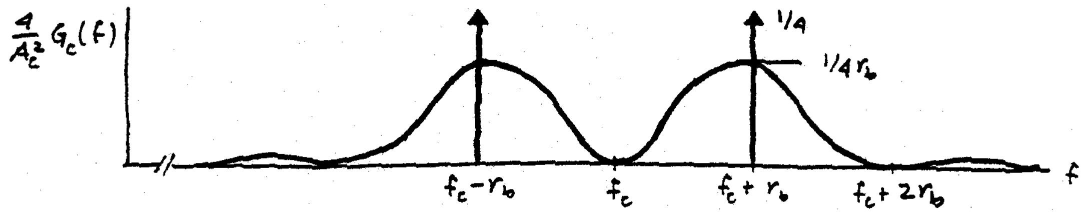
But $e^{-j\pi(f \mp r_b/2)T_b} e^{\mp j\pi/2} = e^{-j\pi f T_b} e^{\pm j(N-1)\pi/2} = (\pm j)^{N-1} e^{-j\pi f T_b}$

Thus $G_q(f) \frac{1}{4r_b}\left| j^{N-1}\text{sinc}\left(\frac{f - Nr_b/2}{r_b}\right) + (-j)^{N-1}\text{sinc}\left(\frac{f + Nr_b/2}{r_b}\right)\right|^2$

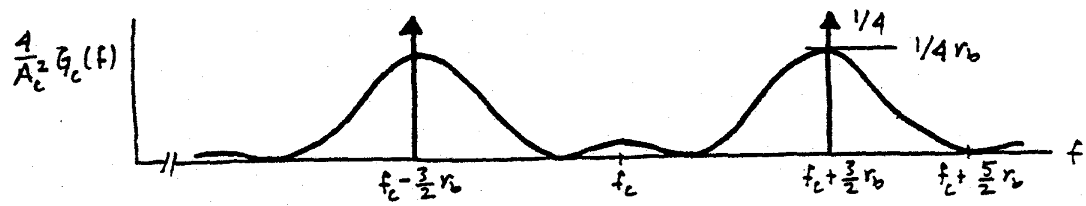
14.1-11

$$N = 2: \quad G_i(f) = \frac{1}{4}[\delta(f - r_b) + \delta(f + r_b)], \quad G_q(f) = \frac{1}{4r_b}\left[\text{sinc}\frac{f - r_b}{r_b} - \text{sinc}\frac{f + r_b}{r_b}\right]^2$$

14.1-11 continued

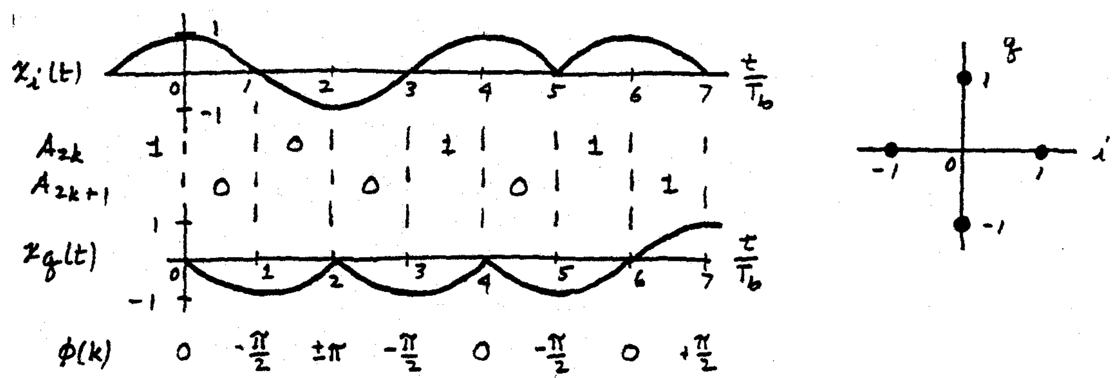


$$N = 3: \quad G_i(f) = \frac{1}{4}\left[\delta\left(f - \frac{3}{2}r_b\right) + \delta\left(f + \frac{3}{2}r_b\right)\right], \quad G_q(f) = \frac{1}{4r_b}\left[\text{sinc}\frac{f - 3r_b/2}{r_b} + \text{sinc}\frac{f + 3r_b/2}{r_b}\right]^2$$



14.1-12

(a)



(b) For $2kT_b < t < (2k+1)T_b$, $x_i(t) = a_{2k}\cos\left[\frac{\pi}{2T_b}(t - 2kT_b)\right]$ and

$$x_q(t) = a_{2k+1} = \cos\left[\frac{\pi}{2T_b}(t - 2kT_b) - \frac{\pi}{2}\right] = a_{2k+1}\sin\left[\frac{\pi}{2T_b}(t - 2kT_b)\right]$$

so $x_i^2(t) + x_q^2(t) = a_{2k}^2\cos^2\left[\frac{\pi}{2T_b}(t - 2kT_b)\right] + a_{2k+1}^2\sin^2\left[\frac{\pi}{2T_b}(t - 2kT_b)\right] = 1$ since $a_k^2 = 1$

Thus, $A(t) = A_c$ for all t

14.1-12 continued

(c) Let $x_i(t) = \sum_{k \text{ even}} I_k\, p'(t-kT_b)$ and $x_q(t) = \sum_{k \text{ odd}} Q_k\, p'(t-kT_b)$

where $I_k = a_k$ for k even and $Q_k = a_k$ for k odd so

$\overline{I_k^2} = \overline{\mathrm{Q}_\mathrm{k}^2} = 1$ and $\overline{I_k} = \overline{\mathrm{Q}_\mathrm{k}} = 0$

$$p'(t-kT_b) = \cos\left[\frac{\pi}{2T_b}(t-2kT_b)\right]\Pi\left(\frac{t-kT_b-T_b}{2T_b}\right)$$

$$p'(t) = \cos\left(\pi r_b t/2\right)[u(t+T_b)-u(t-T_b)]$$

These expressions are identical to MSK, so $G_{lp}(f)$ is as in Eq. (22).

14.1-13

Consider $(k\text{ -}1)T_b < t < (k+1)T_b$ with k odd, so

$x_q(t) = \sin(\phi_{k-1} + a_{k-1}c_{k-1})\, p_{T_b}[t-(k-1)T_b] + \sin(\phi_k + a_k c_k)\, p_{T_b}[t-kT_b]$

since $\cos\phi_k = 0$, $\sin(\phi_k + a_k c_k) = \cos(a_k c_k)\sin\phi_k = \cos c_k \sin\phi_k$

also $\sin\phi_{k\text{-}1} = 0$, $\phi_{k-1} = \phi_k - a_{k-1}\pi/2$, and $c_{k-1} = c_k + \pi/2$ so

$$\sin(\phi_{k-1} + a_{k-1}c_{k-1}) = \sin\left[a_{k-1}\left(c_k + \frac{\pi}{2}\right)\right]\cos\left(\phi_k - a_{k-1}\frac{\pi}{2}\right)$$

$$= a_{k-1}\sin\left(c_k + \frac{\pi}{2}\right)\sin\phi_k\, \sin a_{k-1}\pi/2 = (a_{k-1}\cos c_k)(a_{k-1}\sin\phi_k)$$

$$= \cos c_k \sin\phi_k$$

Thus $x_q(t) = \cos c_k \sin\phi_k$ for $(k\text{ -}1)T_b < t < (k+1)T_b$ with k odd,

and $x_q(t) = \sum_{k \text{ odd}} \mathrm{Q_k}\, p(t-kT_b)$ where $Q_k = \sin\phi_k$ and

$$p(t-kT_b) = \cos\left[\frac{\pi r_b}{2}(t-kT_b)\right]\{u[t-(k-1)T_b] - u[t-(k+1)T_b]\}$$

so $p(t) = \cos\ (\pi r_b t/2)[u(t+T_b)-u(t-T_b)]$

14.1-14

$G_{lp}(f) = \frac{1}{r}\mathrm{sinc}^2(f/r)$ and $r = r_b \Rightarrow G_{lp\max} = G_{lp}(f=0)$

Because we are at baseband $\Rightarrow$ use $f = B_T/2 \Rightarrow f = 1500$ and $G_{lp}(f=0) = 1/r_b$

$$10\log\left(\frac{G_{lp}(f=1500)}{G_{lp}(f=0)}\right) \le -30 \text{ dB} \Rightarrow \log\left(\frac{G_{lp}(f=1500)}{G_{lp}(f=0)}\right) = \log(\mathrm{sinc}^2(1500/r_b)) \le -3$$

$\Rightarrow \mathrm{sinc}^2(1500/r_b) \le 0.001 \Rightarrow$ from sinc tables $\Rightarrow 3.9 = 1500/\mathrm{r_b} \Rightarrow r_b < 385$ bps.

14.1-15

(a) Assume Sunde's FSK $\Rightarrow f_d = r_b/2$; and $f = B_T/2 = 1500$.
Using Eq. 18 and neglecting impulses we have

$$G_{lp}(f=0) = \frac{4}{\pi^2 r_b^2} = G_{lp\,\max}$$

$$\Rightarrow 10\log\left(\frac{G_{lp}(f=1500)}{G_{lp}(f=0)}\right) \le -30 \text{ dB} \Rightarrow \log\left(\frac{\cos(1500\pi/r_b)}{(2 \times 1500/r_b)^2 - 1)}\right)^2 = \le -3$$

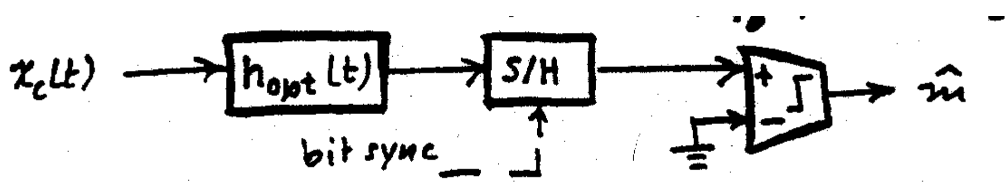
We get $r_b \le 656 \Rightarrow$ since $f_d = r_b/2 \Rightarrow f_d = 328$ Hz.

(b) Using Eqs. (21) and (22) in a similar way as (a), we get $r_b = 1312$ bps and $f_d = 328$ Hz.

14.2-1

Since $-s_0(t) = s_1(t)$, $V_{opt} = 0$ and $h_{opt}(t) = 2KA_c p_{T_b}(T_b - t)\cos w_c(T_b - t)$

$= 2KA_c\cos(w_c t - 2p N_c)\, p_{T_b}(t) = 2KA_c\cos(w_c t) \qquad 0 < t < T_b$

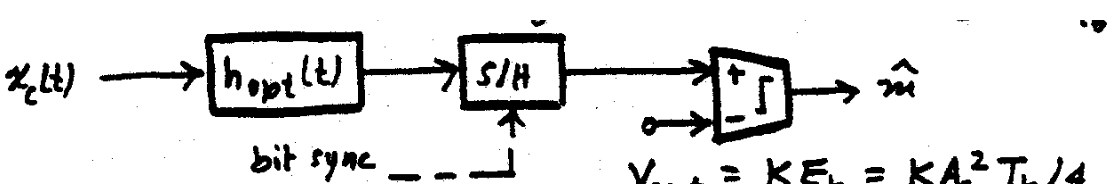


14.2-2

(a) $h_{opt}(t) = KA_c\sin^2\left[\frac{p}{T_b}(T_b - t)\right] p_{T_b}(T_b - t)\cos w_c(T_b - t)$

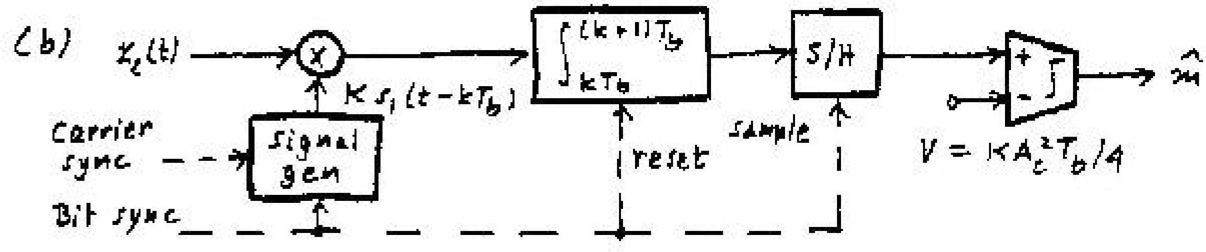
$$= KA_c\sin^2\left(p - \frac{pt}{T_b}\right) p_{T_b}(t)\cos(w_c t - 2p N_c)$$

$$= KA_c\sin^2\left(\frac{pt}{T_b}\right)\cos w_c t \qquad 0 < t < T_b$$



14.2-2 continued

(b)



14.2-3

$$E_1 = \int_0^{T_b} A_c^2 \cos^2 2\pi(f_c + f_d)t\,dt \quad = \frac{A_c^2}{2}\int_0^{T_b}[1+\cos 4\pi(f_c+f_d)t]dt$$

$$= \frac{A_c^2 T_b}{2} + \frac{A_c^2}{2}\frac{\sin 4\pi(f_c+f_d)T_b}{4\pi(f_c+f_d)} = \frac{A_c^2T_b}{2}[1+\sin 4(N_c + f_dT_b)]$$

$$\text{similarly, } E_0 = \int_0^{T_b} A_c^2 \cos^2 2\pi(f_c - f_d)t\,dt \quad = \frac{A_c^2T_b}{2}[1+\text{sinc}4(N_c - f_dT_b)]$$

$$\text{and } E_b = \frac{1}{2}(E_0 + E_1) = \frac{A_c^2T_b}{2}[2+\text{sinc}4(N_c+f_dT_b)+\text{sinc}4(N_c - f_dT_b)]$$

If $N_c - f_dT_b >> 1$ then $N_c - f_dT_b >> 1$ so $|\text{sinc}4(N_c \pm f_dT_b)| << 1$

and $E_b \approx \dfrac{A_c^2T_b}{2}$

14.2-4

$$-\frac{E_{10}}{E_b} = \text{-sinc}(4\text{f}_\text{d}/r_b)$$

$-\text{sinc}\,\lambda\,|_{\max} \approx .216$ at $\lambda \approx 1.4$

so take $4f_d/r_b \approx 0.216 \;\Rightarrow\; f_d \approx .35r_b$

Then $\text{E}_\text{b} - E_{10} = 1.216E_b$ and $P_e = Q\left(\sqrt{1.216\gamma_b}\right)$

14.2-5

$V_{opt} = 0$

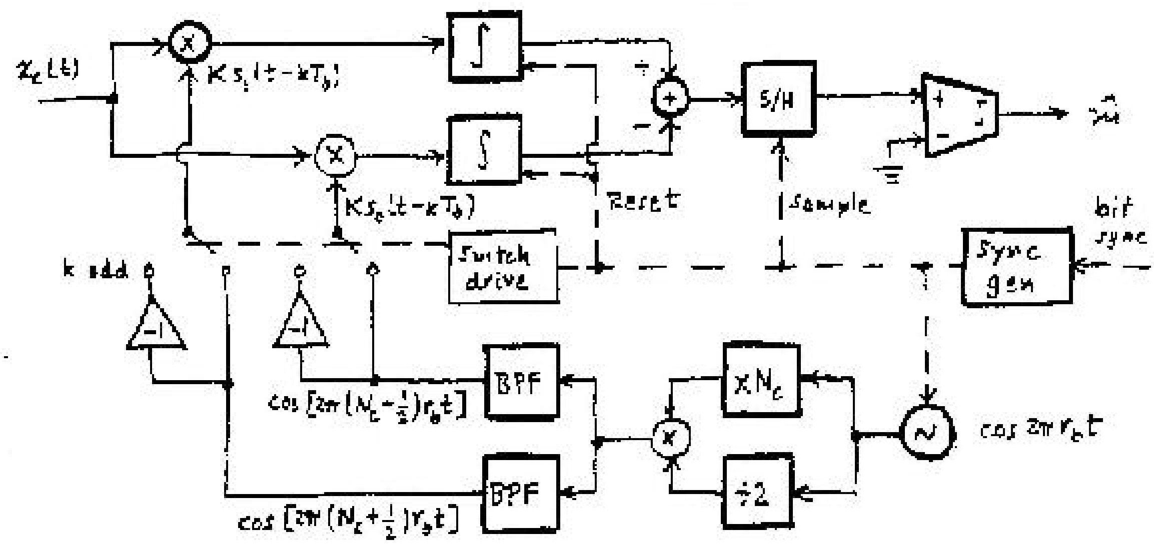
Take $\text{K} = 1/A_c$ so

14.2-5 continued

$$Ks_{1,0}(t-kT_b)=\cos\left[\frac{2p}{T_b}\left(N_c\pm\frac{1}{2}\right)(t-kT_b)\right]\quad \mathrm{k}T_b<t<(k+1)T_b$$

$$=\cos[2p(N_c\pm 1/2)r_b t\mp p\,k]$$

$$=\begin{cases}\cos 2p(N_c r_b\pm r_b/2)t] & k \text{ even}\\ -\cos 2p(N_c r_b\pm r_b/2)t] & k \text{ odd}\end{cases}$$

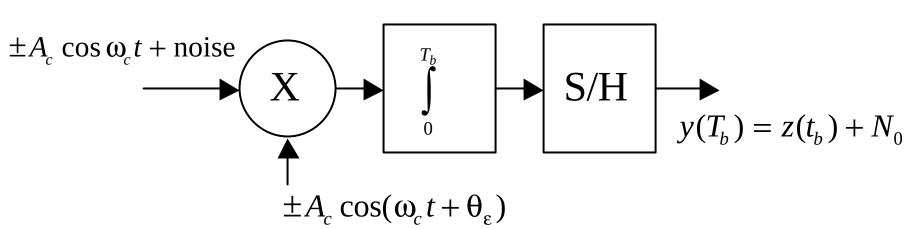


14.2-6

$$P_e=Q\left(\sqrt{2r_b}\right)=10^{-5}\quad\Rightarrow\quad\sqrt{2r_b}=4.27$$

$$Q(4.27\cos q_e)<10^{-4}\quad\Rightarrow\quad q_e<\arccos\frac{3.74}{4.27}\approx 29^0$$

14.2-7



$\omega_c=2\pi N_c/T_b$

14.2-7 continued

$$Z(t_b) = \int_0^{T_b} KA_c \cos(w_c t + q_e)(\pm A_c \cos(w_c t)dt$$

$$= \pm K \frac{A_c^2}{2} \int_0^{T_b} [\cos(q_e) + \cos(2w_c t + q_e)dt]$$

$$= \pm K \frac{A_c^2}{2} T_b \left\{ \cos q_e + \frac{1}{2w_c T_b} [\sin(4p N_c + q_e) - \sin q_e] \right\}$$

$$= \pm KE_b \cos q_e$$

14.2-8

$$s(l) = A_c \cos w_c l \qquad 0 < l < \mathrm{T_b}$$

$$h(t-l) = KA_c \cos w_c (t-l) \qquad t - T_b < l < t$$

for $T_b < t < 0$, $\qquad Z(t) = 0$

for $0 < t < T_b$

$$Z(t) = KA_c^2 \int_0^t \cos w_c l \cos w_c (t-l) dl$$

$$= \frac{KA_c^2}{2} \left[t \cos w_c t + \frac{2 \sin w_c t}{2w_c} \right]$$

$$= KE \frac{t}{T_b} [\cos w_c t + \mathrm{sinc}(2N_c t / T_b)]$$

for $T_b < t < 2T_b$,

$$Z(t) = KA_c^2 \int_{t-T_b}^{t} \cos w_c l \cos w_c (t-l) dl$$

$$= \frac{KA_c^2}{2} \left[(2T_b - t)\cos w_c t + \frac{\sin w_c (2T_b - t) - \sin w_c (t - T_b)}{2w_c} \right]$$

$$= KE \left[\left(2 - \frac{t}{T_b} \right) \cos w_c t - \frac{2 \sin w_c t}{2w_c T_b} \right]$$

$$= KE \left\{ 2 \cos w_c t - \frac{t}{T_b} \left[\cos w_c t + \mathrm{sinc}\left(\frac{2N_c t}{T_b} \right) \right] \right\}$$

14.2-8 continued

If $N_c \gg 1$, then $|\text{sinc}(2N_c t/T_b)| \ll 1$ for $t \neq 0$, so

$$z(t) \approx \begin{cases} KE\dfrac{t}{T_b}\cos w_c t & 0 < t < T_b \\ KE\left(2 - \dfrac{t}{T_b}\right)\cos w_c t & T_b < t < 2\,T_b \end{cases}$$

$$\approx \; KE\Lambda\left(\frac{t - T_b}{T_b}\right)\cos w_c t$$

14.2-9

$$E_1 = A_c^2(1+a)^2\int_0^{T_b}\cos^2(w_c t)dt = (1+a)^2 A_c^2 T_b/2 \qquad \text{since } w_c T_b = 2p\,N_c$$

$$E_0 = A_c^2(1-a)^2\int_0^{T_b}\cos^2(w_c t)dt = (1-a)^2 A_c^2 T_b/2$$

$$E_{10} = -A_c^2(1-a)(1+a)\int_0^{T_b}\cos^2(w_c t)dt = -(1-a^2)A_c^2 T_b/2$$

$$E_b = \frac{(1+a)^2 + (1-a)^2}{2}\,\frac{A_c^2 T_b}{2} = (1+a^2)A_c^2 T_b/2$$

$$E_b - E_{10} = \left[(1+a^2)+(1-a^2)\right]A_c^2 T_b/2 = 2E_b/(1+a^2)$$

$$\Rightarrow \quad P_e = Q\left[\sqrt{2g_b/(1+a^2)}\right]$$

14.2-10

$\cos(w_c t - p/2) = \sin w_c t$ and $w_c T_b = 2p\,N_c$ so

$$E_1 = A_c^2\int_0^{T_b}(\cos^2 w_c t + 2a\sin w_c t\cos w_c t + a^2\sin^2 w_c t)dt$$

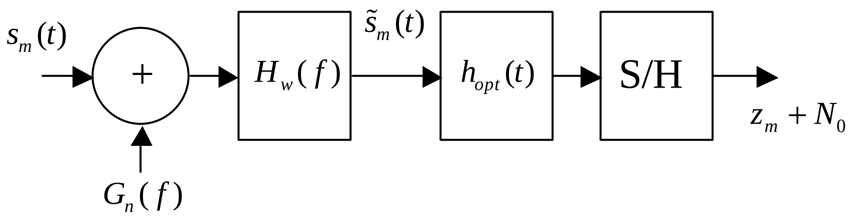
$$= (1+a^2)A_c^2 T_b/2$$

Similarly, $E_0 = E_1$ and $E_b = \dfrac{1}{2}(E_0 + E_1) = (1+a^2)A_c^2 T_b/2$

$$E_{10} = -A_c^2\int_0^{T_b}(\cos^2 w_c t - a^2\sin^2 w_c t)dt = -(1-a^2)A_c^2 T_b/2$$

Thus, $E_b - E_{10} = A_c^2 T_b/2 = \dfrac{E_b}{1+a^2} \quad \Rightarrow \quad P_e = Q\left[\sqrt{2g_b/(1+a^2)}\right]$

14.2-11



$$|H_w(f)|^2 = N_0 / 2G_n(f)$$

$\tilde{S}_m(f) = \Im[\tilde{s}_m(t)] = S_m(f)H_w(f)$ where $S_m(f) = \Im[s_m(t)], \quad m = 0,1$

If $h_{opt}(t) = K[\tilde{s}_1(T_b - t) - \tilde{s}_0(T_b - t)]$ then

$$\left(\frac{z_1 - z_0}{2\sigma}\right)^2_{\max} = \frac{1}{2N_0}(\tilde{E}_1 + \tilde{E}_0 - 2\tilde{E}_{10}) \text{ where}$$

$$\tilde{E}_m = \int_0^{T_b} \tilde{s}_m^2(t)dt = \int_{-\infty}^{\infty} \tilde{s}_m^2(t)dt = \int_{-\infty}^{\infty} |\tilde{S}_m(f)|^2 \, df = \int_{-\infty}^{\infty} |S_m(f)|^2 |H_w(f)|^2 \, df$$

$$\tilde{E}_{10} = \int_0^{T_b} \tilde{s}_1(t)\tilde{s}_0(t)dt = \int_{-\infty}^{\infty} \tilde{s}_1(t)\tilde{s}_0(t)dt$$

$$= \int_{-\infty}^{\infty} \tilde{S}_1(f)\tilde{S}_0^*(f)df = \int_{-\infty}^{\infty} S_1(f)S_0^*(f)\left|H_w(f)\right|^2 df$$

or $$= \int_{-\infty}^{\infty} \tilde{S}_1^*(f)\tilde{S}_0(f)df = \int_{-\infty}^{\infty} S_1^*(f)S_0(f)\left|H_w(f)\right|^2 df$$

Thus, we can write

$$2\tilde{E}_{10} = \int_{-\infty}^{\infty} S^*_1(f)S_0(f)\,|H_w(f)|^2 \, df + \int_{-\infty}^{\infty} S_1(f)S^*_0(f)\,|H_w(f)|^2 \, df$$

so $\tilde{E}_1 + \tilde{E}_0 - 2E_{10}$

$$= \int_{-\infty}^{\infty} \left[\left|S_1(f)\right|^2 + \left|S_0(f)\right| - S_1(f)S_0^*(f) - S_1^*(f)S_0(f)\right] |H_w(f)|^2 \, df$$

$$= \int_{-\infty}^{\infty} |S_1(f) - S_0(f)|^2 \, \frac{N_0}{2G_n(f)} df$$

Hence, $$\left(\frac{z_1 - z_0}{2\sigma}\right)^2_{\max} = \int_{-\infty}^{\infty} \frac{|S_1(f) - S_0(f)|^2}{4G_n(f)} df$$

14.2-12

$$\int_0^{T_b} s_1(t)s_0(t)dt = A_c^2\int_0^{T_b}\cos[2\pi(f_c - f_d)t]\cos[2\pi(f_c + f_d)t]dt$$

$$= \frac{A_c^2}{2}\{\int_0^{T_b}(\cos 4\pi f_c t)dt + \int_0^{T_b}(\cos 4\pi f_d t)dt\} = \frac{A_c^2}{2}[\frac{1}{4\pi f_c}\sin 4\pi f_c T_b + \frac{1}{4\pi f_d}\sin 4\pi f_d T_b]$$

since $f_c T_b$ is an integer $\Rightarrow \sin 4\pi f_c T_b = 0$, and $r_b = 1/T_b$

$$\Rightarrow \int_0^{T_b} s_1(t)s_0(t)dt = \frac{A_c^2}{2}\frac{T_b}{T_b}\frac{\sin 4\pi f_d T_b}{4\pi f_d} = \frac{A_c^2 T_b}{2}\operatorname{sinc}4 f_d / r_b$$

If $E_1 = E_0 = \frac{A_c^2 T_b}{2} \Rightarrow \rho = \frac{1}{\sqrt{E_1 E_0}}\int_0^{T_b} s_1(t)s_0(t)dt = \operatorname{sinc}4 f_d / r_b$

14.2-13

Eq. (9) $\Rightarrow P_e = Q\left[\sqrt{(E_b(1-\rho)/N_0}\right] \Rightarrow$ to minimize $P_e \Rightarrow$ maximize (1- ρ)

$\Rightarrow$ make ρ as negative as possible. With ρ=sinc($4 f_d / r_b$)

From the sinc Table, the maximum negative value of the sinc function =-0.216

$\Rightarrow \rho = \operatorname{sinc}\lambda = -0.216 = \rho$

with $\lambda = 1.4$

14.2-14

(a) Given $P_e = 10^{-5}$ and $N_0 = 10^{-11}$, $E_b = \frac{A_c^2 T_b}{2} = \frac{A_c^2}{2r_b}$, $P_e = Q\left(\sqrt{2E_b / N_0}\right)$

From Table, $P_e = 10^{-5} \Rightarrow 2E_b / N_0 = 18.3 \Rightarrow E_b = \frac{18.3}{2}$ x 10^{-11} = 9.16 x 10^{-11}

$\Rightarrow A_c^2 = 2r_b E_b$ = 2 x 9600 x 9.16 x 10^{-11} = 1.76 x 10^{-6}

$\Rightarrow A_c$ = 0.00133 V.

(b) r_b = 28.8 kpbs $\Rightarrow A_c^2 = 2r_b E_b$ = 2 x 28,800 x 9.16 x 10^{-11} = 5.28 x 10^{-6}

$\Rightarrow A_c$ = 0.0023 V

14.2-15

(a) Given $P_e = 10^{-5}$ and $N_0 = 10^{-11}$, $\mathrm{E_b} = \dfrac{A_c^2 T_b}{2} = \dfrac{A_c^2}{2r_b}$,

Assuming Sunde's FSK, $\Rightarrow P_e = Q\left(\sqrt{E_b / N_0}\right)$

From Table, $P_e = 10^{-5} \Rightarrow E_b / N_0 = 18.3 \Rightarrow E_b = 18.3 \text{ x } 1 \text{ x } 10^{-11} = 1.83 \text{ x } 10^{-10}$

$\Rightarrow A_c^2 = 2r_b E_b = 2 \text{ x } 9600 \text{ x } 1.83 \text{ x } 10^{-10} = 3.51 \text{ x } 10^{-6}$

$\Rightarrow A_c = 0.00187\text{V}.$

(b) $r_b = 28.8$ kpbs $\Rightarrow A_c^2 = 2r_b E_b = 2 \text{ x } 28{,}800 \text{ x } 1.83 \text{ x } 10^{-10} = 1.05 \text{ x } 10^{-5}$

$\Rightarrow A_c = 0.00325$ V

14.3-1

$$\frac{1}{2}\left[e^{-r_b/2} + Q\left(\sqrt{g_b}\right)\right] < 10^{-3} \quad \Rightarrow \quad g_b > 2\ln\left(\frac{1}{2 \text{ x } 10^{-3}}\right) \approx 12.4 \approx 10.9 \text{ dB}$$

so $P_{e1} \approx Q\left(\sqrt{g_b}\right) < 3 \text{ x } 10^{-4}$

14.3-2

$$\frac{1}{2}\left[e^{-r_b/2} + Q\left(\sqrt{g_b}\right)\right] < 10^{-5} \quad \Rightarrow \quad g_b > 2\ln\left(\frac{1}{2 \text{ x } 10^{-5}}\right) \approx 21.6 \approx 13.4 \text{ dB}$$

so $P_{e1} \approx Q\left(\sqrt{g_b}\right) < 2 \text{ x } 10^{-6}$

14.3-3

(a) Given $P_e = 10^{-5}$ and $N_0 = 10^{-11}$, $\mathrm{E_b} = \dfrac{A_c^2 T_b}{2} = \dfrac{A_c^2}{2r_b}$,

Noncoherent FSK, $\Rightarrow P_e = \dfrac{1}{2} e^{-E_b/2N_0} = \dfrac{1}{2} e^{-\gamma_b/2} = 1 \text{ x } 10^{-5}$

$\ln(2 \text{ x } 10^{-5}) = -\gamma_b / 2 \Rightarrow \gamma_b = 21.6 = E_b / N_0 \Rightarrow E_b = 21.6 \text{ x } 1 \text{ x } 10^{-11} = 2.16 \text{ x } 10^{-10}$

$\Rightarrow A_c^2 = 2r_b E_b = 2 \text{ x } 9600 \text{ x } 2.16 \text{ x } 10^{-10} = 4.14 \text{ x } 10^{-6}$

$\Rightarrow A_c = 0.00204\text{V}.$

(b) $r_b = 28.8$ kpbs $\Rightarrow A_c^2 = 2r_b E_b = 2 \text{ x } 28{,}800 \text{ x } 2.16 \text{ x } 10^{-10} = 1.24 \text{ x } 10^{-5}$

$\Rightarrow A_c = 0.00353$ V

14.3-4

(a) Sunde's FSK $\Rightarrow f_d = r_b/2$, $B_T \approx r_b$, and $E_{10} = 0$

$\Rightarrow f_d = 14{,}400/2 = 7200$, S/N=12 dB = 15.9

$$\gamma_b = E_b/N_0 = \left(\frac{S}{N}\right)\left(\frac{B_T}{r_b}\right) = 15.9 \text{ x } 1$$

Coherent FSK $\Rightarrow P_e = Q\left(\sqrt{E_b/N_0}\right) = Q\left(\sqrt{15.9}\right) = 3.4 \text{ x } 10^{-5}$

(b) Noncoherent FSK $\Rightarrow P_e = \frac{1}{2}e^{-\gamma_b/2} = \frac{1}{2}e^{-15.9/2} = 1.76 \text{ x } 10^{-4}$

14.3-5

$KA_c = A_c^2/E_1 = 2/T_b$

$$s^2 = \frac{N_0}{2}\int_{-\infty}^{\infty}|H(f)|^2\,df = \frac{N_0}{2}\int_{-\infty}^{\infty}|h(t)|^2\,dt = \frac{N_0}{2}\frac{2}{T_b}\int_0^{T_b}\cos^2(w_c t)dt$$

$$= \frac{N_0}{T_b}\left[1 + \text{sinc}\left(\frac{4f_c}{r_b}\right)\right] \approx \frac{N_0}{T_b} \text{ since } f_c >> r_b$$

Thus $A_c^2/s^2 = A_c^2 T_b/N_0 = 4E_b/N_0$ where $E_b = A_c^2 T_b/4$

14.3-6

$$s^2 = (N_0/2)*2\int_0^{\infty}|H(f)|^2\,df = N_0\int_0^{\infty}\frac{df}{\left(1+\frac{4(f-f_c)^2}{B^2}\right)}$$

$\approx (N_0 B/2)p$ since $f_c/B >> 1 \Rightarrow \arctan(-2f_c/B) \approx -p/2$

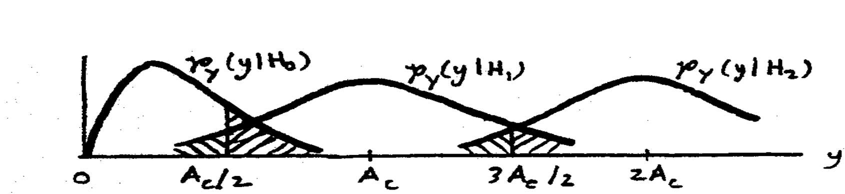
$P_e \approx 1/2P_{eo} = 1/2e^{-A^2{}_c/8s^2}$ where $A^2{}_c/8s^2 = A^2{}_c/4pBN_0$ and

$A^2{}_c = 4E_b r_b$ so $A^2{}_c/8s^2 = 4E_b r_b/4pBN_0 = (2r_b/pB)r_b/2$

$\Rightarrow$ increase r_b by $pB/2r_b \geq p \approx 5$dB

14.3-7

Take $K = A_c/E_1$ and thresholds at $A_c/2$ and $3A_c/2$



14.3-7 continued

Then $P_{eo} = e^{-A_c^2/8\sigma^2}$, $P_{e1} \approx 2Q\left(\frac{A_c}{2\sigma}\right)$, $P_{e2} \approx Q\left(\frac{A_c}{2\sigma}\right)$

$E_0 = 0,\ E_1 \approx A_c^2 D/2,\ E_2 \approx (2A_c)^2 D/2$ where $D = 1/r$

so $\mathrm{E} = \frac{1}{3}(E_0 + E_1 + E_2) = 5A_c^2 D/6 \quad \Rightarrow \quad A_c^2 D = 6E/5$

and $\frac{A_c^2}{\sigma^2} = \frac{4}{N_0}\frac{A_c^2 D}{4} = \frac{6E}{5N_0}$ from Eq.(9) with $E_b = \frac{A_c^2 D}{4}$

Thus, $P_e = \frac{1}{3}\left(P_{e0} + P_{e1} + P_{e2}\right) \approx \frac{1}{3}e^{-3E/20N_0} + Q\left(\sqrt{3E/10N_0}\right)$

14.3-8

$$\frac{S_T}{L} = S_R = E_b r_b = N_0 \gamma_b r_b \quad \Rightarrow \quad r_b = \frac{S_T}{LN_0\gamma_b} = \frac{2 \text{ x } 10^5}{\gamma_b}$$

(a) $\frac{1}{2}e^{-\gamma_b/2} \le 10^{-4} \quad \Rightarrow \quad \gamma_b \ge 2\ln\frac{1}{2 \text{ x } 10^{-4}} \approx 17$ so $r_b \le 2 \text{ x } 10^5/17 = 11.8$ kbps

(b) $\frac{1}{2}e^{-\gamma_b} \le 10^{-4} \quad \Rightarrow \quad \gamma_b \ge \ln\frac{1}{2 \text{ x } 10^{-4}} \approx 8.5$ so $r_b \le 2 \text{ x } 10^5/8.5 = 23.5$ kbps

(c) $Q\left(\sqrt{2\gamma_b}\right) \le 10^{-4} \quad \Rightarrow \quad \gamma_b \ge \frac{1}{2} \text{ x } 3.75^2 \approx 7$ so $r_b \le 2 \text{ x } 10^5/7 = 28.4$ kbps

14.3-9

$$\frac{S_T}{L} = S_R = E_b r_b = N_0 \gamma_b r_b \quad \Rightarrow \quad r_b = \frac{S_T}{LN_0\gamma_b} = \frac{2 \text{ x } 10^5}{\gamma_b}$$

(a) $\frac{1}{2}e^{-\gamma_b/2} \le 10^{-5} \quad \Rightarrow \quad \gamma_b \ge 2\ln\frac{1}{2 \text{ x } 10^{-5}} \approx 21.6$ so $r_b \le 2 \text{ x } 10^5/21.6 = 9.2$ kbps

(b) $\frac{1}{2}e^{-\gamma_b} \le 10^{-5} \quad \Rightarrow \quad \gamma_b \ge 2\ln\frac{1}{2 \text{ x } 10^{-5}} \approx 10.8$ so $r_b \le 2 \text{ x } 10^5/10.8 = 18.4$ kbps

(c) $Q\left(\sqrt{2\gamma_b}\right) \le 10^{-5} \quad \Rightarrow \quad \gamma_b \ge \frac{1}{2} \text{ x } 4.27^2 \approx 9.1$ so $r_b \le 2 \text{ x } 10^5/9.1 = 21.9$ kbps

14.3-10

$$\text{DPSK}: \frac{1}{2}e^{-\gamma_b} \le 10^{-4} \quad \Rightarrow \quad \gamma_b \ge \ln\left(\frac{1}{2 \text{ x } 10^{-4}}\right) = 8.52$$

$$\text{BPSK}: \quad Q\left(\sqrt{2\gamma_b \cos^2\theta_e}\right) \le 10^{-4} \quad \Rightarrow \quad \gamma_b \ge \frac{1}{2}\left(\frac{3.75}{\cos\theta_e}\right)^2 = \frac{7.03}{\cos^2\theta_e}$$

BPSK requires less energy if $|\theta_e| < \arccos\sqrt{7.03/8.52} \approx 25^0$

14.3-11

$$\text{DPSK}: \frac{1}{2}e^{-\gamma_b} \le 10^{-6} \quad \Rightarrow \quad \gamma_b \ge \ln\left(\frac{1}{2 \text{ x } 10^{-6}}\right) = 13.12$$

$$\text{BPSK}: \quad Q\left(\sqrt{2\gamma_b \cos^2\theta_e}\right) \le 10^{-6} \quad \Rightarrow \quad \gamma_b \ge \frac{1}{2}\left(\frac{4..75}{\cos\theta_e}\right)^2 = \frac{11.28}{\cos^2\theta_e}$$

BPSK requires less energy if $|\theta_e| < \arccos\sqrt{11.28/13.12} \approx 22^0$

14.3-12

$$p_y(y) = p_{nq}(y) = \frac{1}{\sqrt{2\pi\sigma^2}}e^{-y^2/2\sigma^2}, \quad p_x(x) = p_{n_i}(x - A_c) = \frac{1}{\sqrt{2\pi\sigma^2}}e^{-(x-A_c)^2/2\sigma^2}$$

$$p_{xy}(x,y) = p_x(x)p_y(y) = \frac{1}{\sqrt{2\pi\sigma^2}}e^{-[(x-A_c)^2+y^2]/2\sigma^2}$$

For polar transformation: $x = A\cos\phi, \quad y = A\sin\phi, \quad dxdy = A\, dA\, d\phi$

so $\quad p_{A\phi}(A,\phi)\, dA\, d\phi = p_{xy}(x,y)dxdy = p_{xy}(A\cos\phi,\, A\sin\phi)A\, dA\, d\phi$

where $(x - A_c)^2 + y^2 = A^2\cos^2\phi - 2AA_c\cos\phi + +A_c^2 + A^2\sin^2\phi = A^2 - 2AA_c\cos\phi + A_c^2$

Thus $\quad p_{A\phi}(A,\phi) = Ap_{xy}(A\cos\phi,\, A\sin\phi) = \dfrac{A}{2\pi\sigma^2}e^{-(A^2-2AA_c\cos\phi+A_c^2)/2\sigma^2}$

14.4-1

$r_b / B_T \ge 1.25 \quad \Rightarrow$ Modulation types with $r_b / B_T = 2$

(a) QAM/QPSK: $Q\left(\sqrt{2\gamma_b}\right) \le 10^{-6} \quad \Rightarrow \quad \gamma_b \ge 1/2 \text{ x } 4.75^2 = 10.5 \text{ dB}$

(b) DPSK with $M = 4$ so $K = 2$: $\dfrac{2}{2}Q\left(\sqrt{4 \text{ x } 2\gamma_b \sin^2\pi/8}\right) \le 10^{-6}$

$$\gamma_b \ge \frac{1}{8}\left(\frac{4.75}{0.383}\right)^2 = 12.8 \text{ dB}$$

14.4-2

$r_b / B_T \geq 2.5 \quad \Rightarrow \quad$ modulation types with $r_b / B_T = 3$

(a) PSK with $M = 8$ so $K = 3$: $\frac{2}{3} Q\left(\sqrt{2 \text{ x } 3\gamma_b \sin^2 \pi/8}\right) \leq 10^{-6}$

$$\gamma_b \geq \frac{1}{6}\left(\frac{4.7}{0.383}\right)^2 = 14.0 \text{ dB}$$

(b) DPSK with $M = 8$ so $K = 3$: $\frac{2}{3} Q\left(\sqrt{\sqrt{4 \text{ x } 3\gamma_b \sin^2 (\pi/16}}\right) \leq 10^{-6}$

$$\gamma_b \geq \frac{1}{12}\left(\frac{4.7}{0.195}\right)^2 = 16.8 \text{ dB}$$

14.4-3

$r_b / B_T \geq 3.2 \quad \Rightarrow$ modulation types with $r_b / B_T = 4$

(a) QAM with $M = 16$ so $K = 4$: $\frac{4}{4}\left(1 - \frac{1}{4}\right) Q\left(\sqrt{\frac{3 \text{ x } 4}{15}\gamma_b}\right) \leq 10^{-6}$

$$\gamma_b \geq \frac{15}{12} 4.7^2 = 14.4 \text{ dB}$$

(b) PSK with $M = 16$ so $K = 4$: $\frac{2}{4} Q\left(\sqrt{2 \text{ x } 4\gamma_b \sin^2 \pi/16}\right) \leq 10^{-6}$

$$\gamma_b \geq \frac{1}{8}\left(\frac{4.6}{0.195}\right)^2 = 18.4 \text{ dB}$$

14.4-4

$r_b / B_T \geq 4.8 \quad \Rightarrow$ modulation types with $r_b / B_T = 5$ or 6

(a) QAM with $M = 64$ so $K = 6$: $\frac{4}{6}\left(1 - \frac{1}{8}\right) Q\left(\sqrt{\frac{3 \text{ x } 6}{63}\gamma_b}\right) \leq 10^{-6}$

$$\gamma_b \geq \frac{63}{18} 4.65^2 = 18.8 \text{ dB}$$

(b) PSK with $M = 32$ so $K = 5$: $\frac{2}{5} Q\left(\sqrt{2 \text{ x } 5\gamma_b \sin^2 (\pi/32}\right) \leq 10^{-6}$

$$\gamma_b \geq \frac{1}{10}\left(\frac{4.55}{0.098}\right)^2 = 23.3 \text{ dB}$$

14.4-5

$$x_c(t) = A_c \cos(w_c t + \phi_k)$$

Upper delay output $= x_c(t-D)2\cos[\omega_c(t-D) + \theta_\varepsilon]$

$$= A_c[\cos(\theta_\varepsilon - \phi_k) + \cos(2\omega_c t - 2\omega_c D + \theta_\varepsilon + \phi_k)]$$

Lower delay output $= x_c(t-D)\{-2\sin[\omega_c(t-D) + \theta_\varepsilon]\}$

$$= -A_c[\sin(\theta_\varepsilon - \phi_k) + \sin(2\omega_c t - 2\omega_c D + \theta_\varepsilon + \phi_k)]$$

LPF input $= A_c\left[\sin\hat{\phi}_k \cos(\theta_\varepsilon - \phi_k) - (-\cos\hat{\phi}_k)\sin(\theta_\varepsilon - \phi_k) + \text{ high frequency terms}\right]$

Thus, $v(t) = A_c\left[\frac{1}{2}\sin(\hat{\phi}_k - \theta_\varepsilon + \phi_k) + \frac{1}{2}\sin(\hat{\phi}_k + \theta_\varepsilon - \phi_k)\right.$

$$\left. + \frac{1}{2}\sin(\theta_\varepsilon - \phi_k - \hat{\phi}_k) + \frac{1}{2}\sin(\theta_\varepsilon - \phi_k + \hat{\phi}_k)\right]$$

$$= A_c \sin(\theta_\varepsilon + \hat{\phi}_k - \phi_k) = A_c \sin(\theta_\varepsilon) \quad \text{when } \hat{\phi}_k = \phi_k$$

14.4-6

QAM: $P_e \approx 3Q\left[\sqrt{\frac{3E}{15N_0}}\right]$ DPSK : $P_e \approx 2Q\left[\sqrt{\frac{4E}{N_0}\sin^2(\pi/32)}\right]$

Since magnitude of P_e is dominated by argument of Q, we want

$$\frac{4E_{DPSK}}{N_0} \text{ x } (0.098)^2 \approx \frac{4E_{QAM}}{15N_0} \quad \Rightarrow \frac{E_{DPSK}}{E_{QAM}} \approx \frac{3}{15 \text{ x } 4 \text{ x } (0.098)^2} = 5.2$$

14.4-7

PSK: $P_e \approx 2Q\left[\sqrt{\frac{2E}{N_o} \text{ x } \left(\frac{\pi}{M}\right)^2}\right]$ QAM : $P_e \approx 4Q\left[\sqrt{\frac{3E}{MN_0}\gamma_b}\right]$

since $\sin\pi/M \approx \pi/M$ since $1-\frac{1}{\sqrt{M}} \approx 1$, $M-1 \approx M$

Magnitude of P_e is dominated by argument of Q, so we want

$$\frac{3E_{QAM}}{MN_0} \approx \frac{2E_{PSK}}{N_o}\left(\frac{\pi}{M}\right)^2 \Rightarrow \frac{E_{QAM}}{E_{PSK}} \approx \frac{2\pi^2}{3M}$$

14.4-8

$$p_f(f) = \int_0^{\infty} p_{Af}(A,f)dA = \frac{e^{-A_c^2/2s^2}}{2ps^2}\int_0^{\infty} Ae^{-(A^2-2AA_c\cos f)/2s^2}dA$$

let $l = (A - A_c \cos f)/s$ and $l_0 = (A_c \cos f)/s$

so $A = s\,(l + l_0)$ and $(A^2 - 2AA_c \cos f)/2s^2 = (l^2/2) - (l_0^2/2)$

$$\text{Then}\quad p_f(f) = \frac{e^{-A_c^2/2s^2}}{2ps^2} e^{l_0^2/2} \int_{-l_0}^{\infty} s\,(l + l_0)e^{-l^2/2} s\,dl$$

$$= \frac{1}{2p} e^{-A_c^2/2s^2} e^{-l^2/2}\left[\int_{-l_0}^{\infty} l e^{-l^2} dl + l_0 \int_{l_0}^{\infty} e^{-l^2} dl\right]$$

$$\text{where}\quad \int_{-l_0}^{\infty} l e^{-l^2/2} dl = e^{-l_0^2/2}$$

$$\int_{-l_0}^{\infty} l e^{-l^2/2} dl = \int_{-\infty}^{\infty} e^{-l^2/2} dl - \int_{-\infty}^{-l_0} e^{-l^2/2} dl = \sqrt{2p}\,[1 - Q(l_0)]$$

$$\text{and}\quad A_c^2/2s^2 - l_0^2/2 = A_c^2(1-\cos^2 f)/2s^2 = A_c^2 \sin^2 f/2s^2$$

$$\text{Thus}\quad p_f(f) = \frac{1}{2p} e^{-A_c^2/2s^2} e^{l_0^2/2}\left\{e^{-l_0^2/2} + l_0\sqrt{2p}\,[1 - Q(l_0)]\right\}$$

$$= \frac{1}{2p} e^{-A_c^2/2s^2} + \frac{A_c \cos f}{\sqrt{2ps^2}} e^{-A_c^2 \sin^2 f/2s^2}\left[1 - Q\left(\frac{A_c \cos f}{s}\right)\right]$$

14.4-9

Use the design of Fig. 14.4-2 and (1) change the 4th law device to a second law device, (2) change the $4f_c$ BPF to a $2f_c$ BPF, (3) change the $\div 4$ block to a $\div 2$ block, (4) eliminate the +90 deg block. $\Rightarrow$ The output of the $\div 2$ block is the reference signal and is $\cos(2\pi f_c t + \pi N)$. The πN term is a phase ambiguity that depends on the lock-in transient and have to be accounted for. This could be done using a known preamble at the beginning of the transmission.

14.4-10

Use the design of Fig. 14.4-2 and (1) change the 4th law device to a Mth-law device, (2) change the $4f_c$ BPF to a Mf_c BPF. The output of the PLL is $\cos[2M\pi t + M\phi_k + 2\pi N]$. The $2\pi N$ term is a phase ambiguity that depends on the lock-in transient and will have to be accounted for. This could be done using a known preamble at the beginning of the transmission (3) At the output of the PLL, change the $\div 4$ block to a $\div M$ block, giving an output of $\cos[2\pi t + \phi_k + 2\pi N/M]$. (4) Replace +90 deg block with an M output phase network.

14.4-11

$\gamma_b = 13$ dB=20, $P_e = P_{be}K$ and $K = \log_2 M$

(a) FSK $\Rightarrow P_e = \frac{1}{2}e^{-\gamma_b/2} = \frac{1}{2}e^{-20/2} = 2.3 \text{ x } 10^{-5}$

(b) BPSK $\Rightarrow P_e = Q\left(\sqrt{2\gamma_b}\right) = Q\left(\sqrt{2 \text{ x } 20}\right) = 1.8 \text{ x } 10^{-10}$

(c) 64-PSK $\Rightarrow P_{be} = \frac{2}{K}Q\left(\sqrt{2K\gamma_b \sin^2\frac{\pi}{M}}\right) = \frac{2}{7}Q\left(\sqrt{2 \text{ x } 7 \text{ x } \gamma_b \text{ x } \sin^2\frac{\pi}{64}}\right)$

$= 0.0571 \Rightarrow P_e = 0.031 \text{ x } 7 = 0.40$

(d) 64-QAM $\Rightarrow P_e = \frac{4}{K}\left(1 - \frac{1}{\sqrt{M}}\right)Q\left(\sqrt{\frac{3K\gamma b}{M-1}}\right)$

$K = 7,\ M = 64 \Rightarrow P_{be} = \frac{4}{7}\left(1 - \frac{1}{\sqrt{64}}\right)Q\left(\sqrt{\frac{3 \text{ x } 7 \text{ x } 20}{63}}\right) = 2.4 \text{ x } 10^{-3}$

$\Rightarrow P_e = P_{be} \text{ x } K = 2.4 \text{ x } 10^{-3} \text{ x } 7 = 0.017$

14.5-1

Eq. (6) $\Rightarrow P_e = N_{\min} Q\left(\sqrt{d^2_{\min}/2N_0}\right) = 1 \text{ x } 10^{-5}$

For uncoded QPSK $\Rightarrow N_{\min} = 2$ and $(d_{\min})_{uncoded} = \sqrt{2}$

$\Rightarrow 1 \text{ x } 10^{-5} = 2Q\left(\sqrt{2/2N_0}\right) \Rightarrow$ solving gives $N_0 = 0.052$

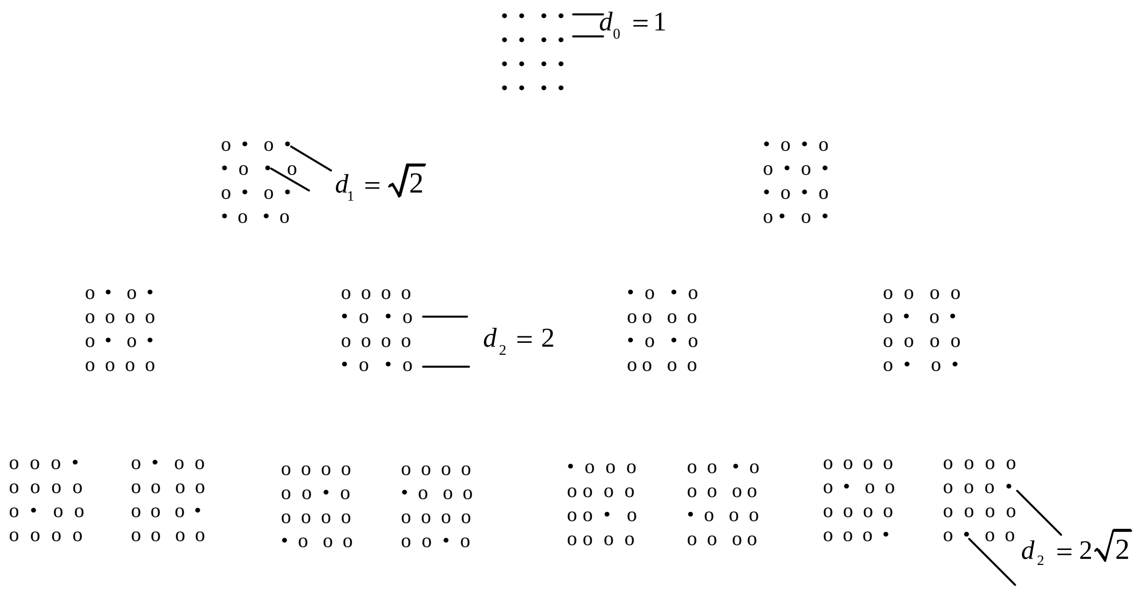
TCM with 8 states, $m = \tilde{m} = 2 \Rightarrow N_{\min} = 2$ and g=3.6 dB=2.3

From Eq. (5) $\Rightarrow g = \frac{(d^2_{\min})_{coded}}{(d^2_{\min})_{uncoded}} = \frac{(d^2_{\min})_{uncoded}}{2} = 2.3 \Rightarrow (d^2_{\min})_{coded} = 4.6$

$\Rightarrow P_e = 2Q\left(\sqrt{\frac{4.6}{2 \text{ x } 0.052}}\right) = 4.0 \text{ x } 10^{-11}$

14.5-2

From Ungerbroeck (1982), we have



14.5-3

Input:	$x_2 x_1$	$00 \rightarrow\ 01 \rightarrow\ 10 \rightarrow\ 01 \rightarrow\ 11 \rightarrow\ 00$
Output:	$y_3 y_2 y_1$	$000 \rightarrow 100 \rightarrow 011 \rightarrow 010 \rightarrow 111 \rightarrow 111$
State:		a b g b h e

14.5-4

What is the distance between paths (0,2,4,2) and (6,1,3,0)?

Using Figs. 14.5-4 and 14.5-7 we have:

$$\sqrt{d^2_{0\to6} + d^2_{2\to1} + d^2_{4\to3} + d^2_{2\to0}} = \sqrt{d_1^2 + d_0^2 + d_0^2 + d_1^2}$$

$$= \sqrt{(\sqrt{2})^2 + (2\sin\pi/8)^2 + (2\sin\pi/8)^2 + (\sqrt{2})^2} = \sqrt{5.2} = 2.3$$

14.5-5

See error event Trellis diagram of Fig. P14.5-5

$$d^2_{\min} = d^2_{0\to6} + d^2_{0\to1} + d^2_{0\to7} + d^2_{0\to2} = d_1^2 + d_0^2 + d_0^2 + d_1^2$$

$$= 2 + 4\sin^2\pi/8 + 4\sin^2\pi/8 + 2 = 5.17$$

$$\Rightarrow \text{coding gain} = 10\log\left(\frac{5.17}{2}\right) = 4.13\ \text{dB}$$

Chapter 15

15.1-1

(a) With DSS $(\mathrm{S/N})_D = (S/N)_R = \dfrac{S_R}{N_R} = \dfrac{E_b r_b}{N_0 r_b} = \dfrac{E_b}{N_0} = 60 \text{ dB} = 10^6$

with $N_0 = 10^{-21} \quad \Rightarrow \quad E_b = 10^{-15}$

$S_R = E_b \times r_b = 10^{-15} \times 3000 = 3 \times 10^{-12} \quad \Rightarrow \quad J = 15 \text{ x } 10^{-12}$

Since $J \gg N_0$ we can neglect noise and use Eq. (13) and $J/S_R = 5$ giving

$$P_e = 10^{-7} = Q\left(\sqrt{\frac{2P_g}{5}}\right) \quad \Rightarrow \quad 5.2^2 = 2P_g/5 \quad \Rightarrow \quad P_g = 67.6$$

Eq. (9) with $W_x = 3000$ $W_c =$ 67.6 x 3000 = 203 kcps

(b) $B_T = 2 \text{ x } W_c = 2 \text{ x } 203 \text{ x } 10^3 = 0.406 \text{ MHz}$

15.1-2

$P_g = 30 \text{ dB} = 1000$ With Eq. (13) we have $P_e = Q\left(\sqrt{\dfrac{2 \text{ x } 1000}{J/S_R}}\right) = 10^{-7}$

$$\Rightarrow \quad 5.2^2 = \frac{2000}{J/S_R} \Rightarrow J/S_R = 74.0 \Rightarrow \text{jamming margain} = 10\log(74) = 18.7 \text{ dB}$$

$$\begin{aligned}\text{Jamming margain} &= 10\times\log(J/S_r) \\ &= 10\times\log(P_g) - 10\times\log(E_b/N_J) \\ &= 10\times\log(1000) - 10\times\log(13.52) = 18.69\end{aligned}$$

15.1-3

(a) $(S/N)_D = 20 \text{ dB} = 100 = E_b/N_0$

$W_c = 10 \text{ x } 10^6$ and $r_b = 6000 = W_x \Rightarrow P_g = 10\times 10^6/6000 = 1.67 \text{ x } 10^3$

With Eq. (19) we have $P_e = 10^{-7} = Q\left(\dfrac{1}{\sqrt{M/(3 \text{ x } 1.67 \text{ x } 10^3) + N_0/2E_b}}\right)$

$$\Rightarrow \quad 5.2^2 = \frac{1}{\dfrac{M}{5000} + \dfrac{1}{200}} \Rightarrow M = 159 \text{ additional users for a total of 160 users.}$$

15.1-3 continued

(b) If each user reduces their power by 6 dB $\Rightarrow$ 6 dB = 4 $\Rightarrow$ $E_b / N_0 = 100/4 = 25$
Using the results of part (a) we have

$$5.2^2 = \frac{1}{\frac{M-1}{5000} + \frac{1}{50}} \Rightarrow M - 1 = 85$$ additional users for a total of 86 users.

15.1-4

Let d = distance between the transmitter and authorized receiver then
$d_m = d + 500$ = distance of the multipath, T_m multipath travel time and c speed of light.
With $d_m = c \times T_m \quad \Rightarrow \quad T_m = d_m/c = 500/3 \text{ x } 10^8 = 1.67\ \mu s$
To avoid multipath interference $T_c < T_m \quad \Rightarrow \quad W_c > 600$ kcps

15.1-5

Using Eq. (2) with $M - 1 = 9$ additional users (10 total users), we have $P_e = Q\left(\sqrt{\frac{3P_g}{9}}\right) = 10^{-7}$

$9 \text{ x } 5.2^2 / 3 = P_g = 81.1 \quad \Rightarrow W_c = P_g \text{ x } r_b = 81.6 \text{ x } 6000 = 487$ kcps = W_c

15.1-6

With $P_e = 10^{-9} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \Rightarrow \frac{2E_b}{N_0} = 6^2 = 36$

Eq. (19), $M - 1 = 9$ additional users and $P_e = 10^{-7} = Q\left(\sqrt{\frac{1}{\frac{9}{3P_g} + \frac{1}{36}}}\right)$

$$\Rightarrow 5.2^2 = \frac{1}{\frac{9}{3P_g} + \frac{1}{36}} \Rightarrow P_g = 326 = \frac{W_c}{6000} \Rightarrow W_c = 1.96 \text{ Mcps}$$

15.1-7

$r_b = 9\text{kbps}, \ J/S_R = 30\text{dB} = 1000$ and $P_e < 10^{-7}$;

Eq. (13) $10^{-7} = Q\left(\sqrt{\frac{2P_g}{1000}}\right) \Rightarrow 5.2^2 = 2P_g/1000 \Rightarrow P_g = 13{,}520$

15.1-8

$P_g = 30\, dB = 1000$ and assuming negligible noise,

Using Eq. (20) $P_e = 10^{-7} = Q\left(\sqrt{\dfrac{3P_g}{M-1}}\right) \Rightarrow 3 \text{ x } 1000/(M-1) = 5.2^2 \Rightarrow M-1 = 111$

$\Rightarrow 112$ total users

15.1-9

$r_b = 6\text{kbps},\ W_c = 10 \text{ Mcps} \Rightarrow P_g = 10 \text{ x } 10^6/6000 = 1667$

For single user $P_e = 10^{-10}$, then Eq. (6) $\Rightarrow$ $10^{-10} = Q\left(\sqrt{\dfrac{2E_b}{N_0}}\right) \Rightarrow 6.4^2 = 41 = \dfrac{2E_b}{N_0}$

If each user reduces their power by 3 dB = 2 $\Rightarrow E_b \to E_b/2 \Rightarrow \dfrac{2E_b}{N_0} \to 41/2 = 20.5$

Eq. (19) we have $10^{-5} = Q\left(\dfrac{1}{\sqrt{\dfrac{M-1}{3 \text{ x } 1667} + \dfrac{1}{20.5}}}\right) \Rightarrow 4.3^2 = \dfrac{1}{\dfrac{M-1}{3 \text{ x } 1667} + \dfrac{1}{20.5}} \Rightarrow$

$M-1 = 26$ additional users $\Rightarrow$ $M = 27$ total users

15.2-1

(a) $E_b/N_0 = 60\text{dB} = 10^6$ and if $N_0 = 10^{-21}$ $\Rightarrow$ $E_b = 10^6/10^{-21} = 10^{-15}$

$S_R = E_b \times r_b = 10^{-15} \times 3000 = 3 \times 10^{-12}$

$\text{J} = 5 \times S_r = 15 \times 10^{-12}$

$P_e = \dfrac{1}{2} e^{-\frac{E_b}{2(N_0 + N_J)}}$ $\Rightarrow$ $10^{-7} = \dfrac{1}{2} e^{-\frac{10^{-15}}{2(10^{-21} + 15 \times 10^{-12})}}$ $\Rightarrow$ $N_J = 3.24 \text{ x } 10^{-17}$

$N_J = \dfrac{J}{W_c}$ $\Rightarrow$ $W_c = \dfrac{15 \times 10^{-12}}{3.2424 \times 10^{-17}} = 4.62 \text{ x } 10^5$

$P_g = \dfrac{W_c}{W_x} = \dfrac{4.62 \text{ x } 10^5}{3 \times 10^3} = 154$

If $k = 7$ $\Rightarrow$ $P_g = 2^7 = 128$

If $k = 8$ $\Rightarrow$ $P_g = 2^8 = 256$

$\Rightarrow$ if $P_g = 256$ $\Rightarrow$ $W_c = 256 \times 3103 = 768$ kHz

(b) $B_T = W_c = 768\text{kHz}$

15.2-2

10 users $\Rightarrow$ $M = 10$

$$P_e = \frac{(1/2)(K-1)}{P_g} + (1/2)\times e^{-Eb/(2\times N_0)}\left[1-\frac{(K-1)}{P_g}\right]$$

With $P_e = 10^{-10}$ for one user $\Rightarrow$ first term in above Eq. dominates.

$$2\times 10^{-5} = \frac{9}{P_g} + 10 - 10\times\left(1 - \frac{9}{P_g}\right) \Rightarrow P_g = 450000$$

But $P_g = 2^k \geq 450{,}000 \Rightarrow k = 19 \Rightarrow P_g = 2^{19} = 524{,}288$

$$P_g = \frac{W_c}{W_x} \Rightarrow W_c = 524{,}288 \text{ x } 3000 = 1.57 \text{ GHz}$$

15.2-3

From problem 15.2.1 $S_R = 3\times 10^{-12}$, $J = 1.5\times 10^{-11}$, W_x=3000

Using Sect 15.1, Eq. (13) we have $10^{-7} = Q\left(\sqrt{\frac{2P_g}{J/S_R}}\right) = Q\left(\sqrt{\frac{2W_c/3000}{5}}\right)$

$$\Rightarrow \frac{2W_c/3000}{5} = 5.2^2 \Rightarrow W_c = 2 \text{ x } 10^5$$

$P_g = W_c/W_x = 2 \text{ x } 10^5/3000 = 67$

$B_T = 2\times W_c = 400$ kHz

15.2-4

With $k = 10$, $\Rightarrow$ $P_g = 2^{10} = 1024$

and with r_b=6000 $\Rightarrow$ $W_c = P_g W_x = 1024\times 6000 = 6144$ kbps

$N_J = J/W_c = 6\times 10^{-3}/1024 = 9.76\times 10^{-10}$

From Eq. (4) $P_e = \frac{1-0.1}{2}e^{-2 \text{ x } 10^{-11}/2 \text{ x} 10^{-12}} + \frac{0.1}{2}e^{\frac{-2 \text{ x } 10^{-11}}{2\left(2 \text{ x } 10^{-12} + 9.766 \text{ x } 10^{-10}/0.1\right)}}$

$P_e = 2.04\times 10^{-5} + 4.99\times 10^{-2} = 0.05$

15.2-5

$d = 5$ miles $\Rightarrow$ $d = 5\times 1610 = 8050$meters

$\Delta d = 2\times 8050 = 16100$

$\Delta d = v\times t$ $\Rightarrow$ $t = \Delta d / t = 16100/\left(2.99\times 10^8\right) = 5.38\times 10^{-9}$ s

$f = 1/t = 1/\left(5.38\times 10^{-9}\right) = 18.6$ kHz

15.3-1

(a) A shift register with [4,1] configuration, and initial state of 0100 has the following contents after each clock pulse:

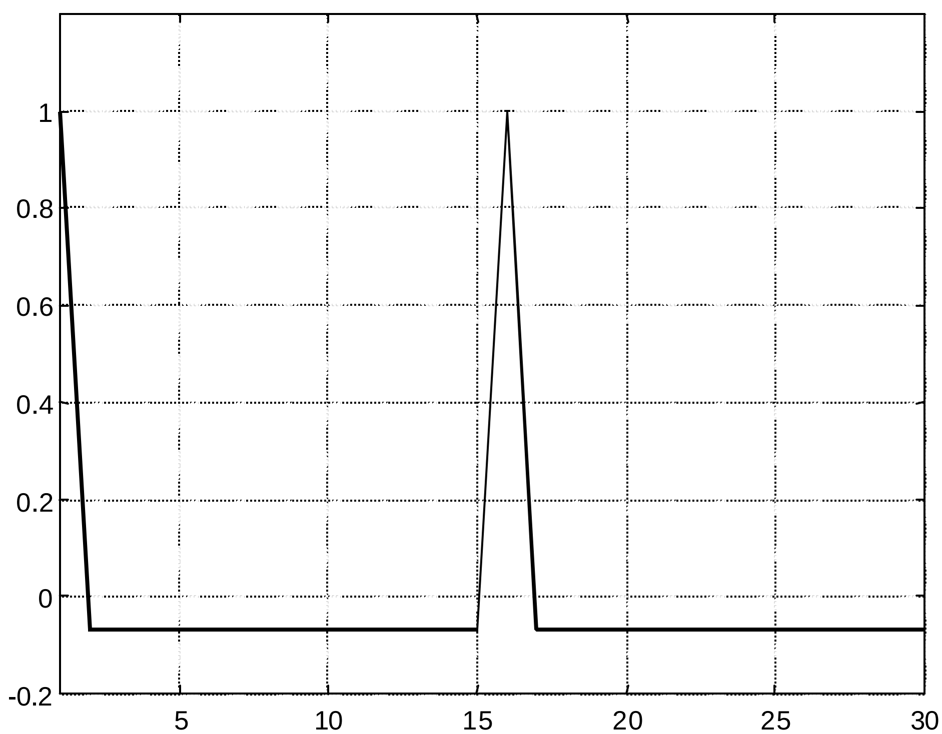
Clock shift	Register contents	Clock shift	Register contents
0	0100	8	1011
1	0010	9	0101
2	0001	10	1010
3	1000	11	1101
4	1100	12	0110
5	1110	13	0011
6	1111	14	1001
7	0111	15	0100

Thus, the output sequence= 001000111101011…

(b) PN sequence length = $N = 15$

(c) $f_c = 10$ MHz $\Rightarrow$ $T_c = 10^{-7}$ s. With $T_{PN \text{ sequence}} = NT_c = 15 \text{ x } 10^{-7}$ =1500 ns

(d) Autocorrelation function, $R_{[4,1],[4,1]}(\tau)$ versus τ



15.3-2

(a) Shift register with [4,2] configuration, and initial state of 0100 has the following contents after each clock pulse:

Clock shift	Register contents
0	0100
1	1010
2	0101
3	0010
4	0001
5	1000
6	0100

The output sequence: 001010…

(b) PN sequence length = $N = 6$

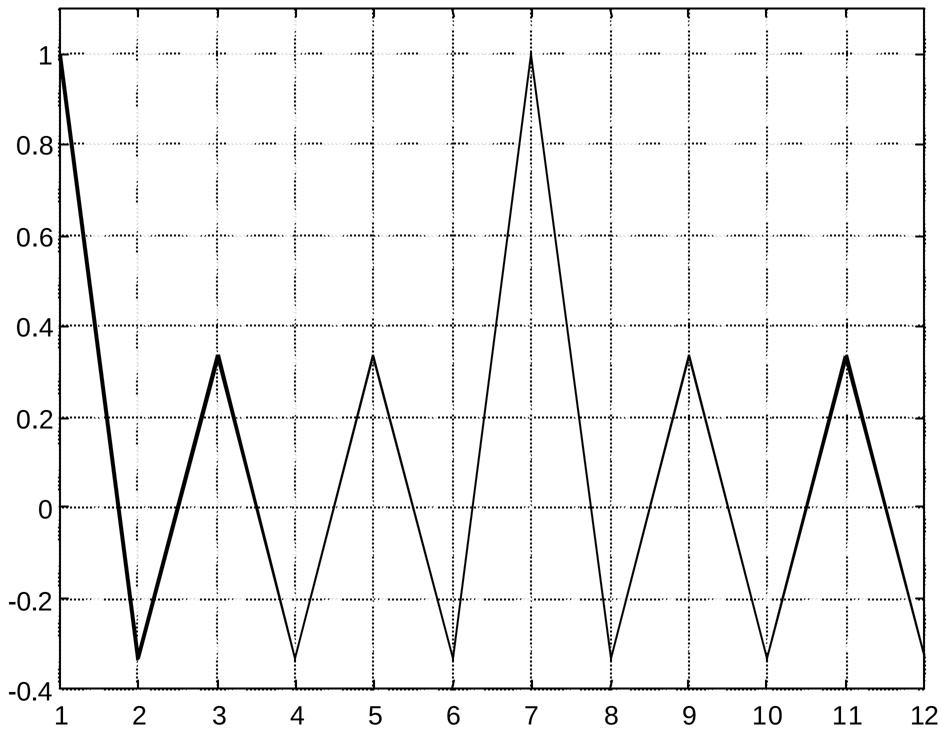
(c) $f_c = 10$ MHz $\Rightarrow$ $T_c = 10^{-7}$ s. With $T_{PN \text{ sequence}} = NT_c$ = 6 x 10^{-7} =600 ns

(d) To calculate the autocorrelation function, we use the method of Example 11.4-1 to get:

τ	original/shifted	$v(\tau)$	$R_{[4,2][4,2]}(t) = v(t) / N$
0	001010 001010	6-0=6	6/6=6
1	001010 000101	2-4=-2	-2/6 =-0.33
2	001010 100010	4-2=2	0.33
3	001010 010001	2-4=-2	-0.33
4	001010 101000	4-2=2	0.33
5	001010 010100	2-4=-2	-0.33
6	001010 001010	6-0=6	1

15.3-2 continued

The plot of $R_{[4,2][4,2]}(\tau)$ versus τ



15.3-3

Given the results of Problem 15.3-1,

(1) Number of 1s= 8, number of 0s: 7 $\Rightarrow$ satisfies balance property.

(2) Length of single type of digit: 4/8 of length 1, 2/8 of length 2, 1/8 of length 3, 1/8 of length 4 $\Rightarrow$ satisfies run property.

(3) Single autocorrelation peak $\Rightarrow$ satisfies autocorrelation property.

(4) Mod 2 addition of the output with a shifted version results in another shifted version

(5) All 15 states exist.

$\Rightarrow$ A [4,1] register produces a ml sequence.

15.3-4

A shift register with [5,4] configuration, and initial state of 11111 has the following contents after each clock pulse:

15.3-4 continued

Clock pulse	register contents	clock pulse	register contents
0	11111	10	11000
1	01111	11	01100
2	00111	12	00110
3	00011	13	10011
4	00001	14	01001
5	10000	15	10100
6	01000	16	01010
7	00100	17	10101
8	00010	18	11010
9	10001	19	11101
10	11000	20	11110
		21	11111

The output sequence is thus: $111110000100011001010 1 \Rightarrow N = 21$

To be a ml sequence, the PN length should be $N = 2^n - 1 = 2^5 - 1 = 31$

$21 \le 31 \quad \Rightarrow$ the shift register does not produce a ml sequence

15.3-5

(a) From Ex. 11.4-1 and Sect. 11.4 we get the output sequences from [5,4,3,2] and [5,2] configurations to obtain [5,2] $\oplus$[5,4,3,2]=

1111100110100100001010111011000

$\oplus$ 1111100100110000101101010001110

0000000010010100100111101010110 = output sequence

(b) $\left|R_{st}\right| = \left(\dfrac{2^{\frac{n+1}{2}} + 1}{N} \right) = \left(2^3 - 1\right)/31 = 0.29$

15.3-6

0.01 miles x 1610 meters/mile = 16.1 meters

From Eq. (7) we have $T_c = \Delta d / c = 16.1/3 \text{ x } 10^8 = 53.7$ ns

$\Rightarrow \quad f_c = 1/T_c = 1/53.8 \text{ x } 10^{-9} = 18.6$ MHz

15.4-1

With 5 Hz/hour drift $\Rightarrow$ chip uncertainty/day = 5 chips/hour x 24 hour/day = 120 chips

15.4-2

f_c = 900 MHz, f_{clock} = 10 MHz, and v = 500 Mph, and light speed = c = 3 x 10^8 M/s

(a) Doppler shift = $\Delta f_c = \pm \dfrac{v f_c}{c} = \dfrac{500 \text{ Mph x } 1610 \text{ M/mile x 1 hour/3600 s x } 900 \text{ x } 10^6 \text{ s}^{-1}}{3 \text{ x } 10^8 \text{ M/s}}$

$= 671$ Hz

(b) Doppler shift = $\Delta f_{clock} = \pm \dfrac{v f_{clock}}{c} = \dfrac{500 \text{ Mph x } 1610 \text{ M/mile x 1 hour/3600 s x } 10 \text{ x } 10^6 \text{ s}^{-1}}{3 \text{ x } 10^8 \text{ M/s}}$

$= 7.45$ chips

15.4-3

(a) Using Eq. (2) with a preamble of $l = 2047$, $T_c = 1/f_{clock}$ =1 x 10^{-7}s α=100, $P_D = 0.9$, $P_{FA} = 0.01$, and assuming that the average phase error is 2048/2 chips we have

$$\overline{T_{acq}} = \frac{2 - P_D}{P_D}(1 + \alpha P_{FA}) N_c l T_c = \frac{2\text{-}0.9}{0.9}(1\text{+}100 \text{ x } 0.01) \text{ x } 1024 \text{ x } 2047 \text{ x } 10^{-7}$$

$= 0.51$

(b) Using Eq. (3) we have

$$\sigma^2_{Tacq} = (2 \text{ x } 1024 \text{ x } 2047 \text{ x } 10^{-7})^2 \text{ x } (1\text{+}100 \text{ x } 0.01)^2 \left(\frac{1}{12} + \frac{1}{0.9^2} - \frac{1}{0.9}\right) = 0.15$$

$\sigma_{Tacq} = 0.38$

15.4-4

(a) 12 stage shift register $\Rightarrow$ $l = 4095$, then using Eq. (2) with $T_c = 1/f_{clock}$ =2 x 10^{-8}s α=10, $P_D = 0.9$, $P_{FA} = 0.001$, and assuming that the average phase error is 4096/2 chips we have

$$\overline{T_{acq}} = \frac{2 - P_D}{P_D}(1 + \alpha P_{FA}) N_c l T_c = \frac{2\text{-}0.9}{0.9}(1\text{+}10 \text{ x } 0.001) \text{ x } 2048 \text{ x } 4097 \text{ x } 2 \text{ x } 10^{-8}$$

$= 0.21$

(b) Using Eq. (3) we have

$$\sigma^2_{Tacq} = (2 \text{ x } 2048 \text{ x } 4095 \text{ x } 2 \text{ x } 10^{-8})^2 \text{ x } (1\text{+}10 \text{ x } 0.001)^2 \left(\frac{1}{12} + \frac{1}{0.9^2} - \frac{1}{0.9}\right) = 0.024$$

$\sigma_{Tacq} = 0.15$

Chapter 16

16.1-1

$P(\text{not } F) = 4/5 \Rightarrow I = \log 5/4 = 0.322$ bits, $P(\text{specific grade}) = 1/5 \Rightarrow I = \log 5 = 2.322$ bits

so $I_{\text{needed}} = 2.322 - 0.322 = 2$ bits

16.1-2

(*a*) $P(\text{heart}) = 13/52 = 1/4 \Rightarrow I = \log 4 = 2$ bits, $P(\text{face card}) = 12/52 = 3/13 \Rightarrow$

$I = \log 13/3 = 2.12$ bits, $I_{\text{heart face card}} = 2 + 2.12 = 4.12$ bits

(*b*) $P(\text{red face card}) = 6/52 \Rightarrow I_{\text{given}} = \log 52/6 = 3.12$ bits, $P(\text{specific card}) = 1/52 \Rightarrow$

$I = \log 52$, $I_{\text{needed}} = \log 52 - 3.12 = 2.58$ bits

16.1-3

Including the direction of the first turn, the number of different combinations is

$2 \times 10^2 \times 10^2 \times 10^2$, assumed to be equally likely. Thus, $I = \log(2 \times 10^6) = 20.9$ bits

16.1-4

$H(X) = \frac{1}{2}\times 1 + \frac{1}{4}\times 2 + \frac{1}{8}\times 3 + \frac{1}{\ln 2}\left(2\times\frac{1}{20}\ln 20 + \frac{1}{40}\ln 40\right) = 1.94$ bits, $6H(X) = 11.64$

$$P(ABABBA) = \left(\tfrac{1}{2}\right)^3 \times \left(\tfrac{1}{4}\right)^3 = \frac{1}{2^9} \Rightarrow I = \log 2^9 = 9 \text{ bits} < 6H(X)$$

$$P(FDDFDF) = \left(\tfrac{1}{40}\right)^3 \times \left(\tfrac{1}{20}\right)^3 = \frac{1}{512\times 10^6} \Rightarrow I = \log(512 \times 10^6) = 28.93 \text{ bits} > 6H(X)$$

16.1-5

$$H(X) = \frac{1}{\ln 2}\left(0.4\ln\frac{1}{0.4} + 0.2\ln\frac{1}{0.2} + 0.12\ln\frac{1}{0.12} + 2\times 0.1\ln\frac{1}{0.1} + 0.08\ln\frac{1}{0.08}\right) = 2.32 \text{ bits,}$$

$6H(X) = 13.90$ bits

$$P(ABABBA) = (0.4)^3(0.2)^3 = 5.12 \times 10^{-4} \Rightarrow I = \log\frac{1}{5.12\times 10^{-4}} = 10.93 \text{ bits} < 6H(X)$$

$$P(FDDFDF) = (0.1)^3(0.08)^3 = 5.12 \times 10^{-7} \Rightarrow I = \log\frac{1}{5.12\times 10^{-7}} = 20.90 \text{ bits} > 6H(X)$$

16.1-6

Since the first symbol is always the same, there are $M = 8 \times 8 = 64$ different blocks, and $H(X) = \log 64 = 6$ bits/block. Thus, $R = 1000$ blocks/sec $\times$ 6 bits/block = 6000 bits/sec

16.1-7

There are $M = 16^{15}$ different blocks, and $H(X) = \log 16^{15} = 60$ bits/block at the rate $r = 1/(15 + 5)$ms = 50 blocks/sec, so $R = 50 \times 60 = 3000$ bits/sec

16.1-8

$$P_{dot} + P_{dash} = \tfrac{3}{2} P_{dot} = 1 \quad \Rightarrow \quad P_{dot} = \tfrac{2}{3},\ P_{dash} = \tfrac{1}{3} \ \text{ so } \ H(X) = \tfrac{2}{3}\log\tfrac{3}{2} + \tfrac{1}{3}\log 3 = 0.920 \ \text{ bits/symbol}$$

$$T_{dash} = 2T_{dot} = 0.4, \quad \tfrac{1}{r} = \overline{T} = \tfrac{2}{3}\times 0.2 + \tfrac{1}{3}\times 0.4 = \tfrac{0.8}{3} \ \text{ sec/symbol}$$

Thus, $R = (3/0.8) \times 0.920 = 3.45$ bits/sec

16.1-9

$$P_3 = 1 - (P_1 + P_2) = \tfrac{2}{3} - p \quad \Rightarrow \quad H(X) = \tfrac{1}{3}\log 3 + p\log\frac{1}{p} + \left(\tfrac{2}{3} - p\right)\log\frac{1}{\left(\tfrac{2}{3} - p\right)}$$

When $p = 0$ or $2/3$, $H(X) = \tfrac{1}{3}\log 3 + \tfrac{2}{3}\log\tfrac{3}{2} = 0.918$ bits

16.1-10

Let $P_1 = \alpha$ so $P_i = (1 - \alpha)/(M - 1)$ for $i = 2, 3, \ldots, M$

$$H(X) = \alpha\log\frac{1}{\alpha} + (M-1)\frac{1-\alpha}{M-1}\log\frac{M-1}{1-\alpha} = \alpha\log\frac{1}{\alpha} + (1-\alpha)\left[\log(M-1) - \log(1-\alpha)\right]$$

$$= \log(M-1) + \alpha\left[\log\frac{1}{\alpha} - \log(M-1)\right] - (1-\alpha)\log(1-\alpha)$$

But $\log(1-\alpha) = \dfrac{1}{\ln 2}\ln(1-\alpha) = \dfrac{1}{\ln 2}\left(-\alpha - \tfrac{1}{2}\alpha^2 - \tfrac{1}{3}\alpha^3 - \cdots\right)$ so

$-(1-\alpha)\log(1-\alpha) \approx \dfrac{(1-\alpha)\alpha}{\ln 2} \approx \dfrac{\alpha}{\ln 2} = \alpha\dfrac{\ln e}{\ln 2} = \alpha\log e$. Thus,

$$H(X) \approx \log(M-1) + \alpha\left[\log\frac{1}{\alpha} - \log(M-1) + \log e\right]$$

$$\approx \log(M-1) + \alpha\log\tfrac{1}{\alpha} \ \text{ if } \ \tfrac{1}{\alpha} \gg M-1 \text{ and } \tfrac{1}{\alpha} \gg e$$

16.1-11

$-\log P_i \oslash N_i < -\log P_i + 1 \;\Rightarrow\; 2^{\log P_i} \geq 2^{-N_i} > 2^{\log P_i}2^{-1}$ so $P_i \geq 2^{-N_i} > \frac{1}{2}P_i$

Thus, $\sum_{i=1}^{M} P_i \geq \sum_{i=1}^{M} 2^{-N_i} > \frac{1}{2}\sum_{i=1}^{M} P_i$ where $\sum_{i=1}^{M} P_i = 1$. Hence, $1 \geq \sum_{i=1}^{M} 2^{-N_i} > \frac{1}{2} \;\Rightarrow\; \frac{1}{2} < K \leq 1$

16.1-12

x_i	P_i	1	2	3	4	5	Codewords	P_iN_i
A	1/2	0					0	0.5
B	1/4	1	0				10	0.5
C	1/8	1	1	0			110	0.375
D	1/20	1	1	1	0		1110	0.2
E	1/40	1	1	1	1	0	11110	0.25
F	1/40	1	1	1	1	1	11111	0.125

$H(X)$ = 1.940 bits from Prob. 16.1-4, and $\overline{N} = 1.950$, so $H(X)/\overline{N} = 1.940/1.950 = 99.5\%$

16.1-13

Since $P_A = 0.4$ and $P_A + P_B = 0.6$, the dividing line at the first coding step can be between *A* and *B* or between *B* and *C*.

x_i	P_i					Code I	P_iN_i				Code II	P_iN_i
A	0.4	0				0	0.4	0	0		00	0.8
B	0.2	1	0	0		100	0.6	0	1		01	0.4
C	0.12	1	0	1		101	0.36	1	0	0	100	0.36
D	0.1	1	1	0		110	0.3	1	0	1	101	0.3
E	0.1	1	1	1	0	1110	0.4	1	1	0	110	0.3
F	0.08	1	1	1	1	1111	0.32	1	1	1	111	0.24

$H(X)$ = 2.32 bits from Prob. 16.1-5, and $\overline{N} \approx 2.4$, so $H(X)/\overline{N} \approx 2.32/2.4 \approx 97\%$

16.1-14

$$H(X) = 0.5 + \frac{1}{\ln 2}\left(0.4\ln\frac{1}{0.4} + 0.1\ln\frac{1}{0.1}\right) = 1.36 \text{ bits}$$

(*a*) $\overline{N} = 1.5$ so $H(X)/\overline{N} = 1.36/1.5 = 90.5\%$

x_i	P_i			Code	P_iN_i
A	0.5	0		0	0.5
B	0.4	1	0	10	0.8

C	0.1	1	1	11	0.2

(*b*) $2\overline{N} = 2.78$ so $H(X)/\overline{N} = 1.36/1.39 \approx 97.8\%$

(cont.)

x_{ij}	P_{ij}							Codewords	$P_{ij}N_{ij}$
AA	0.25	0	0					00	0.5
AB	0.2	0	1					01	0.4
BA	0.2	1	0					10	0.4
BB	0.16	1	1	0				110	0.48
AC	0.05	1	1	1	0	0		11100	0.25
CA	0.05	1	1	1	0	1		11101	0.25
BC	0.04	1	1	1	1	0		11110	0.20
CB	0.04	1	1	1	1	1	0	111110	0.24
CC	0.01	1	1	1	1	1	1	111111	0.06

16.1-15

$$H(X) = \frac{1}{\ln 2}\left(0.8\ln\frac{1}{0.8} + 0.2\ln\frac{1}{0.2}\right) = 0.7219 \text{ bits}$$

(*a*) $2\overline{N} = 1.56$ so $H(X)/\overline{N} = 0.7219/0.78 = 92.6\%$

x_{ij}	P_{ij}				Codeword	$P_{ij}N_{ij}$
AA	0.64	0			0	0.64
AB	0.16	1	0		10	0.32
BA	0.16	1	1	0	110	0.48
BB	0.04	1	1	1	111	0.12

(*b*) $3\overline{N} = 2.184$ so $H(X)/\overline{N} = 0.7219/0.728 = 99.2\%$

x_{ijk}	P_{ijk}						Codeword	$P_{ijk}N_{ijk}$
AAA	0.512	0					0	0.512
AAB	0.128	1	0	0			100	0.384
ABA	0.128	1	0	1			101	0.384
BAA	0.128	1	1	0			110	0.384

ABB	0.032	1	1	1	0	0	11100	0.160
BAB	0.032	1	1	1	0	1	11101	0.160
BBA	0.032	1	1	1	1	0	11110	0.160
BBB	0.008	1	1	1	1	1	11111	0.040

16.1-16

$$H(X) = P_0 H(X|0) + P_1 H(X|1) = \tfrac{1}{2}\left[H(X|0) + H(X|1)\right]$$

$$P_{01} = P(0|1) = 3/4 \quad \Rightarrow \quad P_{11} = 1 - P(0|1) = 1/4$$

$$P_{10} = P(1|0) = 3/4 \quad \Rightarrow \quad P_{00} = 1 - P(1|0) = 1/4$$

Thus, $H(X|0) = H(X|1) = \tfrac{3}{4}\log\tfrac{4}{3} + \tfrac{1}{4}\log 4 = 0.811$ and $H(X) = 2\times\tfrac{1}{2}0.811 = 0.811$ bits

16.2-1

$P(x_i y_j) = P(x_i | y_j)P(y_j)$ and $\sum_x P(x_i y_j) = P(y_j)$ so $H(X,Y) = \sum_{x,y} P(x_i y_j)\log\dfrac{1}{P(x_i | y_j)P(y_j)}$

$$= \sum_y \left[\sum_x P(x_i y_j)\right]\log\frac{1}{P(y_j)} + \sum_{x,y} P(x_i y_j)\log\frac{1}{P(x_i | y_j)} = H(Y) + H(X|Y)$$

16.2-2

$P(x_i | y_j) = P(x_i y_j)/P(y_j)$ so

$$I(X;Y) = \sum_{x,y} P(x_i y_j)\log\frac{P(x_i y_j)}{P(x_i)P(y_j)} = \sum_{x,y} P(x_i y_j)\left[\log\frac{1}{P(x_i)} + \log\frac{1}{P(y_j)} - \log\frac{1}{P(x_i y_j)}\right]$$

where $\sum_{x,y} P(x_i y_j)\log\dfrac{1}{P(x_i)} = \sum_x \underbrace{\left[\sum_y P(x_i y_j)\right]}_{P(x_i)}\log\dfrac{1}{P(x_i)} = H(X)$

and likewise $\sum_{x,y} P(x_i y_j)\log\dfrac{1}{P(y_j)} = H(Y)$

Thus, $I(X;Y) = H(X) + H(Y) - \sum_{x,y} P(x_i y_j)\log\dfrac{1}{P(x_i y_j)} = H(X) + H(Y) - H(X,Y)$

16.2-3

$$P(x_i y_j) = P(y_j | x_i)P(x_i) = \begin{cases} P(x_i) & j = i \\ 0 & j \neq i \end{cases} \quad \Rightarrow \quad P(y_j) = \sum_{i=1}^{M} P(x_i y_j) = P(x_j).$$

(cont.)

Thus, $H(Y|X)=\sum_i\sum_j P(x_i y_j)\log\frac{1}{P(y_j|x_i)}=\sum_i P(x_i)\log\frac{1}{P(y_i|x_i)}=\sum_i P(x_i)\log 1=0$

$I(X;Y)=H(Y)-H(Y|X)=H(X)-0=H(X)$

16.2-4

$P(y_1) = p(1 - \alpha) + (1 - p)\beta = \beta + (1 - \alpha - \beta)p$, $P(y_2) = (1 - p)(1 - \beta) + p\alpha = 1 - \beta - (1 - \alpha - \beta)p$

$= 1 - P(y_1)$ so $H(Y)=P(y_1)\log\frac{1}{P(y_1)}+P(y_2)\log\frac{1}{P(y_2)}=\Omega\left[\beta+\left(1-\alpha-\beta\right)p\right]$

$$H(Y|X)=p\left[(1-\alpha)\log\frac{1}{1-\alpha}+\alpha\log\frac{1}{\alpha}\right]+(1-p)\left[\beta\log\frac{1}{\beta}+(1-\beta)\log\frac{1}{1-\beta}\right]$$

$$= p\Omega(\alpha)+(1-p)\Omega(\beta) \text{ so}$$

$$I(X;Y)=H(Y)-H(Y|X)=\Omega[\beta+(1-\alpha-\beta)p]-p\Omega(\alpha)-(1-p)\Omega(\beta)$$

16.2-5

If $\beta = 1 - \alpha$, then $P(y_1|x_1) = P(y_1|x_2)$ and $P(y_2|x_1) = P(y_2|x_2)$ so the occurrence of y_1 or y_2 gives no information about the source. Analytically, if $\beta = 1 - \alpha$, then $\Omega(\beta) = \Omega(1 - \alpha)$ and $\Omega[\beta + (1 - \alpha - \beta)p] = \Omega(1 - \alpha)$. Thus, $I(X;Y)=\Omega(\alpha)-p\Omega(\alpha)-(1-p)\Omega(\alpha)=0$ so no information is transferred.

16.2-6

$$\Omega(a+bp)=(a+bp)\log\frac{1}{a+bp}+(1-a-bp)\log\frac{1}{1-a-bp}$$

$$=-\left[(a+bp)\log(a+bp)+(1-a-bp)\log(1-a-bp)\right] \text{ and } \frac{d}{dp}\left[\log_2 x(p)\right]=\left(\log_2 e\right)\frac{1}{x}\frac{dx}{dp}$$

$$\text{so } \frac{d}{dp}\Omega(a+bp)=-b\log(a+bp)-(a+bp)(\log e)\frac{b}{a+bp}-(-b)\log(1-a-bp)$$

$$-(1-a-bp)(\log e)\frac{-b}{1-a-bp}$$

$$=-b\left[\log(a+bp)-\log(1-a-bp)\right]=b\log\frac{1-a-bp}{a+bp}$$

(cont.)

$$\frac{d}{dp}I(X;Y)=\frac{d}{dp}\Omega[\alpha+(1-2\alpha)p]+0=(1-2\alpha)\log\frac{1-\alpha-(1-2\alpha)p}{\alpha+(1-2\alpha)p}=0 \text{ so}$$

$$\frac{1-\alpha-(1-2\alpha)p}{\alpha+(1-2\alpha)p}=1 \quad\Rightarrow\quad 1-2\alpha=2(1-2\alpha)p \quad\Rightarrow\quad p=\tfrac{1}{2}$$

16.2-7

$$I(X;Y)=\Omega(p/2)-p\Omega(1/2)-(1-p)\Omega(0)=\Omega(p/2)-p \quad \text{since } \Omega(1/2)=1 \text{ and } \Omega(0)=0$$

and $\frac{d}{dp}\Omega\left(0+\tfrac{1}{2}p\right)=\tfrac{1}{2}\log\frac{1-\frac{1}{2}p}{\frac{1}{2}p}=\tfrac{1}{2}\log\frac{2-p}{p}$ so

$$\frac{d}{dp}I(X;Y)=\tfrac{1}{2}\log\frac{2-p}{p}-1=0 \quad\Rightarrow\quad \frac{2-p}{p}=2^2 \quad\Rightarrow\quad p=\tfrac{2}{5}$$

Thus, $C_s=\Omega\left(\tfrac{1}{5}\right)-\tfrac{2}{5}=\tfrac{1}{5}\log 5+\tfrac{4}{5}\log\tfrac{5}{4}-\tfrac{2}{5}=0.322$ bits/symbol

16.2-8

$$I(X;Y)=\Omega(3p/4)-p\Omega(1/4)-(1-p)\Omega(0)=\Omega(3p/4)-0.811p \quad \text{since } \Omega(0)=0$$

and $\frac{d}{dp}\Omega\left(0+\tfrac{3}{4}p\right)=\tfrac{3}{4}\log\frac{1-\frac{3}{4}p}{\frac{3}{4}p}=\tfrac{3}{4}\log\frac{4-3p}{3p}$ so

$$\frac{d}{dp}I(X;Y)=\tfrac{3}{4}\log\frac{4-3p}{3p}-0.811=0 \quad\Rightarrow\quad \frac{4-3p}{3p}=2^{(4/3)0.811}=2.12 \quad\Rightarrow\quad p=0.428$$

Thus, $C_s=\Omega(0.321)-0.811\times 0.428=0.557$ bits/symbol

16.2-9

$$P(0)=P(1)=\tfrac{1}{2}(1-\alpha),\ P(E)=\tfrac{1}{2}\alpha+\tfrac{1}{2}\alpha=\alpha$$

$$H(Y)=2\frac{1-\alpha}{2}\log\frac{2}{1-\alpha}+\alpha\log\frac{1}{\alpha}=(1-\alpha)\log 2+(1-\alpha)\log\frac{1}{1-\alpha}+\alpha\log\frac{1}{\alpha}=1-\alpha+\Omega(\alpha)$$

$$H(Y|X)=2\times\tfrac{1}{2}\left[(1-\alpha)\log\frac{1}{1-\alpha}+\alpha\log\frac{1}{\alpha}+0\log\frac{1}{0}\right]=\Omega(\alpha)$$

Thus, $C_s=\max I(X;Y)=H(Y)-H(Y|X)=1-\alpha$

16.2-10

$$P(|n-\overline{n}|\geq k\sigma)=P(n-\overline{n}\geq k\sigma)+P(n-\overline{n}\leq -k\sigma)\leq 1/k^2 \text{ so}$$

$$P(n\geq\overline{n}+k\sigma)\leq\tfrac{1}{k^2}-P(n-\overline{n}\leq -k\sigma)\leq 1/k^2$$

Let $d = \bar{n} + k\sigma \quad \Rightarrow \quad k = \dfrac{d - \bar{n}}{\sigma}$ where $d = N\beta, \bar{n} = N\alpha, \sigma^2 = N\alpha(1-\alpha)$

Then $P(n \geq d) \leq \left(\dfrac{\sigma}{d - \bar{n}}\right)^2 = \dfrac{N\alpha(1-\alpha)}{(N\beta - N\alpha)^2} = \dfrac{\alpha(1-\alpha)}{N(\beta-\alpha)^2}$

16.3-1

$p(x) = 1/2a \quad \text{for} \quad |x| \leq a, \quad S = \int_{-\infty}^{\infty} x^2 p(x)\,dx = a^2/3$

$H(X) = \int_{-a}^{a} \dfrac{1}{2a} \log 2a\,dx = \log 2a = \log\sqrt{12S} = \tfrac{1}{2}\log 12S < \tfrac{1}{2}\log 2\pi eS$ since $2\pi e = 17.08 > 12$

16.3-2

$p(x) = \dfrac{\alpha}{2} e^{-\alpha|x|}$ and $S = \dfrac{2}{\alpha^2}$. For $x \geq 0$, $\log\dfrac{1}{p(x)} = \log\dfrac{2}{\alpha} + \alpha x \log e$, so

$$H(X) = 2\left[\int_0^{\infty} \frac{\alpha}{2} e^{-\alpha x} \log\frac{2}{\alpha}\,dx + \int_0^{\infty} \frac{\alpha}{2} e^{-\alpha x} \alpha x \log e\,dx\right] = \alpha\left(\log\frac{2}{\alpha}\right)\frac{1}{\alpha} + \alpha^2 (\log e)\frac{1}{\alpha^2}$$

$$= \log\frac{2}{\alpha} + \log e = \log\frac{2e}{\alpha} = \log\sqrt{2e^2 S} = \tfrac{1}{2}\log 2e^2 S < \tfrac{1}{2}\log 2\pi eS \quad \text{since } \pi > e$$

16.3-3

$S = \int_0^{\infty} ax^2 e^{-ax}\,dx = \dfrac{2}{a^2}, \quad \log\dfrac{1}{p(x)} = ax\log e - \log a$ for $x \geq 0$

$$H(X) = \int_0^{\infty} ae^{-ax}(ax\log e)\,dx - \int_0^{\infty} ae^{-ax}(\log a)\,dx = a^2(\log e)\frac{1}{a^2} - \log a = \log\frac{e}{a}$$

$$= \log\sqrt{e^2 S/2} = \tfrac{1}{2}\log e^2 S/2 < \tfrac{1}{2}\log 2\pi eS \quad \text{since } 2\pi > e/2$$

16.3-4

$p_Z(z) = \dfrac{1}{|a|} p_X\left(\dfrac{z-b}{a}\right)$ (cont.)

$$H(Z)=\int_{-\infty}^{\infty}\frac{1}{|a|}\,p_X\left(\frac{z-b}{a}\right)\left[\log|a|+\log\frac{1}{p_X\left(\frac{z-b}{a}\right)}\right]dz$$

$$=\log|a|\int_{-\infty}^{\infty}p_X\left(\frac{z-b}{a}\right)\frac{dz}{|a|}+\int_{-\infty}^{\infty}p_X\left(\frac{z-b}{a}\right)\log\frac{1}{p_X\left(\frac{z-b}{a}\right)}\frac{dz}{|a|}$$

$$=\log|a|\int_{-\infty}^{\infty}p_X(\lambda)\,d\lambda+\int_{-\infty}^{\infty}p_X(\lambda)\log\frac{1}{p_X(\lambda)}\,d\lambda=\log|a|+H(X)$$

16.3-5

$$\int_0^{\infty}p(x)\,dx=1\ \Rightarrow\ F_1=p,\,c_1=1,\,\frac{\partial F_1}{\partial p}=1\ \text{ and }\ \int_0^{\infty}xp(x)\,dx=m\ \Rightarrow\ F_2=xp,\,c_2=m,\,\frac{\partial F_2}{\partial p}=x$$

Thus, $-\dfrac{\ln p+1}{\ln 2}+\lambda_1+\lambda_2 x=0 \quad\Rightarrow\quad p=e^{(\lambda_1\ln 2-1)}e^{(\lambda_2\ln 2)x}=Ke^{-ax}\quad x\ge 0$

$$\int_0^{\infty}Ke^{-ax}\,dx=K/a=1,\ \int_0^{\infty}xKe^{-ax}\,dx=K/a^2=m\quad\Rightarrow\quad K=a=1/m$$

Hence, $p(x)=\frac{1}{m}e^{-x/m}u(x)$ and $H(X)=\int_0^{\infty}p(x)\left[\log m+\dfrac{x}{m}\log e\right]dx=\log m+\log e=\log em$

16.3-6

$$\int_0^{\infty}p(x)\,dx=1\ \Rightarrow\ F_1=p,\,c_1=1,\,\frac{\partial F_1}{\partial p}=1\ \text{ and }\ \int_0^{\infty}x^2p(x)\,dx=S\ \Rightarrow\ F_2=x^2p,\,c_2=S,\,\frac{\partial F_2}{\partial p}=x^2$$

Thus, $-\dfrac{\ln p+1}{\ln 2}+\lambda_1+\lambda_2 x^2=0 \quad\Rightarrow\quad p=e^{(\lambda_1\ln 2-1)}e^{(\lambda_2\ln 2)x^2}=Ke^{-ax^2}\quad x\ge 0$

$$\int_0^{\infty}Ke^{-ax^2}\,dx=\frac{K}{2}\sqrt{\frac{\pi}{a}}=1,\ \int_0^{\infty}x^2Ke^{-ax^2}\,dx=\frac{K}{a\sqrt{a}}\frac{\sqrt{\pi}}{4}=S\quad\Rightarrow\quad K=\frac{2}{\sqrt{2\pi S}},\,a=\frac{1}{2S}$$

Hence, $p(x)=\dfrac{2}{\sqrt{2\pi S}}e^{-x^2/2S}u(x)$ and

$$H(X)=\int_0^{\infty}p(x)\left[\log\sqrt{\frac{\pi S}{2}}+\frac{x^2}{2S}\log e\right]dx=\log\sqrt{\frac{\pi S}{2}}+\frac{\log e}{2S}S=\tfrac{1}{2}\log\frac{\pi S}{2}+\tfrac{1}{2}\log e=\tfrac{1}{2}\log\frac{\pi e S}{2}$$

16.3-7

$$I(X;Y) = -\iint_{-\infty}^{\infty} p_{XY}(x,y)\log\frac{p_X(x)p_Y(y)}{p_{XY}(x,y)}dxdy \text{ and}$$

$$\log\frac{p_X(x)p_Y(y)}{p_{XY}(x,y)} = \frac{1}{\ln 2}\ln\frac{p_X(x)p_Y(y)}{p_{XY}(x,y)} \le \frac{1}{\ln 2}\left[\frac{p_X(x)p_Y(y)}{p_{XY}(x,y)} - 1\right]$$

Thus, $I(X;Y) \ge \dfrac{1}{\ln 2}\displaystyle\iint_{-\infty}^{\infty} p_{XY}(x,y)\left[1 - \frac{p_X(x)p_Y(y)}{p_{XY}(x,y)}\right]dxdy$

$$= \frac{1}{\ln 2}\left[\iint_{-\infty}^{\infty} p_{XY}(x,y)\,dxdy - \iint_{-\infty}^{\infty} p_X(x)p_Y(y)\,dxdy\right]$$

where $\iint_{-\infty}^{\infty} p_{XY}(x,y)\,dxdy = 1,\ \iint_{-\infty}^{\infty} p_X(x)p_Y(y)\,dxdy = \int_{-\infty}^{\infty} p_X(x)dx\int_{-\infty}^{\infty} p_Y(y)\,dy = 1$

Hence, $I(X;Y)$ ☛ 0

16.3-8

$$R \le C = B\log\left(1 + \frac{S}{N_0 B}\right) = \frac{B}{\ln 2}\ln\left(1 + \frac{10^4}{B}\right)$$

$$B = 10^3 \quad \Rightarrow \quad R \le \frac{10^3}{\ln 2}\ln 11 = 3459 \text{ bits/sec}$$

$$B = 10^4 \quad \Rightarrow \quad R \le 10^4\log_2 2 = 10{,}000 \text{ bits/sec}$$

$$B = 10^5 \quad \Rightarrow \quad R \le \frac{10^5}{\ln 2}\ln 1.1 = 13{,}750 \text{ bits/sec}$$

16.3-9

$$R \le C = 3000\log(1 + S/N) \quad \Rightarrow \quad S/N \ge 2^{R/3000} - 1$$

$$R = 2400 \quad \Rightarrow \quad S/N \ge 2^{0.8} - 1 = 0.741 \approx -1.3 \text{ dB}$$

$$R = 4800 \quad \Rightarrow \quad S/N \ge 2^{1.6} - 1 = 2.03 \approx 3.1 \text{ dB}$$

$$R = 9600 \quad \Rightarrow \quad S/N \ge 2^{3.2} - 1 = 8.19 \approx 9.1 \text{ dB}$$

16.3-10

$$\frac{S}{N_0 R} \ge \frac{B}{R}\left(2^{R/B} - 1\right), \quad N_0 B = 10^{-6} \times 10^3 = 10^{-3}, \quad S \ge 10^{-3}\left(2^{R/1000} - 1\right)$$

$$R = 100 \quad \Rightarrow \quad S \ge 10^{-3}(2^{0.1} - 1) = 0.072 \text{ mW}$$

$$R = 1000 \quad \Rightarrow \quad S \ge 10^{-3}(2 - 1) = 1 \text{ mW}$$

$$R = 10{,}000 \quad \Rightarrow \quad S \ge 10^{-3}(2^{10} - 1) = 1023 \text{ mW}$$

16.3-11

For an ideal system with the same parameters,

$$\left(\frac{S}{N}\right)_D = \left(1 + \frac{S_R}{N_0 B_T}\right)^{B_T/W} - 1 = (1+3)^{10} - 1 = 60.2 \text{ dB}$$

Since the claimed performance approaches an ideal system, the claim is highly doubtful.

16.3-12

$$b = 4: \quad \left(\frac{S}{N}\right)_D = \left(1 + \frac{\gamma}{4}\right)^4 - 1 = 10^4 \quad \Rightarrow \quad \gamma \approx 36$$

$$b = 3 \times 4: \quad \left(\frac{S}{N}\right)_D = \left(1 + \frac{36}{12}\right)^{12} - 1 \approx 4^{12} = 72.2 \text{ dB}$$

16.3-13

$$b = 4: \quad \left(\frac{S}{N}\right)_D = \left(1 + \frac{\gamma}{4}\right)^4 - 1 = 10^4 \quad \Rightarrow \quad \gamma \approx 36$$

$$b = \tfrac{1}{2}: \quad \left(\frac{S}{N}\right)_D = \left(1 + \frac{36}{1/2}\right)^{1/2} - 1 = 7.54 = 8.8 \text{ dB}$$

16.3-14

$$b = 4, \quad LN_0 = 10^6 \times 100 \times 10^{-12} = 10^{-4}$$

$$\left(\frac{S}{N}\right)_D = \left(1 + \frac{\gamma}{4}\right)^4 - 1 = 10^3 \quad \Rightarrow \quad \gamma = 4\left(1001^{1/4} - 1\right) = 18.5 \quad \Rightarrow \quad S_T = \gamma L N_0 W = 5.55 \text{ W}$$

16.3-15

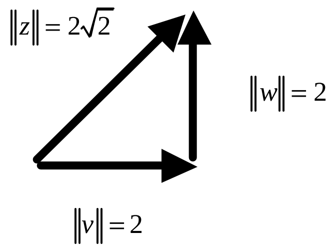
$b = 2, \quad LN_0 = 10^6 \times 100 \times 10^{-12} = 10^{-4}$

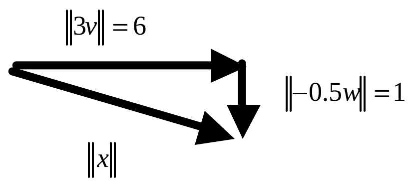
$$\left(\frac{S}{N}\right)_D = \left(1 + \frac{\gamma}{2}\right)^2 - 1 = 10^3 \quad \Rightarrow \quad \gamma = 2\left(\sqrt{1001} - 1\right) = 61.3 \quad \Rightarrow \quad S_T = \gamma L N_0 W = 36.8 \text{ W}$$

16.4-1

$$\|v\|^2 = E_v = 4, \quad \|w\|^2 = E_w = 4, \quad \|z\|^2 = \|v + w\|^2 = E_z = 8$$

$$\|v + w\|^2 = \|v\|^2 + \|w\|^2 \quad \Rightarrow \quad (v, w) = 0, \quad E_x = \|x\|^2 = 6^2 + 1^2 = 37$$

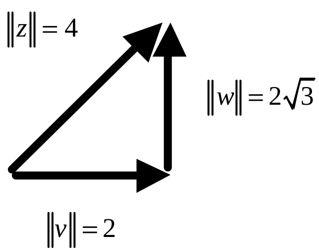


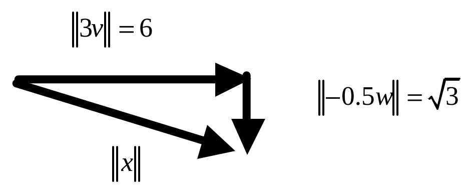


16.4-2

$$\|v\|^2 = E_v = 4, \quad \|w\|^2 = E_w = 12, \quad \|z\|^2 = \|v + w\|^2 = E_z = 16$$

$$\|v + w\|^2 = \|v\|^2 + \|w\|^2 \quad \Rightarrow \quad (v, w) = 0, \quad E_x = \|x\|^2 = 6^2 + \left(\sqrt{3}\right)^2 = 39$$





16.4-3

$\|v + w\|^2 = \|v\|^2 + 2(v, w) + \|w\|^2$ and $|(v, w)| \le \|v\|\|w\|$ so

$2(v, w) \le 2\|v\|\|w\| \quad \Rightarrow \quad \|v + w\|^2 \le \|v\|^2 + 2\|v\|\|w\| + \|w\|^2 = \left(\|v\| + \|w\|\right)^2$. Thus, $\|v + w\| \le \|v\| + \|w\|$

16.4-4

With $v_w = \alpha w$, $(v - v_w, w) = (v - \alpha w, w) = (v,w) - \alpha(w,w)$ so $(v - v_w, w) = 0$

$\alpha = (v,w)/(w,w) = (v,w)/\|w\|^2$

Hence, $v_w = \dfrac{(v,w)}{\|w\|^2} w$ and $\|v_w\|^2 = (\alpha w, \alpha w) = \alpha^2 \|w\|^2 = \left[\dfrac{(v,w)}{\|w\|^2}\right]^2 \|w\|^2 = \dfrac{(v,w)^2}{\|w\|^2}$

But $(v,w)^2 \leq \|v\|^2\|w\|^2$ so $\|v_w\| \leq \left[\|v\|^2\|w\|^2 / \|w\|^2\right]^{1/2} = \|v\|$

16.4-5

$$\|s_1\|^2 = \int_{-1}^{1} 1\,dt = 2 \quad \Rightarrow \quad \varphi_1 = s_1/\|s_1\| = 1/\sqrt{2} \quad |t| \leq 1$$

$$(s_2, \varphi_1) = \int_{-1}^{1} \frac{t}{\sqrt{2}}\,dt = 0 \quad \Rightarrow \quad g_2 = s_2 \quad \Rightarrow \quad \|g_2\|^2 = \int_{-1}^{1} t^2\,dt = \tfrac{2}{3} \quad \Rightarrow \quad \varphi_2 = \frac{s_2}{\sqrt{2/3}} = \sqrt{\frac{3}{2}}\,t \quad |t| \leq 1$$

$$(s_3, \varphi_1) = \int_{-1}^{1} \frac{t^2}{\sqrt{2}}\,dt = \frac{\sqrt{2}}{3}, \quad (s_3, \varphi_2) = \int_{-1}^{1} t^2 \sqrt{\tfrac{3}{2}}\,t\,dt = 0 \text{ so}$$

$$g_3 = s_3 - \frac{\sqrt{2}}{3}\varphi_1 = t^2 - \tfrac{1}{3} \text{ and } \|g_3\|^2 = \int_{-1}^{1} \left(t^2 - \tfrac{1}{3}\right)^2 dt = \tfrac{8}{45} \quad \Rightarrow \quad \varphi_3 = \sqrt{\tfrac{45}{8}}\left(t^2 - \tfrac{1}{3}\right)$$

Thus, $s_1 = \sqrt{2}\varphi_1, \quad s_2 = \sqrt{\tfrac{2}{3}}\varphi_2, \quad s_3 = \sqrt{\tfrac{8}{45}}\varphi_3 + \tfrac{1}{3} = \tfrac{\sqrt{2}}{3}\varphi_1 + \sqrt{\tfrac{8}{45}}\varphi_3$

16.4-6

$$\|s_1\|^2 = \int_{-1}^{1} 1\,dt = 2 \quad \Rightarrow \quad \varphi_1 = s_1/\|s_1\| = 1/\sqrt{2} \quad |t| \leq 1$$

$$(s_2, \varphi_1) = \int_{-1}^{1} \cos\frac{\pi t}{2}\frac{1}{\sqrt{2}}\,dt = \frac{2\sqrt{2}}{\pi} \quad \Rightarrow \quad g_2 = s_2 - \frac{2\sqrt{2}}{\pi}\frac{1}{\sqrt{2}}$$

$$\|g_2\|^2 = \int_{-1}^{1}\left(\cos^2\frac{\pi t}{2} - \frac{4}{\pi}\cos\frac{\pi t}{2} + \frac{4}{\pi^2}\right)dt = 1 - \frac{8}{\pi^2} \text{ so } \varphi_2 = \frac{\left(s_2 - \frac{2}{\pi}\right)}{\|g_2\|} = \frac{\pi}{\sqrt{\pi^2 - 8}}\left(\cos\frac{\pi t}{2} - \frac{2}{\pi}\right) \quad |t| \leq 1$$

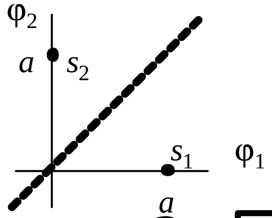
$$(s_3, \varphi_1) = \int_{-1}^{1} \sin \pi t\,dt = 0, \quad (s_3, \varphi_2) = \int_{-1}^{1} \sin \pi t \frac{\pi}{\sqrt{\pi^2 - 8}}\left(\cos\frac{\pi t}{2} - \frac{2}{\pi}\right)dt = 0\text{, so}$$

$$g_3 = s_3, \quad \|g_3\|^2 = \int_{-1}^{1} \sin^2 \pi t\,dt = 1 \quad \Rightarrow \quad \varphi_3 = s_3 = \sin \pi t \quad |t| \leq 1.\text{ Thus,}$$

$$s_1 = \sqrt{2}\varphi_1, \quad s_2 = \frac{\sqrt{\pi^2 - 8}}{\pi}\varphi_2 + \frac{2}{\pi} = \frac{2\sqrt{2}}{\pi}\varphi_1 + \frac{\sqrt{\pi^2 - 8}}{\pi}\varphi_2, \quad s_3 = \varphi_3$$

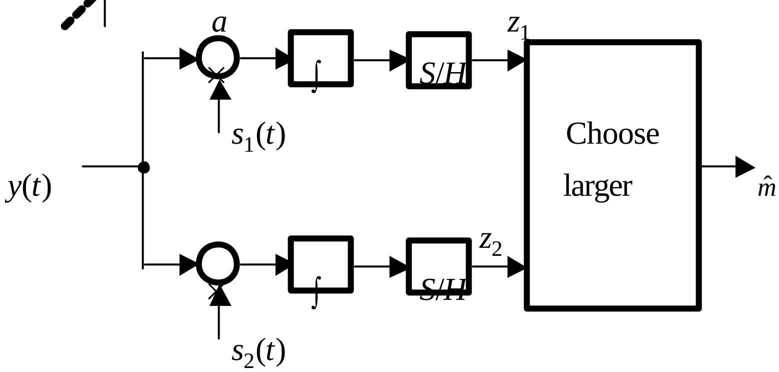
16.5-1

(*a*)

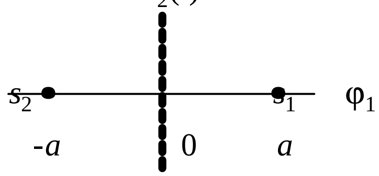


$c_1 = c_2 = a^2/2$ so take

$z_1 = (y,s_1) \quad z_2 = (y,s_2)$



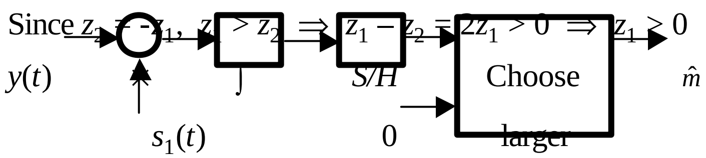
(*b*)



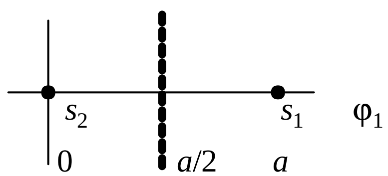
$c_1 = c_2 = a^2$ so take

$z_1 = (y,s_1) \quad z_2 = (y,s_2) = -(y,s_1)$

Since $z_2 = -z_1$, $z_1 > z_2 \Rightarrow z_1 - z_2 = 2z_1 > 0 \Rightarrow z_1 > 0$



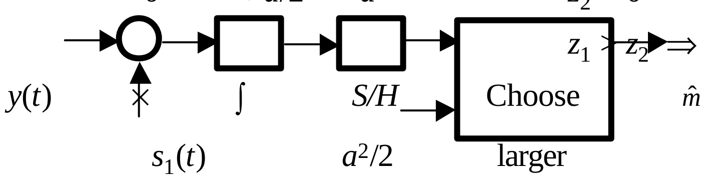
(*c*)



$z_1 = (y,s_1) - a^2/2$

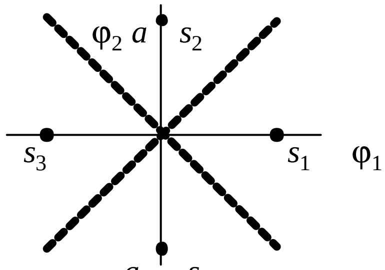
$z_2 = 0$

$z_1 > z_2 \Rightarrow (y,s_1) > a^2/2$



16.5-2

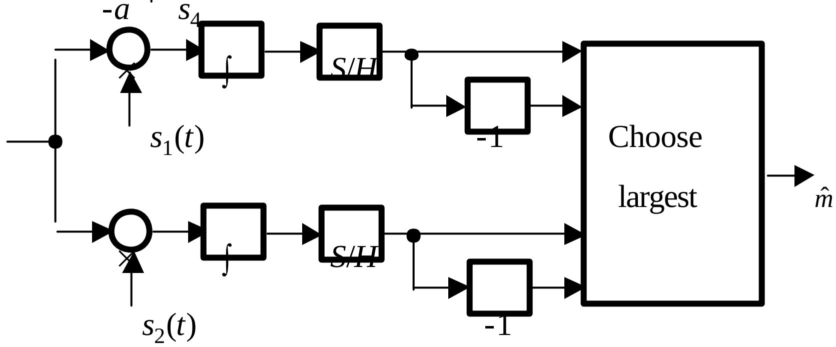
(*a*)



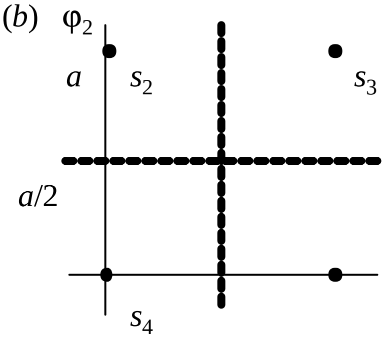
Since all $c_i = a^2/2$ take

$z_1 = (y,s_1)$, $z_2 = (y,s_2)$

$z_3 = (y,s_3) = -(y,s_1)$, $z_4 = (y,s_4) = -(y,s_2)$



(*b*)

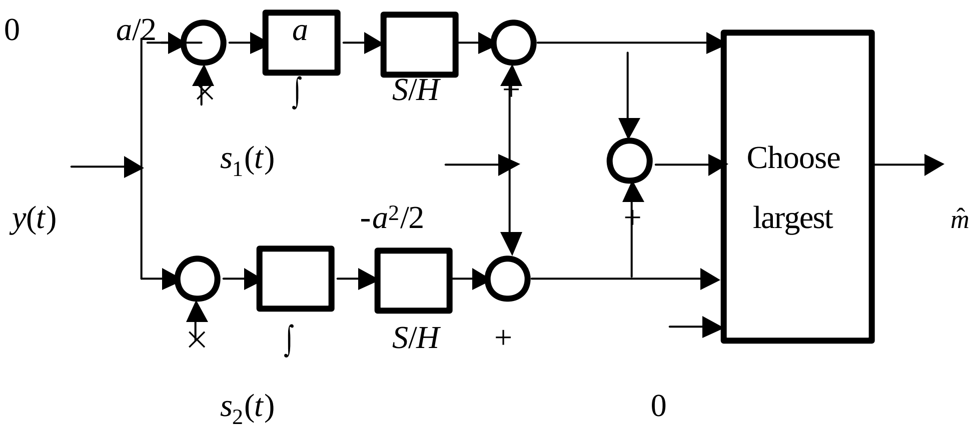


$c_1 = c_2 = a^2/2$, $c_3 = a^2$, $c_4 = 0$

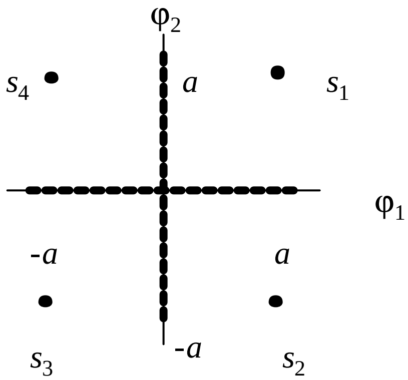
$z_1 = (y,s_1) - a^2/2$, $z_2 = (y,s_2) - a^2/2$

$z_3 = (y,s_1 + s_2) - a^2 = z_1 + z_2$

$z_4 = 0$



16.5-3

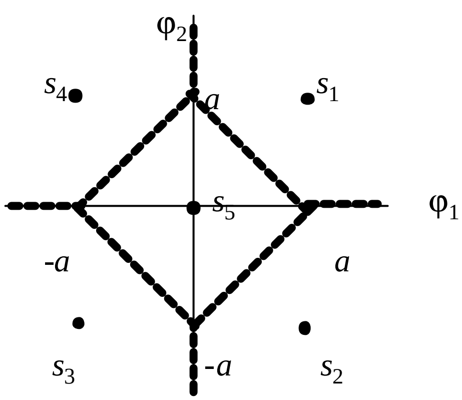


$E_i = \left(\sqrt{2}a\right)^2$ for all i so $E = \frac{1}{4}\times 4\left(\sqrt{2}a\right)^2 = 2a^2$

For all i, $P(c|m_i) = \int_{-a}^{\infty} p_\beta(\beta_1)\,d\beta_1 \int_{-a}^{\infty} p_\beta(\beta_2)\,d\beta_2 = \left[1 - Q\left(\frac{a}{\sqrt{N_0/2}}\right)\right]^2$ where $a = \sqrt{E/2}$

Thus, $P_e = 1 - \frac{1}{4}\times 4\left[1 - Q\left(\sqrt{\frac{E}{N_0}}\right)\right]^2 = 2Q\left(\sqrt{\frac{E}{N_0}}\right) - Q^2\left(\sqrt{\frac{E}{N_0}}\right)$

16.5-4



$E_i = (\sqrt{2}a)^2 \quad i = 1,2,3,4$

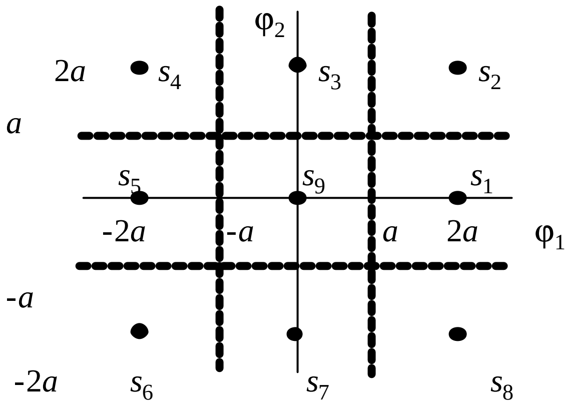
$E_5 = 0$

$E = \frac{1}{5}\times 4(\sqrt{2}a)^2 = \frac{8}{5}a^2$

Since s_5 is nearest neighbor to all other s_i, $d_j = \sqrt{2}a = \sqrt{5E/4} \quad j = 1,2,\ldots,5$

Thus, $P_e \le \frac{4}{5}\times 5Q\left(\frac{\sqrt{2}a}{\sqrt{2N_0}}\right) = 4Q\left(\sqrt{\frac{5E}{8N_0}}\right)$

16.5-5



$E_i = (2a)^2 \quad i = 1, 3, 5, 7$

$\quad = 2(2a)^2 \quad i = 2, 4, 6, 8$

$\quad = 0 \quad i = 9$

$$E = \tfrac{1}{9}\left[4\times(2a)^2 + 4\times 2(2a)^2\right] = \tfrac{48}{9}a^2$$

$$\text{Let } q = Q\left(\frac{a}{\sqrt{N_0/2}}\right) = Q\left(\sqrt{\frac{9E}{24N_0}}\right)$$

$$P(c|m_9) = \int_{-a}^{a} p_\beta(\beta_1)\,d\beta_1 \int_{-a}^{a} p_\beta(\beta_2)\,d\beta_2 = (1-2q)^2$$

For $i = 1, 3, 5, 7$ $\quad P(c|m_i) = \int_{-a}^{\infty} p_\beta(\beta_1)\,d\beta_1 \int_{-a}^{a} p_\beta(\beta_2)\,d\beta_2 = (1-q)(1-2q)$

For $i = 2, 4, 6, 8$ $\quad P(c|m_i) = \int_{-a}^{\infty} p_b(b_1)\,db_1 \int_{-a}^{\infty} p_b(b_2)\,db_2 = (1-q)^2$

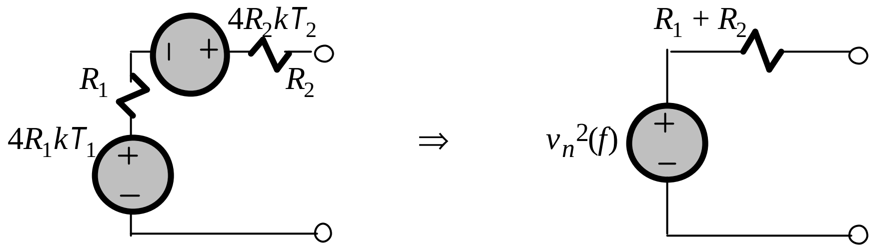
Thus, $P_c = \tfrac{1}{9}\left[4(1-q)(1-2q) + 4(1-q)^2 + (1-2q)^2\right] = \tfrac{1}{9}\left(9 - 24q + 16q^2\right)$ and

$$P_e = 1 - P_c = \tfrac{24}{9}q - \tfrac{16}{9}q^2 = \tfrac{24}{9}Q\left(\sqrt{\frac{9E}{24N_0}}\right) - \tfrac{16}{9}Q^2\left(\sqrt{\frac{9E}{24N_0}}\right)$$

For union bound note that $d_j = 2a$, $j = 1, 2, \ldots, 9$ so $P_e \le \tfrac{8}{9}\times 9Q\left(\frac{2a}{\sqrt{2N_0}}\right) = 8Q\left(\sqrt{\frac{9E}{24N_0}}\right)$

Appendix

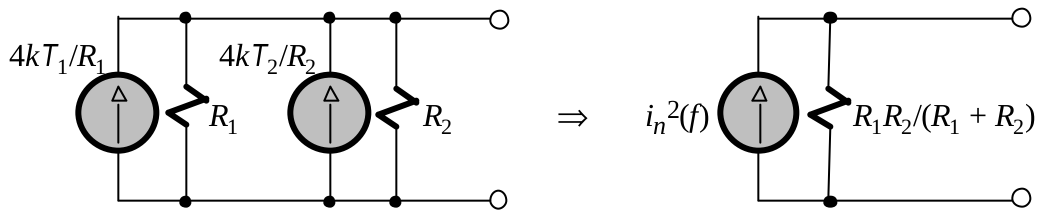
A-1



$$v_n^2(f) = 4R_1kT_1 + 4R_2kT_2, \quad i_n^2(f) = \frac{v_n^2(f)}{|Z(f)|^2} = \frac{4k(R_1T_1 + R_2T_2)}{(R_1^2 + R_2^2)}$$

If $T_1 = T_2 = T$, then $v_n^2(f) = 4(R_1 + R_2)kT$, $\quad i_n^2(f) = \dfrac{4kT}{R_1 + R_2}$

A-2



$$i_n^2(f) = \frac{4kT_1}{R_1} + \frac{4kT_2}{R_2}$$

$$v_n^2(f) = |Z(f)|^2 i_n^2(f) = \left(\frac{R_1R_2}{R_1 + R_2}\right)^2 \frac{4kR_2T_1 + 4kR_1T_2}{R_1R_2} = \frac{4kR_1R_2}{(R_1 + R_2)^2}(R_2T_1 + R_1T_2)$$

If $T_1 = T_2 = T$, then $i_n^2(f) = 4\dfrac{R_1 + R_2}{R_1R_2}kT$ and $v_n^2(f) = 4\dfrac{R_1R_2}{R_1 + R_2}kT$

A-3

$$Z(f) = \frac{9(1 + jf)}{10 + jf} \quad \Rightarrow \quad R(f) = 9\frac{10 + f^2}{100 + f^2} \quad \Rightarrow \quad v_n^2(f) = 36kT\,\frac{10 + f^2}{100 + f^2}$$

A-4

$$Z(f) = \frac{R(R - j/\omega C)}{2R - j/\omega C} \quad \Rightarrow \quad R(f) = R\frac{2R^2 + (1/\omega C)^2}{4R^2 + (1/\omega C)^2} \quad \Rightarrow \quad v_n^2(f) = 4RkT\,\frac{1 + 2(2\pi RCf)^2}{1 + 4(2\pi RCf)^2}$$

A-5

If qV/kT » 1, then I » I_S so $i_n^2(f) \approx 2qI$ and $r \approx kT/qI$. Thus, $i_n^2(f) \approx 2rkT = 4(r/2)kT$, which looks like thermal noise from resistance $R = r/2$.

A-6

With $R_S = r_i = r_o = 50\ \Omega$, Eq. (8) yields $g_a(f) = \left(\dfrac{|H(f)|50}{100}\right)^2 \dfrac{50}{50} = \dfrac{1}{4}|H(f)|^2$. Then,

since $\Pi^2(\,) = \Pi(\,)$, Eq. (10) becomes $gB_N = \int_0^\infty g_a(f)\,df = \dfrac{200^2}{4}\int_0^\infty \Pi\left(\dfrac{f - f_c}{B}\right)df = 10^4 B = 10^{10}$.

Then, from Eq. (11), $kT_e = \dfrac{1}{gB_N}\int_0^\infty \eta_{\text{int}}(f)\,df = \dfrac{2\times 10^{-16} B}{10^{10}} = 2\times 10^{-20}$. Finally, with $T_s = T_0$,

Eq. (12) becomes

$$N_o = gB_N(kT_s + kT_e) \approx 10^{10}(4\times 10^{-21} + 2\times 10^{-20}) = 2.4\times 10^{-10} = 240 \text{ pW.}$$

A-7

$T_S = T_0$: $N_o = 10^5 k(T_0 + T_e)\times 20\times 10^3 = 8\times 10^{-12}\,\dfrac{T_0 + T_e}{T_0} = 80\times 10^{-12}$ so $(T_0 + T_e)/T_0 = 10$ and

$T_e = 9T_0$, $F = 1 + 9 = 10$

$T_s = 2T_0$: $N_o = 10^5 \times 4\times 10^{-21}\,\dfrac{2T_0 + 9T_0}{T_0}\times 20\times 10^3 = 88$ pW

A-8

$T_S = T_0$: $N_o = gk(T_0 + T_e)B_N$,

$T_s = 2T_0$: $N_o = gk(2T_0 + T_e)B_N = \frac{4}{3}gk(T_0 + T_e)B_N$ so

$2T_0 + T_e = \frac{4}{3}(T_0 + T_e)$. Thus, $T_e = 2T_0$, $F = 1 + 2 = 3$.

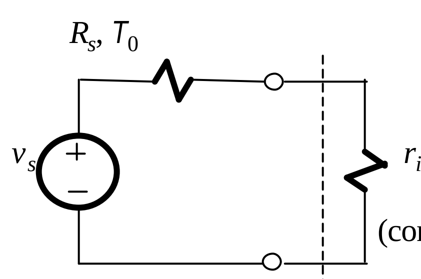
A-9

Assume matched impedances ($R_s = r_i$), so

$N_1 = CN_o = Cgk(T_0 + T_e)B_N$

$N_2 = N_1 + CgS = 2N_1 \;\Rightarrow\; S = k(T_0 + T_e)B_N$

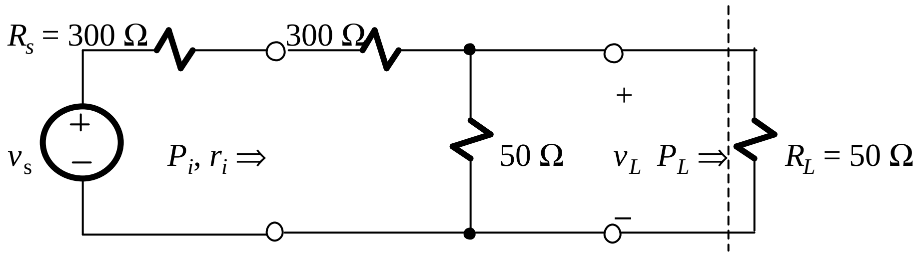
Thus, $T_e = \dfrac{S}{kB_N} - T_0 \quad\Rightarrow\quad F = \dfrac{S}{kT_0B_N}$



(cont.)

To determine F by this method, we need an *absolute* power meter to measure S, and we must also know B_N. Both requirements are disadvantages compared to the noise-generator method.

A-10



$$r_i = 300 + \frac{50\times 50}{50+50} = 325\ \Omega \quad P_i = \left(\frac{325}{300+325} v_s\right)^2 / r_i$$

$$v_L = \frac{50\|50}{300+300+50\|50} v_s \quad P_L = \left(\frac{25}{625} v_s\right)^2 / R_L$$

$$g = \frac{P_L}{P_i} = \left(\frac{25v_s}{625}\right)^2 \frac{1}{50} \times \left(\frac{625}{325 v_s}\right)^2 \times 325 = \frac{1}{26}.$$ Thus, $F = L = 1/g = 26$.

A-11

$T_e = T_{e1} + T_{e2}/g$ where $F_2 = 13.2$ dB $= 21$ $\Rightarrow$ $T_{e2} = (21 - 1)T_0$, so

$T_e = 3T_0 + 20T_0/10 = 5T_0$ and

$$\left(\frac{S}{N}\right)_o = \frac{S_s}{k(T_s + T_e)B_N} = \frac{S_s}{15kT_0B_N} = 10^3 \quad \Rightarrow \quad S_s = 6\times 10^{-12} = 6\ \text{pW}$$

A-12

Since $T_s = T_0$, $\left(\frac{S}{N}\right)_o = \frac{1}{F}\left(\frac{S}{N}\right)_s \geq 0.05\left(\frac{S}{N}\right)_s \quad \Rightarrow \quad F \leq 20$

Since $F_1 = 1/g_1 = L$ and $F_2 = 7$ dB ≈ 5, $F = F_1 + \frac{F_2 - 1}{g_1} = L + L(5-1) = 5L$

Thus, we want $5L$ ⦸ 20 $\Rightarrow$ L ⦸ 4 = 6 dB $\Rightarrow$ L ⦸ 6 dB/2 dB/km = 3 km

A-13

Preamp: $F = 2$ and $g = 100$; Cable: $F = 4$ and $g = 1/4$; Receiver: $F = 20$

(cont.)

$$(a)\ F = 2 + \frac{4-1}{100} + \frac{20-1}{100\times 1/4} = 2.79 = 4.5\ \text{dB}$$

$$(b)\ F = 4 + \frac{2-1}{\frac{1}{4}} + \frac{20-1}{\frac{1}{4}\times 100} = 8.76 = 9.4\ \text{dB}$$

A-14

$L_1 = 10^{0.15} = 1.413$, $g_2 = 100$, $F_3 = 10$

$$T_e = 0.413T + 1.413\times 50\text{K} + \frac{1.413}{100}(10-1)290\text{K} = 0.413T + 107.5\text{K}$$

so $T_e \oslash 150$ K $\Rightarrow$ $T \oslash 103$ K

A-15

With unit #1 first, cascade has $F_{12} = F_1 + (F_2 - 1)/g_1$. Similarly, interchange the subscripts for unit #2 first. Thus, $F_{12} - F_{21} = \left(F_1 + \frac{F_2 - 1}{g_1}\right) - \left(F_2 + \frac{F_1 - 1}{g_2}\right) = \left(F_1 - \frac{F_1 - 1}{g_2}\right) - \left(F_2 - \frac{F_2 - 1}{g_1}\right)$

If unit #1 first yields $F_{12} < F_{21}$, then $F_{12} - F_{21} < 0$ and

$$F_1 - \frac{F_1 - 1}{g_2} = (F_1 - 1)\left(1 - \frac{1}{g_2}\right) + 1 < F_2 - \frac{F_2 - 1}{g_1} = (F_2 - 1)\left(1 - \frac{1}{g_1}\right) + 1 \text{ so}$$

$$\frac{F_1 - 1}{1 - 1/g_1} < \frac{F_2 - 1}{1 - 1/g_2} \quad \Rightarrow \quad M_1 < M_2$$